

10[7].—RUDOLPH ONDREJKA, 1273 *Exact Factorials*, one ms. volume of 671 unnumbered computer sheets deposited in the UMT file.

This large volume gives the exact values of $n!$ for $n = 1(1)1273$, with no explanatory text. However, private correspondence with this reviewer revealed that the calculation of the table was performed on a Honeywell 200 computer system and the first 108 of each entry were checked with the corresponding entry in the table of Reid & Montpetit [1].

Furthermore, the value of $1000!$ has been found by the reviewer to agree with the final entry in the table of Lal & Russell [2], which was not available to the author.

J. W. W.

1. J. B. REID & G. MONTPETIT, *Table of Factorials 0! to 9999!*, Publication 1039, National Academy of Sciences—National Research Council, Washington, D. C., 1962. (See *Math. Comp.*, v. 17, 1963, p. 459, RMT 67.)

2. M. LAL & W. RUSSELL, *Exact Values of Factorials 500! to 1000!*, Department of Mathematics, Memorial University of Newfoundland, St. John's, Newfoundland, Canada, undated. (See *Math. Comp.*, v. 22, 1968, pp. 686–687, RMT 68, and the references cited therein.)

11[7].—RUDOLPH ONDREJKA, *Tables of Double Factorials*, two ms. volumes, each of 618 unnumbered computer sheets deposited in the UMT file.

The first of these impressive volumes gives the exact values of $(2n - 1)!!$ for $n = 1(1)1162$; the second gives the companion values of $(2n)!!$ for $n = 1(1)1161$, all computed on a Honeywell 200 computer system.

The author collated these tables with the corresponding entries in his earlier table [1] of double factorials and found complete agreement.

The present elaborate tables should prove more than adequate to satisfy the requirements of most users of such data.

J. W. W.

1. RUDOLPH ONDREJKA, *The First 100 Exact Double Factorials*, ms. in the UMT file. (See *Math. Comp.*, v. 21, 1967, p. 258, RMT 16.)

12[7].—JAMES D. TALMAN, *Special Functions, A Group Theoretic Approach*, W. A. Benjamin, Inc., New York, 1969, xii + 260 pp., 24 cm. Price \$13.50 cloth, \$5.95 paper.

An intriguing aspect of mathematical thinking is the variegated approaches and analyses used to extend well-established theories and construct new ones. The properties of the special functions of mathematical physics are usually studied on the basis of their analytic character, the principal tools being the theory of analytic functions. Thus special functions can be studied as the solution of differential and difference equations. They can be characterized by definite integrals, power series, addition theorems and so on.

The volume under review is based on lectures by E. P. Wigner and approaches the subject of special functions from the group theoretic standpoint. The general introduction is written by E. P. Wigner. He points out that the lectures began with the observation that the results of the analytic theory are more general than those derived from the group-theoretic analyses. Since the time of the lectures, this drawback has been considerably reduced by subsequent developments. "This in no way

diminishes the beauty and elegance of the analytic theory, or the inventiveness that was necessary to its development. Rather, the claim of the present volume is to point to a role of the 'special functions' which is common to all, and which leads to a point of view which permits the classification of their properties in a uniform fashion."

Wigner continues as follows:

"The role which is common to all the special functions is to be matrix elements of representations of the simplest Lie groups, such as the group of rotations in three-space, or the Euclidean group of the plane. The arguments of the functions are suitably chosen group parameters. The addition theorems of the functions then just express the multiplication laws of the group elements. The differential equations which they obey can be obtained either as limiting cases of the addition theorems or as expressions of the fact that multiplication of a group element with an element in the close neighborhood of the unit element furnishes a group element whose parameters are in close proximity to the parameters of the element multiplied. The integral relationships derive from Frobenius' orthogonality relations for matrix elements of irreducible representations as generalized for Lie groups by means of Hurwitz's invariant integral. The completeness relations have a similar origin. Further relations derive from the possibility of giving different equivalent forms to the same representation by postulating that the representatives of one or another subgroup be in the reduced form. Finally, some of the Lie groups can be considered as limiting cases of others; this furnishes further relations between them. Thus, the Euclidean group of the plane can be obtained as a limit of the group of rotations in three-space. Hence, the elements of the representations of the former group (Bessel functions) are limits of the representations of the latter group (Jacobi functions)."

The author has attempted to make the book self-contained. Thus, about the first third of the volume is a preliminary discussion of abstract groups, Lie groups and algebras, group representations and related topics (Chapters 1-7). The remaining chapters consider various groups, and the special functions are studied in connection with the group with which they are related. Chapters 8 and 9 take up rotation in 2-space and 3-space, respectively. Some associated special functions are the so-called $3 - j$ coefficients, harmonic polynomials and spherical harmonics. Chapter 10 is called Rotation in Four Dimensions, and properties of Gegenbauer polynomials are considered. Chapter 11, called Euclidean Group in the Plane, studies properties of Bessel functions of integer order and Chapter 12, called The Euclidean Group in Space, studies properties of spherical Bessel functions. Finally, the title of Chapter 13 is The Quantum-Mechanical Group, and here the Laguerre polynomials enter.

The acid test of any scientific theory is that it should include existing knowledge and produce results not forthcoming from extant developments. To date I know of no results on the special functions which follow from the group approach that cannot be deduced from the conventional analytic approaches. On the other hand, there are numerous results on the special functions which do not arise from the group-theoretic analyses. Thus there is a challenge—can future developments alter this situation? The present volume is very readable and should prove valuable to workers in this area.

Y. L. L.