35[7].—Henry E. Fettis & James C. Caslin, *A 20-D Table of Jacobi's Nome and its Inverse*, Report ARL 69-0050, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, March 1969, iv + 30 pp., 28 cm. [Copies obtainable from the Clearinghouse, U. S. Department of Commerce, Springfield, Va. 22151. Price $3.00.]

Jacobi's nome q is defined by the equation $q = \exp(-\pi K'/K)$, where K and K' are the quarter-periods of the Jacobian elliptic functions. The importance of this function derives from the fact that the Jacobian elliptic functions and theta functions possess well-known rapidly convergent expansions in terms of it.

This report consists of four tables: Table 1 gives q to 20D for $k^2 = 0.001(0.001)0.999$; Table 2 gives q to 20D for $\alpha = 0(0.1°)89°(0.01°)89.99°(0.0002°)90°$, where $\alpha = \sin^{-1} k$ is the so-called modular angle; Table 3 consists of 20D values of k and k' for $q = 0(0.001)0.5$; and Table 4 gives 20D values of q, $\bar{q}$, and $\bar{q}/q$ for $k' = 0.0001(0.0001)0.2$, as well as the second central differences of this tabulated ratio. The quantity $\bar{q}$ is defined as $\exp\{-\pi^2/[2\ln(4/k')]\}$; it approximates q with an error of the order of $(k')^2$, as the authors note in their explanatory remarks.

The tables were computed on an IBM 1620 system by means of modular reduction using Gauss's transformation, all arithmetical operations being carried to 23D prior to rounding the final results to 20D.

The user of these tables will probably be disconcerted by a series of derangements of tabular entries in Table 1 (p. 8), due to a corresponding disorder in the output cards used in the automatic printing.

Moreover, two of the five listed references contain errors. For example, the title of the important table of Curtis [1] is misquoted and the relevant paper of Salzer [2] is located erroneously in the *Journal*, instead of the *Communications*, of the *ACM*. The authors have informed this reviewer that they are planning to issue an appropriate errata sheet listing these corrections.

Despite such regrettable typographical imperfections, these tables constitute a significant improvement both in range and size of tabular interval over earlier tables of the Jacobi nome.

J. W. W.

1. A. R. Curtis, *Tables of Jacobian Elliptic Functions whose Arguments are Rational Fractions of the Quarter Period*, National Physical Laboratory, *Mathematical Tables*, Vol. 7, Her Majesty's Stationery Office, London, 1964. (See *Math. Comp.*, v. 19, 1965, pp. 154–155, RMT **10**.)
2. H. E. Salzer, "Quick calculation of Jacobian elliptic functions," *Comm. ACM*, v. 5, 1962, p. 399.

36[7].—Henry E. Fettis & James C. Caslin, *Tables of Toroidal Harmonics*, I: *Orders 0–5, All Significant Degrees*, Report ARL 69-0025, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, February 1969, iv + 209 pp., 28 cm. [Copies obtainable from the Clearinghouse, U. S. Department of Commerce, Springfield, Va. 22151. Price $3.00.]

This report contains tables of both 11S and 16S values (all in floating-point form) of the toroidal harmonics, which are identifiable with the Legendre function of the second kind, $Q_\nu^\mu(s)$, when the order μ is an integer m, the degree ν is a half-odd integer $n - 1/2$, and the argument s exceeds unity.

The first table gives $Q_{n-1/2}^m(s)$ to 11S for $m = 0(1)5$, n varying from 0 through consecutive integers to a value (at least 29) for which the value of the function relative to that when n is zero is less than 10^{-12}, and $s = 1.1(0.1)10$.

The second table differs from the first with respect to the argument, which here is $\cosh \eta$, where $\eta = 0.1(0.1)3$. As noted in the abstract and explained in the introduction, this form of the argument appears naturally in the solution of the potential problem in toroidal coordinates.

The last two tables consist of 16S values of $Q_{n-1/2}^m(s)$ for $n = 0$ and 1, for the same range of values of m, s, and η as in the first two tables. These more extended decimal approximations were calculated independently by means of well-known formulas relating these toroidal functions to the complete elliptic integrals of the first and second kinds.

A useful introduction of 12 pages gives the derivation of these functions as solutions of Laplace's equation in toroidal coordinates, enumerates their principal properties, develops a continued fraction for the ratio of such functions of consecutive degree, and discusses the mathematical methods used in calculating the tables on IBM 1620 and IBM 7090 systems. Appended to the introduction is a list of five references.

Photographic offset printing of these tables from the computer sheets has not been completely satisfactory, as may be inferred from the two pages of corrigenda inserted to clarify a number of indistinctly printed tabular digits.

Despite such typographical imperfections, these extensive tables should prove generally useful to applied mathematicians.

J. W. W.

37[7].—M. Kumar & G. K. Dhawan, *Numerical Values of Certain Integrals Involving a Product of Two Bessel Functions*, Maulana Azad College of Technology, Bhopal, report and tables deposited in the UMT file.

In numerous applied problems, one encounters

$$I(\mu, \nu, \lambda) = \int_0^\infty e^{-pt} t^\lambda J_\mu(at) J_\nu(bt)\, dt.$$

A discussion of this integral with references to tables is given by Luke [1]. Let $a = t/h$; $b = u/h$; $u, t = 0.2(0.2)1.0$; $p = 2$; and $h = 1.05$, 1.10, 1.30 and 1.50. For all possible combinations of these parameters the authors tabulate $I(\mu, \nu, \lambda)$ to 6D for $\mu = \nu = 0, 1$, $\lambda = 1, 2, 3$, and for $\mu = 1$, $\nu = 0$ and $\lambda = 3$. All the integrals can be expressed in terms of the complete elliptic integrals of the first and second kinds. These expressions are delineated in an introduction to the tables.

Y. L. L.

1. Y. L. Luke, *Integrals of Bessel Functions*, McGraw-Hill Book Co., New York, 1962, pp. 314–318. (See also *Math. Comp.*, v. 17, 1963, pp. 318–320.)

38[9].—L. M. Chawla & S. A. Shad, "On a trio-set of partition functions and their tables", Table, *J. Natur. Sci. and Math.*, v. 9, 1969, pp. 87–96.