

22[2.05].— THEODORE J. RIVLIN, *An Introduction to the Approximation of Functions*. Blaisdell Publishing Co., Waltham, Mass., 1969, viii + 150 pp., 24 cm. Price \$7.50.

This is a well written, enjoyable book. It requires only a good undergraduate mathematics program as background. The author is quite successful in his compromise of restricting the discussion to concrete interpolation and approximation procedures, without losing sight of the basic mathematical ideas, which can be carried over to more general situations. It is an elementary book in the best sense of the word.

The main emphasis of the book is on approximation by polynomials and by piecewise polynomials. What follows is a short description of the various chapters.

A short introduction gives the abstract existence theory, discusses the role of uniform convexity and introduces the necessary spaces.

Chapter 1. "Uniform Approximation," is a skillful presentation of Weierstrass' theorem. Jackson's theorems and the Chebyshev characterization of the best approximation. The corresponding problem on finite point sets is discussed as well as two numerical procedures.

Chapter 2. "Least Squares Approximation," contains a treatment of approximation with orthonormal polynomials on a bounded interval as well as on a finite set of points. Also included is a discussion of the effectiveness, as a uniform approximation, of least squares approximation.

Chapter 3. This chapter develops a theory of least-first-power approximation on intervals and finite point sets. It is shown that the solution of the discrete problem converges to that of the continuous one under appropriate conditions. The solution of the discrete problem by linear programming is discussed.

Chapter 4. Here polynomial and spline interpolation are treated. The chapter starts with an illuminating discussion of Lagrange interpolation, the convergence problem, and how the convergence depends on the location of the nodes. Next follows a discussion of the effectiveness of interpolation polynomials in the least-square and least-first-power sense. The end of the chapter discusses cubic splines. Existence of a cubic spline interpolant is proved as well as extremal properties. The use of splines for least-square and uniform approximation is discussed and an error estimate is given in the uniform norm.

Chapter 5. This last chapter discusses the characterization of rational approximation and interpolation on finite intervals and finite point sets. The author also treats numerical procedures for these problems.

A series of exercises increases the usefulness of this excellent textbook.

OLOF WIDLUND

Courant Institute of Mathematical Sciences
New York University
New York, New York 10012

23[2.05, 2.10, 2.55, 4, 5, 13.05].— L. COLLATZ, G. MEINARDUS & H. UNGER, Editors, *Numerische Mathematik, Differentialgleichungen, Approximationstheorie*, Birkhäuser Verlag, Basel, 1968, 401 pp., 25 cm. Price SFR 48—.

Functional analysis, as a unifying agent, has become of increasing significance in many disciplines of mathematics, numerical mathematics being no exception. The

desire to elucidate the interactions between functional analysis and numerical mathematics, particularly the numerical treatment of differential equations and approximation theory, led in 1966 to two working conferences being held at the Mathematics Research Institute Oberwolfach (Black Forest). The first conference, under the direction of L. Collatz and H. Unger, was devoted to differential equations, the second, under the direction of L. Collatz and G. Meinardus, to numerical analysis, especially approximation theory. The present volume contains the proceedings of these conferences. The contributions vary in length and content, ranging from short abstracts to preliminary research reports, expository articles, and complete original research papers. The authors and their titles are listed below:

Conference on the Numerical Treatment of Differential Equations, June 20–25, 1966

- H. Amann, Monte-Carlo Methoden zur Lösung elliptischer Randwertprobleme
 R. Ansorge, Zur Frage der Verallgemeinerung des Äquivalenzsatzes von P. D. Lax
 I. Babuška, Optimierungsprobleme numerischer Methoden
 G. Bruhn, Ein Charakteristikenverfahren für instationäre Strömungen entlang bewegter Wände
 B. Dejon, Stabilitätskriterien in Abhängigkeit von den Normen für die Startwerte
 S. Filippi, Neue Lie-Reihen-Methode
 F. Krückeberg, Defekterfassung bei gewöhnlichen und partiellen Differentialgleichungen
 K. Nickel und P. Rieder, Ein neues Runge-Kutta-ähnliches Verfahren
 J. Nitsche, Zur Konvergenz des Ritzschen Verfahrens und der Fehlerquadratmethode I
 G. Opitz, Einheitliche Herleitung einer umfassenden Klasse von Interpolationsformeln und Anwendung auf die genäherte Integration von gewöhnlichen Differentialgleichungen
 P. Rózsa, Ein Rekursionsverfahren zur Lösung linearer Differentialgleichungssysteme mit singulären Koeffizientenmatrizen
 J. W. Schmidt und H. Schönheinz, Fehlerschranken für die genäherte Lösung von Rand- und Eigenwertaufgaben bei gewöhnlichen Differentialgleichungen durch Differenzenverfahren
 H. Schwermer, Zur Fehlererfassung bei der numerischen Integration von gewöhnlichen Differentialgleichungssystemen erster Ordnung mit speziellen Zweipunktverfahren
 H. J. Stetter, Stabilitätsbereiche bei Diskretisierungsverfahren für Systeme gewöhnlicher Differentialgleichungen
 W. Törnig, Über Konvergenzbereiche von Differenzapproximationen bei quasilinearen hyperbolischen Anfangswertproblemen
 W. Walter, Wärmeleitung in Systemen mit mehreren Komponenten
 W. Wendland, Zur numerischen Behandlung der Randwertaufgaben für elliptische Systeme
 W. Wetterling, Lösungsschranken beim Differenzenverfahren zur Potentialgleichung

Conference on Numerical Analysis, especially Approximation Theory, November 13–19, 1966

- J. Blatter, Zur stetigen Abhängigkeit der Menge der besten Approximierenden eines Elementes in einem normierten reellen Vektorraum
 B. Brosowski, Rationale Tschebyscheff-Approximation differenzierbarer Funktionen
 E. W. Cheney and A. A. Goldstein, A Note on Nonlinear Approximation Theory
 E. Gröbner, Approximationen durch Umordnungen von Lie-Reihen
 D. Henze, Über nichtlineare Approximationen in linearen normierten Räumen
 W. Krabs, Über ein Kriterium von Kolmogoroff bei der Approximation von Funktionen
 J. Meinguet, Optimal Approximation and Error Bounds in Normed Spaces
 K. Nickel, Anwendungen einer Fehlerschranken-Arithmetik
 R. Nicolovius, Extrapolation bei monoton zerlegbaren Operatoren
 M. J. D. Powell, On Best L , Spline Approximations
 J. Schröder, Monotonie-Aussagen bei quasilinearen elliptischen Differentialgleichungen und anderen Problemen
 F. Schurer and F. W. Steutel, Approximation with Singular Integrals of the Jackson Type
 P. C. Sikkema, Über Potenzen von verallgemeinerten Bernsteinoperatoren
 J. J. Sopka, Über verallgemeinerte numerische Integrationen
 H. Werner, Diskretisierung bei Tschebyscheff-Approximation mit verallgemeinerten rationalen Funktionen
 W. Wetterling, Lösungsschranken bei elliptischen Differentialgleichungen.

W. G.

24[2.10].—ROBERT PIESSENS, *Gaussian Quadrature Formulas for the Integration of Oscillating Functions*, 2 pages of tables and 1 page of explanation, reproduced on the microfiche card attached to this issue. Review taken from author's explanation.