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Remark on a Conjecture of Erdos on Binomial Coefficients

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Abstract. A conjecture attributed to Erdös concerning the Diophantine equation

$$2\binom{x+n-1}{n} = \binom{y+n-1}{n}$$

is shown to be false.

M. Wunderlich [2] attributes the following conjecture to P. Erdös:
The equation

$$(1)\qquad 2\binom{x+n-1}{n} = \binom{y+n-1}{n}$$

has only one solution in positive integers: $x = n$, $y = n + 1$.

Because (1) has infinitely many solutions for $n = 2$ (cf. [1, p. 30]) the assumption $n \geqq 3$ must surely be added. But that does not suffice.

Observe that for $b - a \geqq 3$ the equality

$$(2)\qquad s\binom{a}{2} = t\binom{b}{2}$$

implies

$$s\binom{b-2}{b-a} = t\binom{b}{b-a}.$$

Because (2) has infinitely many solutions in integers a, b for $s = 2$, $t = 1$, we obtain infinitely many counterexamples to the conjecture of Erdös, viz. $n = b - a$, $x = a - 1$, $y = a + 1$, where

$$2\binom{a}{2} = \binom{b}{2}.$$

For example,

$$2\binom{19}{6} = \binom{21}{6}$$

is a solution of (1).

Probably the conjecture is true when we require $y - x \geqq 3$.

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1. L. E. DICKSON, *History of the Theory of Numbers*. Vol. II, reprint, Chelsea, New York, 1952.

2. M. WUNDERLICH, "Certain properties of pyramidal and figurate numbers," *Math. Comp.*, v. 16, 1962, pp. 482–486. MR **26** #6115.

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