This is a text for an introductory course in ordinary differential equations, designed for students with no previous experience in the subject. Chapter 1 uses some typical mechanical systems to motivate the study of differential equations. Chapter 2 brings elementary methods of solution, including Euler's method as an example of an approximate method. Chapter 3 is devoted to the general theory of linear differential equations, while Chapter 4 concentrates on linear second-order equations, and presents a detailed treatment of solutions in power series, including a study of Bessel's equation, and an introduction to asymptotic expansions (arising naturally in connection with irregular singular points). Chapter 5 discusses boundary value problems, proceeding from elementary examples to general Sturm-Liouville problems. Up to this point, the material can be mastered by students with no background in linear algebra. A knowledge of the elements of vector algebra will be needed for Chapter 6, which deals with systems of linear and nonlinear differential equations, and briefly with the qualitative theory of autonomous systems. The basic existence and uniqueness theorems, as well as theorems on the continuous dependence on initial data and parameters are proved in Chapter 7, both for single differential equations and systems of differential equations. Chapter 8 gives an introduction to numerical methods with a good discussion of error accumulation. Chapter 9, finally, is on Laplace transforms and their use for solving initial value problems for linear differential equations. There are numerous exercises, some scattered throughout the chapters, others collected at the end of each chapter.

While maintaining a high standard of mathematical exposition, the authors have succeeded in blending theory with applications so as to impart to the student an appreciation not only of the mathematical coherence of the subject, but also of its usefulness as a tool to "explain and help him understand various physical phenomena in the physical world".

W. G.

33[5].—DONALD GREENSPAN, *Lectures on the Numerical Solution of Linear, Singular, and Nonlinear Differential Equations*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1968, 185 pp., 26 cm. Price $6.95.

This book is a survey of numerical methods for the solution of differential equations, based on the author's lectures at summer conferences at the University of Michigan. According to the preface, "Scientists and technologists should be able to determine easily from the text what the latest methods are and whether those methods apply to their problems". There are 486 references that "will enable teachers to adapt the material for classroom presentation".

The reviewer would hesitate very much to use this book as a textbook or to suggest it as reading for applied scientists asking for advice on numerical methods. The material is quite specialized, and only a few methods are discussed. No proofs are given. In the discussion of elliptic problems, the problem of convergence and accuracy is not even mentioned, and the reader is left without any guidance as to why one difference approximation might be preferable to another. Instead, a lot of space is taken up by a detailed and repetitive discussion of how the replacement of differential operators by finite differences leads to systems of algebraic equations. Frequently, numerical values for the coefficients are given for some specific mesh-size.

The only method discussed for hyperbolic and parabolic problems is one invented by the author, which changes the problem into a boundary value problem. This is odd in a book which claims to be a survey. For several decades, the more conventional marching procedures have been used, often with great success, by an enormous number of people. The basic algorithmic ideas behind these methods, as well as a simplified stability theory, could have been presented easily, even to an audience which is not very sophisticated mathematically.

The last chapter deals with the author's method for the steady state Navier-Stokes problem. The author describes a series of numerical experiments, using only 81 interior meshpoints for Reynold's numbers up to 10^5. There is no discussion of accuracy; in fact, it should be obvious that the flow described by the discrete model in such a case has only a formal connection with the differential equations. Because of the fact that such flows cannot be described with so few parameters, the treatment of these calculations should have been omitted or else put into proper perspective.

The author is well known as a master of the very sophisticated art of obtaining numerical solutions to difficult applied problems. But this book does not fulfill the promise indicated in the preface, because it concentrates on his own work and neglects too many important methods.

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34[7].—B. S. Berger & H. McAllister, *A Table of the Modified Bessel Functions $K_n(x)$ and $I_n(x)$ to at Least 60S for $n = 0, 1$, and $x = 1, 2, \ldots, 40$*, ms. of 4 typewritten sheets + 8 computer sheets (reduced) deposited in UMT file. (Copies also obtainable from Professor Berger, Department of Mechanical Engineering, University of Maryland, College Park, Md. 20742.)

Assisted by R. Carpenter, the authors have here produced a table of the modified Bessel functions of orders 0 and 1, for integer arguments ranging from 1 through 40. The tabular entries are presented in floating-point form and range in precision from 61S to 98S.

Standard power series for these Bessel functions were used in the underlying calculations, which were performed by multiple-precision arithmetic routines on an IBM 7094 system, the number of terms retained in the series ranging from 80 to 122, with increasing argument. The tabulated figures were subjected to the appropriate Wronskian check, which as the authors note, however, is satisfied even when erroneous values of Euler's constant and $\ln(x/2)$ have been used in the calculation of $K_n(x)$. Accordingly, the value of Euler's constant was carefully checked against several independent sources, and the computed natural logarithms were compared with those of Mansell [1].

The reviewer has compared the present basic tables with the tables of Aldis [2], and has thereby discovered in the latter, several errors which are listed elsewhere in this journal.

J. W. W.

1. W. E. Mansell, *Tables of Natural and Common Logarithms to* 110 *Decimals*, Royal Society Mathematical Tables, Vol. 8, Cambridge Univ. Press, New York, 1964. (See *Math. Comp.*, v. 19, 1965, p. 332, RMT **35**.)

2. W. S. Aldis, "Tables for the solution of the equation $d^2y/dx^2 + (1/x)\,dy/dx - (1 + n^2/x^2)y = 0$," *Proc. Roy. Soc. London*, v. 64, 1899, pp. 203–223.