

desire to elucidate the interactions between functional analysis and numerical mathematics, particularly the numerical treatment of differential equations and approximation theory, led in 1966 to two working conferences being held at the Mathematics Research Institute Oberwolfach (Black Forest). The first conference, under the direction of L. Collatz and H. Unger, was devoted to differential equations, the second, under the direction of L. Collatz and G. Meinardus, to numerical analysis, especially approximation theory. The present volume contains the proceedings of these conferences. The contributions vary in length and content, ranging from short abstracts to preliminary research reports, expository articles, and complete original research papers. The authors and their titles are listed below:

Conference on the Numerical Treatment of Differential Equations, June 20–25, 1966

- H. Amann, Monte-Carlo Methoden zur Lösung elliptischer Randwertprobleme
 R. Ansorge, Zur Frage der Verallgemeinerung des Äquivalenzsatzes von P. D. Lax
 I. Babuška, Optimierungsprobleme numerischer Methoden
 G. Bruhn, Ein Charakteristikenverfahren für instationäre Strömungen entlang bewegter Wände
 B. Dejon, Stabilitätskriterien in Abhängigkeit von den Normen für die Startwerte
 S. Filippi, Neue Lie-Reihen-Methode
 F. Krückeberg, Defekterfassung bei gewöhnlichen und partiellen Differentialgleichungen
 K. Nickel und P. Rieder, Ein neues Runge-Kutta-ähnliches Verfahren
 J. Nitsche, Zur Konvergenz des Ritzschen Verfahrens und der Fehlerquadratmethode I
 G. Opitz, Einheitliche Herleitung einer umfassenden Klasse von Interpolationsformeln und Anwendung auf die genäherte Integration von gewöhnlichen Differentialgleichungen
 P. Rózsa, Ein Rekursionsverfahren zur Lösung linearer Differentialgleichungssysteme mit singulären Koeffizientenmatrizen
 J. W. Schmidt und H. Schönheinz, Fehlerschranken für die genäherte Lösung von Rand- und Eigenwertaufgaben bei gewöhnlichen Differentialgleichungen durch Differenzenverfahren
 H. Schwermer, Zur Fehlererfassung bei der numerischen Integration von gewöhnlichen Differentialgleichungssystemen erster Ordnung mit speziellen Zweipunktverfahren
 H. J. Stetter, Stabilitätsbereiche bei Diskretisierungsverfahren für Systeme gewöhnlicher Differentialgleichungen
 W. Törnig, Über Konvergenzbereiche von Differenzapproximationen bei quasilinearen hyperbolischen Anfangswertproblemen
 W. Walter, Wärmeleitung in Systemen mit mehreren Komponenten
 W. Wendland, Zur numerischen Behandlung der Randwertaufgaben für elliptische Systeme
 W. Wetterling, Lösungsschranken beim Differenzenverfahren zur Potentialgleichung

Conference on Numerical Analysis, especially Approximation Theory, November 13–19, 1966

- J. Blatter, Zur stetigen Abhängigkeit der Menge der besten Approximierenden eines Elementes in einem normierten reellen Vektorraum
 B. Brosowski, Rationale Tschebyscheff-Approximation differenzierbarer Funktionen
 E. W. Cheney and A. A. Goldstein, A Note on Nonlinear Approximation Theory
 E. Gröbner, Approximationen durch Umordnungen von Lie-Reihen
 D. Henze, Über nichtlineare Approximationen in linearen normierten Räumen
 W. Krabs, Über ein Kriterium von Kolmogoroff bei der Approximation von Funktionen
 J. Meinguet, Optimal Approximation and Error Bounds in Normed Spaces
 K. Nickel, Anwendungen einer Fehlerschranken-Arithmetik
 R. Nicolovius, Extrapolation bei monoton zerlegbaren Operatoren
 M. J. D. Powell, On Best L , Spline Approximations
 J. Schröder, Monotonie-Aussagen bei quasilinearen elliptischen Differentialgleichungen und anderen Problemen
 F. Schurer and F. W. Steutel, Approximation with Singular Integrals of the Jackson Type
 P. C. Sikkema, Über Potenzen von verallgemeinerten Bernsteinoperatoren
 J. J. Sopka, Über verallgemeinerte numerische Integrationen
 H. Werner, Diskretisierung bei Tschebyscheff-Approximation mit verallgemeinerten rationalen Funktionen
 W. Wetterling, Lösungsschranken bei elliptischen Differentialgleichungen.

