chapter, where one finds not only a formulation of the basic problems and methods of the calculus of variations, but also a detailed discussion of how to construct a variational problem from a given boundary-value, eigenvalue, or integral equations problem, and a thorough exposition of direct methods (due to Ritz, Galerkin, Friedrichs, Trefftz, Synge, and others) for solving variational problems. The remaining three chapters are largely method-oriented, but draw frequently upon the problems discussed earlier for illustration. Chapter VI begins with closed-form solutions by means of series (power series, orthogonal and asymptotic expansions), and then illustrates some general principles and approaches toward the numerical treatment of problems. The method of finite differences for differential equations, and the quadrature method for integral equations, of course, hold a central position in the context of this section, and are therefore discussed very thoroughly in Chapter VII. Iteration methods, finally, are the subject of Chapter VIII, which contains general contraction and fixed point theorems as well as a formulation of Newton's method and the method of false position in a Banach space.

W. G.

16[2.05, 2.10, 2.15, 2.20, 2.35, 2.40, 2.55, 3, 4].—CHARLES B. TOMPKINS & WALTER L. WILSON, JR., *Elementary Numerical Analysis*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1969, xvi + 396 pp., 24 cm. Price $10.50.

The book is directed to a wide audience of beginning students and offers them a sound introduction into the techniques and underlying philosophies of numerical analysis. A commendable effort has been made to motivate all subjects discussed, and to emphasize general principles involved. While the selection of topics is fairly standard, it is an unusual feature of the book that all formulas are displayed in a one-line format not unlike that of present-day computer outputs. Table of contents: 1. Introduction, 2. Taylor's formula: truncation error, 3. Iteration processes: Newton's method, 4. Systems of linear equations, 5. Eigenvalues and eigenvectors, 6. Finite differences, 7. Interpolation, 8. Least squares estimates, 9. Numerical differentiation, 10. Numerical integration, 11. Difference equations, 12. Numerical solution of differential equations.

W. G.

17[2.05, 7].—GÉZA FREUD, *Orthogonale Polynome*, Birkhäuser Verlag, Basel, Switzerland, 1969, 294 pp., 25 cm. Price Sfr. 42.00.

To indicate the general character of this important book and how it relates to earlier monographs on the subject, it is best to quote (in free translation) from the author's preface.

"This book is concerned with the general theory of orthogonal polynomials relative to a nonnegative measure on the real line. For prerequisites, it is assumed that, beyond the usual basic analysis courses, the reader has completed an introductory course in real analysis. Only the last chapter requires some knowledge of complex analysis. I hope to offer something useful to every reader, regardless of whether he is interested in applications of finished results, in lecture material, or in further research in the subject. To the expert, too, I hope to say much which is new to him.

"Since the appearance of G. Szegö's monograph, thirty years have passed. In these three decades, his excellent book has served as a guiding line for further research. The second edition of Szegö's work, published in 1959, contained relatively few additions. More recent publications, such as the books of F. Tricomi and G. Sansone, as well as the relevant portions of the 'Bateman Project', are mostly concerned with special orthogonal polynomials. The monograph of Ja. L. Geronimus treats, exclusively, Szegö's theory. An up-to-date survey of the general theory of orthogonal polynomials was not available. I hope, with my book, to be able to fill this gap. By 'general theory', we mean that all the results are derived from the two facts that we are dealing with polynomials, and that the sequence of these polynomials forms an orthogonal system relative to a given measure. I hope to convince the reader that in the framework of this general theory, it is possible to prove many theorems concerning special orthogonal polynomials (e.g., on the convergence of interpolation processes and series of orthogonal polynomials) much simpler, and in a logically more transparent manner, than by considering these polynomials as special functions.

