7	251	1.38807	5923	0.28574
9	419	1.38129	10627	0.27428
11	659	1.34617	15667	0.27609
13	1019	1.27940	20563	0.28481
15	971	1.51228	34483	0.25377
17	1091	1.61691	37123	0.27719
19	2099	1.30286	38707	0.30340
21	1931	1.50134	61483	0.26607
23	1811	1.69792	90787	0.23981
25	3851	1.26562	93307	0.25712
27	3299	1.47680	103387	0.26380
29	2939	1.68054	166147	0.22351
31	3251	1.70806	133387	0.26666
33	4091	1.62087	222643	0.21971
35	4259	1.68486	210907	0.23943
37	8147	1.28781	158923	0.29158
39	5099	1.71582	253507	0.24334
41	9467	1.32382	296587	0.23651
43	6299	1.70209	300787	0.24631
45	6971	1.69323	308323	0.25460
47	8291	1.62160	375523	0.24095
49	8819	1.63922	393187	0.24550

A second table deposited is listed on 9 pairs of sheets in the same format. This is a subtable, which includes only those p having

$$m^2 | h(-p) \qquad (m > 1).$$

It therefore includes all p having $h(-p) = 9, 25, 27, 45$, and 49, together with (incomplete) sets of p having $h(-p) = 63, 75$, etc. As indicated in our previous reviews, [1], [2], the desire to examine all 25's and 27's was the motivation for computing those tables. By examining the present table, I find, for example, that the two fields $Q(\sqrt{-p})$ have $h(-p) = 81$ for $p = 430411$ and 298483 (among many others). But these two have a class group $C(9) \times C(9)$, an elegant, but very unusual structure. I can now add these to $p = 134059$, that has the same group, which I found earlier.

D. S.

1. UMT **29**, *Math. Comp.*, v. 23, 1969, p. 458.
2. UMT **50**, *Math. Comp.*, v. 23, 1969, p. 683.
3. D. H. LEHMER ET AL., "Integer sequences having prescribed quadratic character," *Math. Comp.*, v. 24, 1970, pp. 433–451.

40[9].—ELVIN J. LEE, *The Discovery of Amicable Numbers*, a 28-page history together with a computer-listed table of the 977 pairs of amicable numbers then known, Oak Ridge National Laboratory, Oak Ridge, Tenn., June 4, 1969, deposited in the UMT file.

Although this version is deposited in the Unpublished Mathematical Tables file, a revision will in fact be published, perhaps in several parts, in the *Journal of Recreational Mathematics*.

Since a revision will be published, we can be somewhat brief here. The text discusses all known methods of discovering amicable pairs, all known tables published, all the numerous errata and duplications therein, most of the theory developed, most of the unsolved problems announced, and it concludes with a 61-item bibliography.

Of the 977 pairs, the oldest (220 and 284) has an unknown discoverer, but each of the remaining pairs is attributed to its (apparent) discoverer. The grand totals for each discoverer are: Fermat, 1; Descartes, 1; Euler, 59; Legendre, 1; Paganini, 1; Seelhoff, 2; Dickson, 2; Mason, 14; Poulet, 104; Gerardin, 5; Poulet & Gerardin, 4; Escott, 218; Brown, 1; Garcia, 150; Rolf, 1; Alanen, Ore & Stemple, 8; Lee (himself), 390; Bratley & McKay, 14.

In the revision, it appears that Garcia gets three more, and 59 new pairs (at least) will be credited to Cohen, and, almost simultaneously, to Bratley, McKay and Lunnon.

Besides these attributions, each pair in the table has an identifying number, and the ratio of its two members is also listed. The 977 pairs are listed by type, not magnitude. The first type is

$$E\,p, \qquad E\,qr$$

which means that the common factor E is prime to p, q, and r, and these latter are distinct primes. The next type is

$$E\,pq, \qquad E\,rs,$$

etc. Within types the ordering is, first, by the power of 2 in E, and then by some similar rules. However, the revision will reorder some of these pairs, since in this edition there is no explicit set of rules that unequivocally defines the location of each pair.

While such a listing (by types) is more analytical than a numerical listing in that it is more amenable to theory, it does have the drawback that new discoveries must be interpolated, thereby destroying the identifying numbers.

While the text and table will be revised, owing mainly to the new surge of work in this field, it certainly is a must for everyone interested in this field, and Lee has a few extra copies.

One observation (conjecture?) here has already bitten the dust because of this new work. The operator

$$\sigma_0(N) = \sigma(N) - N$$

defines perfect numbers by $\sigma_0(N) = N$, amicable numbers by $\sigma_0^2(N) = N$, and sociable numbers of order k by $\sigma_0^k(N) = N$. Poulet long ago found one example each of $k = 5$ and $k = 28$, and until recently no other $k > 2$ were known. Now it happens that there is no solution of $\sigma_0(x) = 2$ or $\sigma_0(x) = 5$ or $\sigma_0(x) = 28$, and it was thought that this fact was somehow associated with the known existing k's: 2, 5, 28. On the other hand, one has $\sigma_0(16) = \sigma_0(33) = 15$, $\sigma_0(15) = 9$, $\sigma_0(9) = 4$, and $\sigma_0(4) = 3$, and no sociables of order $k = 4$ or 3 were known. That was the observed pattern. But Cohen then found nine examples of $k = 4$.

D. S.