

associated with dictionaries and encyclopedias, to which one rarely turns for inspiration and vitality.

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52 [2.05].—AMERICAN MATHEMATICAL SOCIETY TRANSLATIONS, *Fourteen Papers on Series and Approximation*, American Mathematical Society, Providence, R. I., 1968, iv + 266 pp., 25 cm. Price \$13.60.

Except for a paper of I. M. Vinogradov (Estimation of Trigonometric Sums), motivated by additive number theory, all papers of this volume belong to the theory of approximation or to related branches of analysis (orthogonal series). Short reviews follow.

*Balasov* has very neat theorems about series of Rademacher functions and about series of the form  $\sum a_k f(n_k x)$ .

*Osipov* generalizes work of Ul'janov and R. P. Agnew and shows that if  $\sum_{n=1}^{\infty} a_n^2 = \infty$ , and if  $f$  is measurable on  $(0, 1)$ , then there exists an orthonormal system  $\phi_n$  for which  $\sum_{n=1}^{\infty} a_n \phi_n(x)$  converges everywhere to  $f(x)$  for any rearrangement of its terms. The paper of *Jastrebova* deals with Walsh-Fourier series.

Among the papers on Fourier series, *Bojanić* and *Tomić* deal with the absolute convergence of Fourier series with gaps for which  $n_{k+1} - n_k \geq \text{const}$ . *M. F. Timan* discusses the approximation in spaces  $L^p$  of  $f$  by the  $\lambda$ -means of its Fourier series, where  $\lambda$  stands for many classical summability matrices. *Berdysev* estimates  $\sup_f |a_n(f)|$ ,  $\sup_f \|f - s_n(f)\|_{\infty}$ , when the modulus of continuity of  $f$  is given. Two papers deal with the degree of approximation, in a Banach function space  $X$ , of a function  $f$  by trigonometric polynomials. *Cyganok* has generalizations of Jackson's estimate (involving moduli of continuity of the function  $f$  or of its derivatives) for the degree of approximation of  $f$  in an Orlicz space norm. *A. V. Efimov* relates the lower estimate for the degree of approximation of a class  $M \subset X$  to the supremum of  $\|\phi\|_s$ , where  $\phi$  are all functions of  $M$  which are "cos  $nx$ -symmetric," and finds this supremum for several classes  $M$ .

*Teljakovskii* answers positively a question proposed by this reviewer, and proves that for  $f \in C^r[-1, 1]$  there exists a sequence of algebraic polynomials  $P_n$  for which

$$|f(x) - P_n(x)| \leq C((1 - x^2)^{1/2}/n)^r \omega(f^{(r)}), \quad n \geq r.$$

*G. C. Tumarkin* in 2 papers treats the possibility of approximation, in the norm of  $L^p$ , of a function by rational functions with prescribed poles. *Lizorkin* has inequalities of Bernštein type for fractional derivatives. Finally, *Suetin* discusses uniqueness properties of interpolation series for certain analytic functions.

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