58 [7].—G. BLANCH & D. S. CLEMM, *Mathieu's Equation for Complex Parameters, Tables of Characteristic Values*, U. S. Government Printing Office, Washington, D.C., 1969, xix + 273 pp., 27 cm. Price $4.50.

Mathieu's equation

$$d^2y/dx^2 + (a - 2q \cos 2x)y = 0$$

admits of periodic solutions of period π and 2π for four denumerable sets of characteristic values $a(q)$ for each assigned value of q. There are extensive tabulations for real values of q.* But very little is available in the complex plane.

The present work provides a tabulation in the complex plane (Tables I and II) for $q = \rho \exp(i\varphi)$, $\varphi = 5°(5°)90°$, $\rho = 0(0.5)25$, 4D and additional values for $\varphi = 90°$ for ρ up to 100, 8D (Tables III, IV and V). The data in Tables I and II is sufficient to depict the eigenvalues over the range covered though the mesh lengths in φ and ρ are not sufficiently small for satisfactory interpolation. As the authors correctly remark, excessive computation just to provide for easy interpolation is no longer of paramount importance in view of the availability of high speed computing equipment. Indeed a shorter table that presents a general overview of the function may be more serviceable. An exception was made on the 90° ray due to its importance in both theoretical and applied problems. For this ray, the tables are interpolable and extend beyond the singular point (if there is one) for each order, and always at least up to $\rho = 100$. In one of the sets, the last entry is for $\rho = 130$.

Power series expansions for the characteristic values are known, but the radii of convergence have been for the most part an unknown quantity since these depend on a knowledge of the multiple eigenvalues. Mulholland and Goldstein** gave six sets of characteristic values for $q = is$, $s \leqq 2$. They found that $a_0(q)$ and $a_2(q)$ have a common value when $s = 1.468$. Bouwkamp*** gave the more accurate value 1.468769. The present authors have confirmed that the latter singular point is nearest the origin in Euclidean distance. In addition, the double points in the complex plane for orders $r \leqq 15$ are tabulated to 8D (Table VI). It is believed that all tabular values are accurate to within a unit in the last place.

An introduction gives the background of the problem, derivation of the basic equations, method of computation, useful auxiliary functions near a singular point, method of computing the singular points and remarks on interpolation. There is a complete bibliography and some reliefs of the data.

The table typography is clear but not uniform. As the tables were prepared by offset from machine printout, the nonuniformity appears due to a poor ribbon. This slight imperfection aside, the tables are a definitive and marked contribution to the literature.

Y. L. L.

* See National Bureau of Standards, *Tables Relating to Mathieu Functions*, Second Edition AMS 59. U. S. Govt. Printing Office, Washington, D. C., 1967. See also *Math Comp.*, v. 22, 1968, p. 466.

** H. P. Mulholland and S. Goldstein, "The characteristic numbers of the Mathieu equation with purely imaginary numbers," *Philos. Mag.*, v. 8, 1929, pp. 834–840.

*** C. J. Bouwkamp, "A note on Mathieu functions," *Kon. Nederl. Akad. Wetensch. Proc.*, v. 51, 1948, pp. 891–893.