**41[9].**—D. H. LEHMER, *Tables of Ramanujan's Function* $\tau(n)$, 1963, ms. of 164 pages of computer printout deposited in the UMT file.

There are listed here the values of Ramanujan's $\tau(n)$ for $n = 1(1)10^4$. This is defined [1] by

$$\sum_{n=0}^{\infty} \tau(n)x^n = x\left\{\prod_{n=1}^{\infty} (1 - x^n)\right\}^{24} \tag{1}$$

and begins: $\tau(1) = 1$, $\tau(2) = -24$, $\tau(3) = 252, \cdots$. It was computed by the known [1] recurrence formula obtained by logarithmically differentiating (1). This recurrence calculation was run (about 1963) on Stanford's IBM 7094, and values were checked by the relation

$$\tau(mn) = \tau(n)\tau(m) \qquad \text{for } (m, n) = 1.$$

Earlier tables of $\tau(n)$ were by Watson [2] to 1000, Lehmer [3] to 2500, and Ashworth & Atkin [4] to 1921.

Watson also listed

$$\tau^*(n) = \tau(n)n^{-11/2}$$

to test Ramanujan's conjecture [1]:

$$|\tau^*(n)| < d(n),$$

where $d(n)$ is the number of divisors of $n$. For primes $p$, this becomes

$$|\tau^*(p)| < 2,$$

and Watson found

$$\tau^*(103) = -1.91881, \qquad \tau^*(479) = +1.90410.$$

Lehmer does not list $\tau^*(n)$ here, but casting an eye along his (long) column of numbers I found

$$\tau^*(3371) = +1.94978, \qquad \tau^*(3967) = +1.95144,$$

$$\tau^*(7451) = -1.95502, \qquad \tau^*(7589) = +1.96053.$$

The apparent existence of sequences $p$ with

$$\tau^*(p) \to +2 \quad \text{and} \quad \tau^*(p) \to -2,$$

from below, and above, respectively, does suggest the stronger conjecture that $+2$ and $-2$ are the *best possible* bounds. The function

$$\cos\theta_p = \tfrac{1}{2}p^{-11/2}\tau(p)$$

enters the theory, (see [5] for an introductory account), and we record

$$\theta_{7589} = 11°24'$$

for the smallest angle noted.

Also of interest would be values of

$$|\tau^*(p)| \approx 1,$$

or

$$\theta_p \approx \pi/3, 2\pi/3,$$

since

$$\tau(p^2) = (\tau(p))^2 - p^{11}$$

and therefore $\tau(p^2)$ is relatively very small. An early example is

$$\tau^*(11) = 1.0087, \qquad \tau^*(121) = 0.00175,$$

but these values are more difficult to pick out merely by glancing down the column of $\tau(n)$.

Subsequently, the original table deposited was replaced by a second that also lists the summatory function $\sum^N \tau(n)$.

D. S.

1. S. RAMANUJAN, "On certain arithmetical functions," *Trans. Cambridge Philos. Soc.*, v. 22, 1916, pp. 159–184; see especially §§16–18. A short table of $\tau(n)$ for $n = 1(1)30$ is given here.
2. G. N. WATSON, "A table of Ramanujan's function $\tau(n)$," *Proc. London Math. Soc.*, (2), v. 51. 1950 (paper is dated 1942), pp. 1–13.
3. D. H. LEHMER, *Tables of Ramanujan's* $\tau(n)$, UMT **101**, *MTAC*, v. 4, 1950, p. 162.
4. MARGARET ASHWORTH & A. O. L. ATKIN, *Tables of* $p_k(n)$, UMT **1**, *Math. Comp.*, v. 21, 1967, p. 116.
5. G. H. HARDY, *Ramanujan*, Chelsea reprint, New York, 1959, Chapter X and §§9.17, 9.18.

**42[9].**—PAUL TURÁN, Editor, *Number Theory and Analysis—A Collection of Papers in Honor of Edmund Landau* (1877–1938), Plenum Press, New York, 1969, 355 pp., 24 cm. Price $19.50.

There are 22 papers here in number theory and analysis in honor of Landau by E. Bombieri, H. Davenport, B. M. Bredihin, J. V. Linnik, N. G. Tschudakoff, J. G. van der Corput, M. Deuring, P. Erdös, A. Sárközi, E. Szemerédi, H. Heilbronn, E. Hlawka, A. E. Ingham, V. Jarnik, S. Knapowski, P. Turán, J. Kubilius, J. E. Littlewood, L. J. Mordell, G. Pólya, J. Popken, H. Rademacher, A. Rényi, I. J. Schoenberg, C. L. Siegel, Arnold Walfisz, and Anna Walfisz.

There also is a joint paper by Davenport and Landau himself: "On the representation of positive integers as sums of three cubes of positive rational numbers". Davenport explains: "This paper was written, in a rough form, in February 1935, when Landau visited Cambridge . . . As far as I can recollect, the reason why the paper was not published. . . ." Unfortunately, Landau is not the only departed author here, since one must add Ingham, Knapowski, Arnold Walfisz, Rademacher, and Davenport.

The papers are in English and German. The volume first appeared in Germany with the title "Abhandlungen aus Zahlentheorie und Analysis zur Erinnerung an Edmund Landau (1877–1938)". It includes a photograph of Landau, a short foreword by Turán, and a list of Landau's seven books and his 254 papers (not counting the one here).

Landau's first two papers (published at age 18) were on chess, but with the third he begins his real life work. It is his well-known 1899 Inaugural Dissertation: "Neuer Beweis der Gleichung $\sum_1^\infty \mu(k)/k = 0$". Landau liked to joke about this paper. "Gordan pflegte etwa zu sagen: 'Die Zahlentheorie ist nützlich, weil man nämlich mit ihr promovieren kann'. Ich habe mit einer Antwort auf diese Frage 1899 promoviert."