W. G.

24[2.10].—ROBERT PIESSENS, *Gaussian Quadrature Formulas for the Integration of Oscillating Functions*, 2 pages of tables and 1 page of explanation, reproduced on the microfiche card attached to this issue. Review taken from author's explanation.

A table of weights and abscissas (to sixteen significant figures) for the $2n$ -point Gaussian integration formula

$$\int_{-\pi}^{\pi} f(x) \sin x \, dx = \sum_{j=1}^n [w_j f(x_j) - w_j f(-x_j)], \quad 0 < x_j < \pi,$$

is given for $n = 1(1)9$. The abscissas x_j and weights w_j are given by

$$x_j = \sqrt{u_j}, \quad w_j = b_j/x_j, \quad j = 1, 2, \dots, n,$$

where u_j and b_j are the abscissas and weights of the n -point Gaussian quadrature formula

$$\int_0^{\pi^2} \frac{1}{2} \sin \sqrt{u} g(u) \, du = \sum_{j=1}^n b_j g(u_j).$$

The author chose the method of Anderson [1], to compute the u_j and b_j . The calculations were performed on the IBM 1620 of the Computing Center of the University of Leuven.

E. I.

1. D. G. ANDERSON, "Gaussian quadrature formulae for $\int_0^1 -\ln(x)f(x) \, dx$," *Math. Comp.*, v. 19, 1965, pp. 477-481.

25[2.10, 7].—DAVID M. BISHOP, *Evaluation of Certain Integrals of Reduced Modified Bessel Functions of the Second Kind*, ms. of 6 typewritten pages (undated) deposited in the UMT file.

The main table in this manuscript consists of 8S floating-point values of the integral

$$I_n(\mu) = \int_0^{\infty} [x^\mu K_\mu(x)]^4 x^n \, dx$$

for $n = 0(1)4$ and $\mu = 0.1(0.1)5$. The numerical integration was accomplished by means of Gauss-Laguerre quadrature, using 32 points.

When μ is half an odd integer (and n is a nonnegative integer) the value of $I_n(\mu)$ is expressible as a rational multiple of π^2 ; accordingly, the author presents in a supplementary table these rational coefficients (with their decimal equivalents to 8S) for $n = 0(1)4$ and $\mu = 0.5(1)2.5$. From a comparison of corresponding entries in the two tables the author concludes that the main table is accurate to at least 7S. (This reviewer noted one instance of accuracy to 6S, corresponding to $n = 0, \mu = 1.5$.)

The author states that this integral appears in certain atomic and molecular calculations where the electronic wave function is chosen as a combination of Bessel functions.

J. W. W.

26[2.30, 2.45, 8, 9, 12].—DONALD E. KNUTH, *The Art of Computer Programming*, Vol. II: *Seminumerical Algorithms*, Addison-Wesley Publishing Co., Reading, Mass., 1969, xi + 624 pp., 25 cm. Price \$18.50.

In this impressive sequel to the first volume [1] of his projected seven-volume series on the art of computer programming the author considers those aspects of

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ABRAMOWITZ & STEGUN 458, ALDIS 459, ERDÉLYI, MAGNUS, OBERHETTINGER & TRICOMI 460, GRÖBNER & HOFREITER 461, KNUTH 462, KOBER 463, MAGNUS, OBERHETTINGER & SONI 464, NICHOL, SELFRIDGE & MCKEE 465, VEGA 466.

MICROFICHE SUPPLEMENT

Tables of Gaussian Quadrature Rules for the Calculation of Fourier Coefficients, addendum to MOC this issue, p. 245. WALTER GAUTSCHI
Gaussian Quadrature Formulas for the Integration of Oscillating Functions, see review #24, p. 479 ROBERT PIESSENS

The editorial committee would welcome readers' comments about this microfiche feature. Please send comments to Professor Eugene Isaacson, MATHEMATICS OF COMPUTATION, Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, New York 10012.

Mathematics of Computation

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