the character function of any representation describes its composition completely. But statements of this and of a similar type (e.g., concerning the importance of the regular representation) which could serve as landmarks for the reader, in what amounts to a very wide range of facts, are not, or certainly not conspicuously, displayed.

There are, of course, the nearly unavoidable small inaccuracies. Theorem 12, p. 68 uses the term "algebraically closed" for the ring of algebraic integers—whereas not even all linear equations with coefficients in this ring have a solution that is in the ring.

Also, on p. 4, it would be better to say that there are groups without finite dimensional representations (in spite of the author's definition of a representation at the top of p. 4 which excludes infinite dimensional representations if n is tacitly assumed to be finite). After all, there exists a large and growing literature on infinite dimensional representations of some groups. On p. 12, a reference [G. Higman, B. H. Neumann, H. Neumann, *J. London Math. Soc.*, v. 24, 1949, pp. 247–254] could be given for the construction of infinite groups with two conjugacy classes.

Finally, there are some questions of method. In proving Theorem 3, p. 64, the author refers to van der Waerden for the theory of symmetric functions. But the proof given in van der Waerden for the same theorem just uses a little linear algebra—and would have fitted nicely into the text. Also, it is not clear to the reviewer why the author, after abstaining from using ideals (and, e.g., Wedderburn's Theorem) in the main part of the text, proves the unique factorization into prime ideals for the integers of algebraic number fields in the Appendix.

WILHELM MAGNUS

Courant Institute of Mathematical Sciences
New York University
New York, New York 10012

29[3, 12].—T. J. DEKKER & W. HOFFMAN, *ALGOL* 60 *Procedures in Numerical Algebra*, *Part* II, Mathematical Centre Tracts 23, Mathematisch Centrum, Amsterdam, 1968, 95 pp., 24 cm. Price $3.00.

The authors present the programs which they have developed at the Mathematical Center in Amsterdam for calculating eigenvalues and eigenvectors of real matrices which may be stored in high speed memory. Their procedures make use of the basic subroutines for matrix and vector operations which the authors presented in Part I.

This is an excellent collection of programs developed with understanding and loving care, fully comparable with the Handbook series of Numerische Mathematik. Documentation is thorough. To each page of code, there is at least a page of description. However, this book does not claim to be a textbook on these numerical methods. Familiarity with the subject matter is assumed in the descriptions, whose purpose is to present the all important programming details which can make or break a procedure.

Each chapter begins with a survey of its subdivisions. Each subdivision corresponds to a particular procedure, explains the numerical method and gives the necessary details.

The first chapter (Chapter 23 of the series) concerns symmetric matrices. These are reduced to tridiagonal form by Householder's method. The pth step introduces zeros into the $(n - p + 1)$th row and column. What was the reason for reversing the usual order, one wonders? The pth step is skipped if the elements to be annihilated are smaller than the machine precision times the norm of the matrix. This reviewer has serious reservations about this decision. For example, the off diagonal 1's would be ignored in the matrix

$$\begin{pmatrix} 10^{12} & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}.$$

This produces answers with errors which are large relative to the true values and negligible compared with the matrix norm. The point, however, is that there is no difficulty in producing answers which are correct and it seems a pity not to do this when possible. The solution is to skip a step only when the elements to be annihilated are already zero. Occasionally, unnecessary transformations will be performed, but they will do no harm.

Two methods are given for calculating the eigensystems of tridiagonal matrices: iterated linear interpolation and the QR transformation.

The linear interpolation routine, called zeroin, is a clever combination of the secant method, the method of regula falsi, and bisection. Yet it takes only 18 lines of ALGOL to define. It is a fine example of the art of coding numerical methods. It avoids the pitfalls which beset earlier implementations.

The characteristic polynomial is evaluated by triangular factorization of the tridiagonal matrix $T - xI$ (without interchanges). This technique avoids the under/overflow problems which can plague the Sturm sequence of principal minors. Eigenvectors are found by inverse iteration. Multiple (or very close) eigenvalues are not tolerated (à la Wilkinson) and a Gram-Schmidt process is invoked, if necessary, to force orthogonality on the eigenvectors.

