

where the  $a_i(p)$  are known coefficients and  $\sigma_{n,p}(x)$  is the converging factor. This may itself be expanded in a series of the form

$$\sigma_{n,p}(x) \sim \sum_{s=0}^{\infty} b_s(n, p) C_{n-p}^{(s)}(2x) (-2x)^s.$$

It is shown that the function  $C_{n-p}(2x)$  is closely related to the converging factor for the probability integral, considered by Murnaghan in [2], where  $C_m(m)$  is tabulated to 63D for  $m = 2(1)64$ .

It is advantageous to choose a value of  $n$  such that  $2x$  is close to  $n - p$ ; then, writing  $m = n - p$  and  $2x = m + h$ ,  $C_m(2x)$  may be computed from the Taylor series

$$C_m(m + h) = C_m(m) + d_1(m)h + d_2(m)h^2 + \dots$$

An appendix to the report contains 30D values of  $C_m(m)$  and its reduced derivatives  $d_i(m)$  for  $m = 10(1)40$ . Once  $C_m(2x)$  is known, its reduced derivatives  $C_m^{(s)}(2x)$  can be calculated with the aid of a three-term recurrence relation, and hence the series for  $\sigma_{n,p}(x)$  can be summed.

As examples of the use of the procedure described, the values of  $K_0(2\pi)$  and  $K_1(10)$  are evaluated to an accuracy of approximately 17D and 26D, respectively. This represents a substantial improvement on the accuracy of 10D and 14D, respectively, obtainable from the asymptotic series without the converging factor.

Attention may be drawn to an alternative method of computation; namely, the use of a Chebyshev series representation, which appears less laborious than the foregoing and permits higher accuracy to be attained over a considerably extended range of the argument. (Indeed, there is no theoretical limit to the attainable accuracy.) The corresponding coefficients are easily calculated by backward recurrence. In particular, Luke [3] has tabulated to 20D the coefficients for the auxiliary function  $(2x/\pi)^{1/2} e^x K_p(x)$  as a function of the reciprocal argument  $5/x$  in the range  $x \geq 5$ , for  $p = 0$  and 1 and several fractional values of  $p$ ; in each case approximately 20S are obtainable throughout the relevant range with the use of 21 terms.

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1. F. D. MURNAGHAN & J. W. WRENCH, JR., *The Converging Factor for the Exponential Integral*, DTMB Report 1535, David Taylor Model Basin, Washington, D.C., January 1963.
2. F. D. MURNAGHAN, *Evaluation of the Probability Integral to High Precision*, DTMB Report 1861, David Taylor Model Basin, Washington, D.C., July 1965.
3. Y. L. LUKE, *The Special Functions and Their Approximations*, Vol. II, Academic Press, New York, 1969.

**8[9].**—HANS RIESEL, *En Bok om Primtal*, Studentlitteratur, Denmark, 1968, 174 pp., (Swedish), 23 cm. Price \$6.95 equivalent. (Paperback.)

This monograph on prime numbers will be of special interest to readers of *Math. Comp.* since its number theory is very close to that which appears here. In fact, many of its references are to papers that have appeared here.

There are four chapters and five appendices, the titles of which (translated into English) are: I, "The exact number of primes less than a given limit." II, "The approximate number of primes." III, "How to identify large primes." IV, "Factorization." 1, "Some algebraic theorems." 2, "Some number theoretic theorems." 3, "Quadratic residues." 4, "Arithmetic of quadratic fields." 5, "Algebraic factors of  $a^n \pm b^n$ ."

Chapter I deals in such topics as Meissel's formula and Lehmer's formula. Chapter II discusses distribution questions: the Gauss, Legendre, and Riemann approximations; the remainder term in the prime number theorem; twins, large gaps, etc. The last two chapters are concerned with primality tests and factorization methods. The appendices have background material.

There are 12 extensive tables.

1. All primes  $p < 10^4$ .
2. All primes  $10^n < p < 10^n + 1000$  for  $n = 4(1)15$ .
3.  $\pi(x)$  and the Gauss and Riemann approximations to  $x = 10^{10}$ .
4. Composites  $104395289 < c < 2 \cdot 10^8$ , satisfying  $2^{c-1} \equiv 1 \pmod{c}$  and having no prime divisor  $\leq 317$ .
5. Known factors of Fermat numbers.
6. Complete factorization of  $2^n - 1$ . All  $n < 137$  and others  $\leq 540$ .
7. Factors of  $(10^n - 1)/9$  and  $10^n + 1$ .
8. Primes of form  $h \cdot 2^n + 1$  for  $h = 1(2)99$ . [Note,  $h = 1, n = 8$  is missing.]
9. Primes of form  $h \cdot 2^n - 1$  for  $h = 1(2)151$ .
10. Primes of form  $n^4 + 1$  for  $n \leq 4002$ .
11. Miscellaneous.
12. Quadratic residues: for square-free  $a$ ,  $|a| < 100$ , all  $k$  and  $l$  such that primes  $p = kx + l$  have  $(a | p) = +1$ .

An English translation may be published in the future, but even readers with no Swedish will be able to grasp much of the present text. Try this:

SATS B1.3 Om  $A$  är element i en ändlig grupp  $G$  med  $n$  element,  
 är  $A^n = I$ .

The tables are in Arabic numerals and will cause no difficulty at all.

D. S.

**9[10].**—JOHN LEECH, Editor, *Computational Problems in Abstract Algebra*, Pergamon Press, Ltd., Oxford, 1970, x + 402 pp., 23 cm. Price \$18.50.

This book consists of thirty-five papers, most of which were presented at a conference on the use of computers in solving problems in algebra, held in 1967 at the University of Oxford under the auspices of the Science Research Council Atlas Computer Laboratory.

Over one-half of the papers are concerned with the application of computers to problems in group theory. The balance of the book includes papers on the use of computers in such areas as word problems in universal algebras, nonassociative algebras, latin squares, Galois theory, knot theory, algebraic number theory, algebraic topology and linear algebra. The first paper, a survey of the methods used and the