69[6, 7, 13.15, 13.35].—PIERRE VIDAL, *Non-linear Sampled-Data Systems*, Gordon and Breach Science Publishers, New York, 1969, xv + 346 pp., 24 cm. Price $24.50.

This volume is concerned with the analysis of nonlinear systems with discrete information where the sampling is caused by, or inherent in, the selected method of control. The book is a practical working tool for engineers and research workers, and, although it leans heavily on numerous mathematical concepts, it is not, nor is it meant to be, a thorough-going, rigorous, and mathematically sophisticated treatment of the subject. Numerous block diagrams and figures are presented to illustrate the various automatic control problems and their mathematical analogs. Unfortunately, it is assumed that the reader is already familiar with the ideas such diagrams and related notations convey, as no definitions are provided. Indeed, a glossary of terms would have considerably enhanced the book. The volume does contain numerous examples and this should add to its usefulness.

Chapter 1 takes up the fundamentals of the calculus of finite differences. Topics treated include the Carson transform (the Laplace transform multiplied by p) and its inverse. Emphasis is on step-like functions. Table 1.1 gives the transform of numerous step functions and Table 1.2 gives some commonly occurring difference equations which can be solved with the aid of such transforms. The stability of solutions of difference equations is considered. The z-transform and its application for the solution of linear difference equations is the subject of Chapter 2. Applications to certain nonlinear difference equations and linear difference equations with periodic coefficients are given. A modified z-transform is introduced. Table 2.1 gives the Laplace transform, z-transform and modified z-transform of some commonly occurring transcendents.

The solution of a system of two nonlinear difference equations by the so-called discrete phase plane method is taken up in Chapter 3.

Chapters 4 and 5 consider the analysis of nonlinear difference equations by various methods. The stability of certain nonlinear sampled-data systems and related functions are treated in Chapters 6, 7, 8 and 9.

There is a notation index. The printing and typography are satisfactory save that pages 162, 163, 166, 167, 170, 171, 174 and 175 of my copy are blank.

Y. L. L.

70[7].—HENRY E. FETTIS & JAMES C. CASLIN, *Tables of Toroidal Harmonics*, II: *Orders* 5–10, *All Significant Degrees*, Report ARL 69–0209, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, December 1969, iv + 179 pp., 28 cm. [Copies obtainable from the Clearinghouse, U. S. Department of Commerce, Springfield, Virginia 22151 at $3.00 each.]

The tables comprising this report are a continuation of the two main tables in a previous report [1] by the same authors.

Thus, in this second report on toroidal harmonics we find a table of 11S values of $Q^m_{n-1/2}(s)$ for $m = 5(1)10$, $s = 1.1(0.1)10$, and n varying from 0 through consecutive integers to a value ranging from 35 to 160 for which the value of the function relative to that when n is zero is less than 10^{-21}.

Also, as in the first report, this is immediately followed by the tabulation of the same function to the same precision and for the same orders, m, but for arguments $s = \cosh \eta$, where $\eta = 0.1(0.1)3$. Here the upper limit for the degree, n, varies from 34 to 450.

No explanatory text accompanies these tables; accordingly, the user should consult the first report for a mathematical discussion of these functions and the various methods used in the preparation of the tables, as well as for additional references.

J. W. W.

1. Henry E. Fettis & James C. Caslin, *Tables of Toroidal Harmonics*, I: *Orders* 0–5, *All Significant Degrees*, Report ARL 69–0025, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, February 1969. (See *Math Comp.*, v. 24, 1970, p. 489, RMT 36.)

71 [7].—K. A. Karpov & E. A. C͡histova, *Tablit͡sy Funkt͡siĭ Vebera*, Tom III (*Tables of Weber Functions*, v. III), Computing Center, Acad. Sci. USSR, Moscow, 1968, xxiv + 215 pp., 27 cm. Price 2.05 rubles.

Weber functions are defined as solutions of the differential equation

$$\frac{d^2y}{dz^2} + (p + \tfrac{1}{2} - \tfrac{1}{4}z^2)y = 0. \tag{1}$$

Whittaker's solution $D_p(z)$ of (1) may be defined by the initial values

$$D_p(0) = \frac{2^{p/2}\sqrt{\pi}}{\Gamma\left(\frac{1-p}{2}\right)}, \qquad D_p'(0) = -\frac{2^{p/2}(2\pi)^{1/2}}{\Gamma\left(-\frac{p}{2}\right)}$$

and is characterized by the asymptotic behavior

$$D_p(z) \sim e^{-z^2/4}z^p \quad \text{as } z \to \infty \text{ in } |\arg z| < \pi/2.$$

If p is not an integer, then $D_p(z)$, $D_p(-z)$ and $D_{-p-1}(iz)$, $D_{-p-1}(-iz)$ are pairs of linearly independent solutions of (1).

The function $D_p(z)$ for real p and $z = x(1 + i)$, x real, has been tabulated in two earlier volumes [1], [2]. The present volume tabulates $D_p(z)$ for z real and purely imaginary, and p real, and completes the tabulation of Weber functions undertaken by the Computing Center of the U.S.S.R. Academy of Sciences.

There are three principal tables in the present volume. The first gives $D_p(x)$ for $0 \leqq x < \infty$; the second, $\exp(-x^2/4)D_p(x)$ for $-\infty < x \leqq 0$; and the third, the real and imaginary parts of $\exp(-x^2/4)D_p(ix)$ for $0 \leqq x < \infty$. The tabular interval in x is 0.01 for $|x| \leqq 5$, and 0.001, or 0.0001, in $y = 1/x$ for $|x| > 5$. The range in p is $-1(0.1)1$ throughout, but can be extended with the aid of recurrence relations. All tabular entries are given to 7D, if less than 1 in absolute value; otherwise they are given to 8S.