Two versions of the QR transformation for tridiagonals are given: a standard version which uses plane rotations and also gradually builds up the matrix of eigenvectors and, when only eigenvalues are required, the square-root-free version due to Ortega and Kaiser.

Chapter 24 gives procedures for calculating the eigensystems of real matrices with the aid of the QR transformation. The matrices are first balanced and then converted to upper Hessenberg form by stabilized elementary transformations. Both the single and double QR iterations are available for reducing the Hessenberg matrix to block triangular form. The former is used on matrices whose eigenvalues are known to be real. Eigenvectors may be found by using inverse iteration or by saving the product of the Q-matrices and finding the eigenvectors of the final matrix of the QR sequence directly. The latter method is only given for matrices with real eigenvectors. The row eigenvectors are not calculated explicitly. They may be found by inverting the matrix of column eigenvectors produced by these procedures.

One might well wonder whether the problem of computing eigenvalues is now essentially solved. The answer is no (or, at least, not quite). We do not yet understand the convergence properties well enough and we do not know completely satisfactory and economical tolerances and termination criteria for iterations. Sometimes our answers suffer a quite unneccessary lack of accuracy. On the other hand, the publication of these algorithms demonstrates the tremendous progress which has been made during the last fifteen years. For example, no multiple precision is needed anywhere in this booklet. The extent of the details which had to be mastered by the authors is indicated by the fact that nearly 100 pages are required to describe the programs.

BERESFORD N. PARLETT

University of California, Berkeley
Department of Computer Science
Berkeley, California 94720

30[3, 13, 15].—LOUIS A. PIPES & SHAHEN A. HOVANESSIAN, *Matrix Computer Methods in Engineering*, John Wiley & Sons, Inc., New York, 1969, xi + 333 pp., 24 cm. Price $12.95.

This book is written at the junior-senior level; it includes a number of numerical examples and exercises, as well as a number of programs in FORTRAN and in BASIC; about half a dozen references are listed in each chapter. The first four chapters, about 140 pages, are concerned with general theory and general numerical techniques; the remaining five chapters, about 180 pages, take up a variety of applications: electricity, time-frequency domain, vibrations (conservative and nonconservative systems), and structures. There is a five-page index.

The objectives of a course that would use this as a text are hard to imagine. Perhaps it could give the students some feeling for the range of applicability of matrices. But according to the title, the book has to do with methods, and, as already mentioned, there are a fair number of actual programs. But the theory is minimal, and the basic computational techniques described are largely obsolete (barring the inevitable Gaussian elimination) except for very small matrices, say, of order four or five.

The power method is described and illustrated in Chapter 3. Justification is presented in Chapter 7. The power method for roots of smallest modulus is given also in Chapter 3, using the explicit inverse. The Danilevski method for obtaining the characteristic polynomial is given, and so is a version of the method of Le Verrier, attributed to Bocher. Nothing is said about rounding errors, and the name of Wilkinson nowhere occurs. Nothing is said about the Hessenberg or the tridiagonal form. And naturally nothing is said about the use of the inverse power method to refine an approximate root. Among the references, the most recent one is the very poor English translation of the first edition of Faddeev and Faddeeva, the translation having the date 1963, whereas the original appeared in 1960. And yet, in seeing the various programs, the student could easily get the idea that this is the last word.

A. S. H.

31[4, 5, 6, 13.05, 13.15].—S. G. MIKHILIN & K. L. SMOLITSKIY, *Approximate Methods for Solution of Differential and Integral Equations*, American Elsevier Publishing Co., New York, 1967, vii + 308 pp., 24 cm. Price $14.00.

This is a translation from the Russian of a reference book published in 1965 by Nauka Press, Moscow, in its series "Spravočnaja Matematičeskaja Biblioteka." The work gives an excellent exposition, on an advanced level, of the most important approximate methods for solving boundary-value problems for differential equations, both ordinary and partial. It also considers the numerical solution of Fredholm and