

54 [2.15].—D. S. MITRINOVIC, R. S. MITRINOVIC & S. S. TURAJLIC, "A table of coefficients for numerical differentiation," *Publ. Fac. Elect. Univ. Belgrade (Série: Math. et Phys.)*, No. 247–No. 273, 1969, pp. 115–122.

This table appears in the publication numbered 264 in this compilation of recent papers in mathematics and physics by members of the faculty of the University of Belgrade. It consists of exact (rational) values of the coefficients  $A_{r,n}$  for  $r = 1(1)30$ ,  $n = r(1)30$  in Markoff's formula for the  $r$ th derivative in terms of forward differences. These values were computed (partly on an Olivetti desk calculator and partly on an IBM 1130 system) by use of the formula  $A_{r,n} = r!S_n^r/n!$ , where  $S_n^r$  is a Stirling number of the first kind.

In the preparation of this extensive table the authors confirmed the accuracy of the table of Lowan, Salzer and Hillman [1], which was previously the most extensive in the literature.

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1. A. N. LOWAN, H. E. SALZER & A. HILLMAN, "A table of coefficients for numerical differentiation," *Bull. Amer. Math. Soc.*, v. 48, 1942, pp. 920–924.

55 [2.35, 8, 13. 25, 13.35].—M. T. WASAN, *Stochastic Approximation*, Cambridge Univ. Press, New York, 1969, viii + 202 pp., 22 cm. Price \$9.50.

A stochastic approximation method is an iterative procedure by means of which successive values of a parameter are generated and corresponding random samples are observed. For example, the parameter may be the dosage of a new drug and the observations are then the responses. A scheme is set up so that these values converge stochastically in some sense to the desired quantity, such as the mean of an unknown quantity (the critical dosage). A typical scheme, due to Monro-Robbins (1951) and having its origin in bioassay, looks like this:

$$x_{n+1} = x_n - cn^{-1}[\psi(x_n) - \alpha].$$

Older methods go back to Newton in numerical solutions of equations and to the nonstochastic up-and-down method. Kiefer and Wolfowitz considered a stochastic method for determining the location of the maximum of a regression function. A lot of work was done from the fifties down to recent years. This monograph collects the results of many authors, discusses (i) the conditions under which the method gives a valid approximation to the required solution, (ii) methods for optimal choice of parameters to hasten convergence, and (iii) comparisons with other techniques. The stochastic process involved is usually a nonstationary Markov chain of a very special kind, and the mathematical content of the results boils down to upper and lower bounds for moments and related quantities, and sometimes their asymptotic behavior. The prototypes of such inequalities, given by the reviewer to establish asymptotic normality, and ramified by Burkholder, Derman, Sacks, and others, are collected with detailed proofs in an appendix. There are many examples, applications and references to sources. Although the book is not for browsing, by the nature of the subject, the author has written a readable account for readers who are ready

to plow through the abundant formulas. There are a few minor misprints including the spelling of Loeve on p. 155 and some little *o*'s for big *O*'s on p. 176. But, on the whole, the printing is pleasant to look at.

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56 [7].—MURRAY GELLER & EDWARD W. NG, "A table of integrals of the exponential integral," *J. Res. Nat. Bur. Standards—B. Mathematical Sciences*, v. 73B, 1969, pp. 191–210.

The main part of this exhaustive compilation consists of a total of 202 definite and indefinite integrals of products of the exponential integral with both elementary and transcendental functions.

Also included in this paper is a brief introduction enumerating the procedures followed in obtaining the tabular entries and citing applications in diffusion theory, transport problems, astrophysics, and quantum mechanics. This is followed by sections giving, respectively, a glossary of relevant functions and notations, the definition, special values, and integral representations of the exponential integral.

A list of 15 references is appended; these include several standard collections of integrals, as well as publications relating specifically to the exponential integral and its applications.

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57 [7].—EDWARD W. NG & MURRAY GELLER, "A table of integrals of the error functions," *J. Res. Nat. Bur. Standards—B. Mathematical Sciences*, v. 73B, 1969, pp. 1–20.

The greater part of the definitive table in this paper consists of the systematic tabulation of a total of 179 definite and indefinite integrals of products of the error function and its complement with both elementary and transcendental functions.

Preliminary sections of the paper include an introduction enumerating the procedures followed in obtaining these integrals and citing several applications, a glossary of pertinent functions and notation, and analytic definitions and integral representations of the error function and related functions.

Appended to the main table is a table of 16 relevant integrals of elementary functions. A list of 16 references includes several standard tables of integrals from which many of the tabular entries were taken and a number of papers relating to applications in atomic physics, astrophysics, and statistical analysis.

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