75[7].—D. A. TAGGART & F. W. SCHOTT, *Mathematical Tables of Integrals Involving Spherical Bessel Functions*, Electrical Sciences and Engineering Department, School of Engineering and Applied Science, University of California, Los Angeles, California, ms. of 16 typewritten pp. deposited in the UMT file.

Let $j_n(r)$ be the usual notation for the spherical Bessel function of order n. Herein are given six one-page tables, respectively, of 5S values (in floating-point form) of the integrals:

$$I_1 = \int_0^{r_0} j_t(ar)j_n(br)\,dr,$$

$$I_2 = \int_0^{r_0} \frac{\partial}{\partial r}[rj_t(ar)]\frac{\partial}{\partial r}[rj_n(br)]\,dr,$$

$$I_3 = \int_0^{r_0} j_t(ar)\frac{\partial}{\partial r}[rj_n(br)]\,dr,$$

$$I_4 = \int_0^{r_0} rj_t(ar)j_n(br)\,dr,$$

$$I_5 = \int_0^{r_0} rj_t(ar)\frac{\partial}{\partial r}[rj_n(br)]\,dr,$$

$$I_6 = \int_0^{r_0} r^2j_t(ar)j_n(br)\,dr.$$

Here $ar_0 = \mu_{np}$, the pth zero of $j_n(ar)$, and p is limited to 1, 2, and 3 throughout the tables. Also, these zeros are tabulated to 4 or 5S.

In the first three tables the orders t and n are restricted to 2 and 4. In Tables 4 and 5, $n = 2$ and 4, whereas $t = 1$, 3, and 5. Finally, in Table 6, $n = 1(1)5$ and $t = 1$, 3, and 5.

In the prefatory text the authors describe the computational procedure followed in the preparation of these tables, including several checks on the accuracy of the results to 5S.

Related tables to which the authors refer are those of Butler & Pohlhausen [1].

Y. L. L.

1. T. BUTLER & K. POHLHAUSEN, *Tables of Definite Integrals Involving Bessel Functions of the First Kind*, WADC Technical Report 54–420, Wright Air Development Center, 1954. (See *MTAC*, v. 9, 1955, p. 79, RMT 50.)

76[7].—HENRY E. FETTIS & JAMES C. CASLIN, *A Table of the Complete Elliptic Integral of the First Kind for Complex Values of the Modulus*, Part I, Report ARL 69–0172, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, December 1969, iv + 298 pp., 27 cm. [Released to the Clearinghouse, U. S. Department of Commerce, Springfield, Virginia 22151.] and

HENRY E. FETTIS & JAMES C. CASLIN, *A Table of the Complete Elliptic Integral of the First Kind for Complex Values of the Modulus*, Part II, Report ARL 69–0173, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, 1969, iv + 250 pp., 27 cm. [Released to the Clearinghouse, U. S. Department of Commerce, Springfield, Virginia 22151.]

These reports tabulate

$$K(k) = \int_0^{\pi/2} (1 - k^2 \sin^2 \varphi)^{-1/2}\, d\varphi, \qquad k = Re^{i\theta},$$

$$K(k') = K'(k) \quad \text{and} \quad iK'(k)/K(k), \qquad k' = (1 - k^2)^{1/2}$$

to 11D for $\theta = 0(1°)90°$ and $R = 0(0.01)1.0$. In Part I, the tables are arranged with θ as the parameter and R as the variable while in Part II, the tables are arranged with R as the parameter and θ as the variable. The introductory portion to Part I gives a discussion of the complete elliptic integrals of the first and second kinds, their relation to the Gaussian hypergeometric function and Legendre functions, and other properties such as analytic continuation, Jacobi's nome, and the Gauss or Landen transformation formulas. The latter were employed to produce the tables. This introduction is not given in Part II.

An errata sheet is included for Part I. This has to do with the introductory comments noted above. There is also an errata page for Part II which calls attention to the fact that the column of numbers headed by K'/K should be headed by iK'/K.

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77 [8].—U. NARAYAN BHAT & IZZET SAHIN, *Transient Behavior of Queueing Systems $M/D/1$, $M/E_k/1$, $D/M/1$ and $E_k/M/1$-Graphs and Tables*, Technical Memorandum No. 135, Department of Operations Research, Case Western Reserve University, Cleveland, Ohio, January 1969, iii + 323 pp., 28 cm. One copy deposited in the UMT file.

This report presents tables [to 5D] and graphs for the imbedded Markov-chain behavior of the infinite-source, single-server queueing systems with (i) Poisson arrivals and constant service time ($M/D/1$); (ii) Poisson arrivals and Erlangian service times ($M/E_k/1$), with $k = 1(1)5(5)15$; (iii) regular arrivals and exponential service times ($D/M/1$); and (iv) Erlangian arrivals and exponential service times ($E_k/M/1$), with $k = 1(1)5(5)15$. The characteristics considered are the busy-period distribution and its mean, transition probabilities and the time-dependent mean and variance of queue length, effective steady-state and time-dependent measures of utilization and effectiveness.

The graphs and tables follow a discussion of the need for the construction of such tables in the study of the behavior of queueing systems and some details of the method employed in constructing the tables.

AUTHORS' SUMMARY