

The introduction contains detailed comments on interpolation and on methods for extending the tabular range. Many worked examples are included, as well as auxiliary tables. Also included are eight graphs and three reliefs illustrating the behavior of the functions tabulated.

W. G.

1. I. E. KIREVA & K. A. KARPOV, *Tablitsy Funktsii Vebera*, v. I, Computing Center, Acad. Sci. USSR, Moscow, 1959. Edition in English, Pergamon Press, New York, 1961. (See *Math. Comp.*, v. 16, 1962, pp. 384-387, RMT 38.)

2. K. A. KARPOV & E. A. CHISTOVA, *Tablitsy Funktsii Vebera*, v. II, Computing Center, Acad. Sci. USSR, Moscow, 1964.

72 [7].—DZH. CH. P. MILLER, *Tablitsy Funktsii Vebera* (J. C. P. MILLER, *Tables of Weber Functions*), Computing Center, Acad. Sci. USSR, Moscow, 1968, cxvi + 143 pp., 27 cm. Price 1.69 rubles.

Weber functions (or parabolic cylinder functions) in Whittaker's standardization are solutions of

$$(1) \quad \frac{d^2 y}{dx^2} - \left(\frac{1}{4}x^2 + a\right)y = 0$$

or solutions of

$$(2) \quad \frac{d^2 y}{dx^2} + \left(\frac{1}{4}x^2 - a\right)y = 0.$$

Although the second equation may be obtained from the first by simultaneous replacement of a by $-ia$ and x by $xe^{i\pi/4}$, it is convenient to consider each equation separately when dealing with the real-variable theory of these equations.

Both equations arise naturally in the solution of Helmholtz's equation upon separation of variables in parabolic cylinder coordinates. They also occur in the asymptotic theory of second-order differential equations with turning points. For special values of a the solutions of (1) are related to the normal error function and its repeated integrals and derivatives.

One owes to J. C. P. Miller [1] a thorough mathematical treatment of Weber functions, covering both equations (1) and (2), and the first attempt at systematic tabulation of the solutions of (2) for real x and a .

The volume under review is a Russian translation of [1] by M. K. Kerimov. All tables and mathematical formulas appear to have been reproduced photographically from the original.

A supplementary section added by the translator contains additional material on Weber functions, mostly of recent origin. In particular, one finds an account of the asymptotic theory of these functions and their zeros as developed by Olver in the late 1950's miscellaneous results such as integral representations, limit relations, addition theorems, infinite series and integrals involving Weber functions, as well as a survey of recent tables and computer programs. The bibliography of cited references contains some 200 items.

W. G.

1. NATIONAL PHYSICAL LABORATORY, *Tables of Weber Parabolic Cylinder Functions*, Computed by Scientific Computing Service Limited, Mathematical Introduction by J. C. P. Miller, Editor. Her Majesty's Stationery Office, London, 1955. (See *MTAC*, v. 10, 1956, pp. 245–246, RMT 101.)

73 [7].—A. R. KERTIS, *Volnovye Funktsii Kulona* (A. R. CURTIS, *Coulomb Wave Functions*), Computing Center, Acad. Sci. USSR, Moscow, 1969, li + 209 pp., 27 cm. Price 2.23 rubles.

This is a translation into Russian of the Royal Society Mathematical Tables of Coulomb wave functions [1]. The original has been reviewed in this journal (v. 19, 1965, pp. 341–342, RMT 46).

The tables, as well as all formulas and mathematical characters contained in the preface, appear to have been reproduced photographically from the original. A few minor blemishes in the original printing have been corrected. According to the editor, all tabular entries were checked by differencing, and no errors were found.

The translator, M. K. Kerimov, has added a supplementary section in which he gives further relationships for the Coulomb wave functions (in the notation of the NBS tables [2]) and a comprehensive survey of published tables in the field.

W. G.

1. A. R. CURTIS, *Coulomb Wave Functions*, Royal Society Mathematical Tables, Volume 11, Cambridge Univ. Press, New York, 1964.

2. NATIONAL BUREAU OF STANDARDS, *Tables of Coulomb Wave Functions*, Volume I, Applied Mathematics Series, No. 17, U. S. Government Printing Office, Washington, D. C., 1952.

74 [7].—ANNE E. RUSSON & J. M. BLAIR, *Rational Function Minimax Approximations for the Bessel Functions $K_0(x)$ and $K_1(x)$* , Report AECL-3461, 1969, Atomic Energy of Canada Limited, Chalk River, Ontario. Price \$1.50.

Consider

$$x^{-r} \left[K_r(x) + (-1)^r \ln x I_r(x) - \frac{r}{x} \right] = F_r(x^2),$$

$$x^{-r} I_r(x) = G_r(x^2),$$

$$x^{1/2} e^z K_r(x) = H_r(z), \quad z = x^{-1},$$

where $r = 0$ or 1 . Let $F_r(x^2)$ and $G_r(x^2)$ be approximated by $P_n(x^2)/Q_m(x^2)$ where $P_n(x^2)$ and $Q_m(x^2)$ are polynomials in x^2 of degree n and m respectively. For the range $0 \leq x \leq 1$, the coefficients in these polynomials corresponding to the 'best' approximation in the Chebyshev sense are tabulated for $m = 0$, $n = 1(1)8$, $m = 1$, $n = 2(1)6$ and $m = 3$, $n = 3, 4$. Define precision as $P = -\log |\text{maximum error in the range}|$. Then P ranges from about 3 to 23. Similarly coefficients for the 'best' Chebyshev approximation for $H_r(x)$ in the form $P_n(z)/Q_m(z)$ are given for the range $0 \leq z \leq 1$, where $m = 1(1)12$, $n = m - 1$ and $n = m$ if $r = 0$; and where $m = 1(1)12$, $n = m$ and $n = m + 1$ if $r = 1$. Again P ranges from about 3 to 23.

Y. L. L.