"I was not satisfied with merely grouping together new theorems, but attempted to provide a new framework for the whole theory. In this framework, a whole chapter was devoted to the moment problem (in a form free of continued fractions). To the great classical investigators of orthogonal polynomials, like Chebyshev and Stieltjes, the close connection between the moment problem and orthogonal polynomials was still self-evident. This connection was reinforced also through more recent investigations in this century. It suffices to point to the beautiful theorem of M. Riesz characterising all measures $d\alpha$ for which the orthogonal polynomials are complete in $L^2_{d\alpha}$. However, I penetrated into the theory of the moment problem only as far as seemed useful to the applications in this book. The exposition was simplified with the aid of some results of my own."

The book has five chapters. Chapter I develops the basic properties of orthogonal polynomials $p_n(d\alpha; x)$ relative to a measure $d\alpha$. Many results, such as the Gauss-Jacobi quadrature formula and the Markov-Stieltjes inequalities, are formulated in terms of the more general "quasi-orthogonal" polynomials of M. Riesz, $\psi_n(x, \xi) = p_{n-1}(\xi)p_n(x) - p_n(\xi)p_{n-1}(x)$, where ξ is an arbitrary real number. The chapter also contains a brief account of Chebyshev and Legendre polynomials and some elementary estimates for orthogonal polynomials. Chapter II is devoted to the Hamburger-Stieltjes moment problem. It begins with Hamburger's fundamental result on the solubility of the moment problem, and then proceeds to develop the uniqueness theory due to Hamburger, M. Riesz, and others. Related questions of one-sided approximation by polynomials, and the completeness in $L^2_{d\alpha}$ of orthogonal polynomials, are also discussed. Chapter III first treats the convergence of interpolatory quadrature formulas and the convergence in $L^2_{d\alpha}$ of interpolation polynomials, when the nodes are zeros of orthogonal polynomials, or zeros of certain related polynomials. There are also results on pointwise and uniform convergence. For measures $d\alpha$ with support in $[-1, 1]$, there follows a discussion of the behaviour of $p_n(d\alpha; z)$ for large n and for z in the complex plane cut along the segment $[-1, 1]$. One finds, in particular, the result of P. Erdös and P. Turán, according to which $\lim_{n\to\infty} [p_n(d\alpha; z)]^{1/n} = z + (z^2 - 1)^{1/2}$, if the support of $d\alpha$ coincides with $[-1, 1]$ and

$\alpha'(x) > 0$ a.e. in $[-1, 1]$. This is then applied to discuss interpolation of analytic functions at the zeros of orthogonal polynomials. The chapter concludes with a result on the equidistribution of zeros (more precisely, their projections on the unit circle) of orthogonal polynomials. In Chapter IV, the author then turns his attention to the convergence and summability theory of orthogonal series, assuming measures with support in $[-1, 1]$. The final Chapter V presents Szegö's theory, i.e., the theory of orthogonal polynomials on the unit circle, and contains further important asymptotic results on orthogonal polynomials, Christoffel numbers, and the distance between consecutive zeros.

Each chapter is followed by a collection of exercises, which form an integral part of the book. These in turn are followed by historical notes. In an epilogue, the author points toward certain parts of the theory which are not as yet completely developed and lists a series of important open problems. This should be especially valuable for the young mathematician seeking research problems in the area of orthogonal polynomials.

It is impossible, in a brief review, to convey the extraordinary wealth and beauty of the results presented. Any reader who seriously studies this book will find his efforts richly rewarded.

It is only to be regretted that the printing is not up to the high standards one has come to expect from the publisher and that there are a disturbing number of typographical errors.

W. G.

18[2.35].—J. M. Ortega & W. C. Rheinboldt, *Iterative Solution of Nonlinear Equations in Several Variables*, Academic Press, New York, 1970, xx + 572 pp., 24 cm. Price \$24.00.

This mature presentation is said by the authors to be the outgrowth of their research and graduate teaching over the last five years. Their "aim is to present a survey of the basic theoretical results about nonlinear equations in n dimensions as well as an analysis of the major iterative methods for their numerical solution—to provide here a text for graduate numerical analysis courses—to make the work useful as a reference source".

The authors succeed admirably. They supply numerous exercises to extend the text, many with references to research articles by other workers. Another outstanding feature is the addition of a supplement, called "Notes and Remarks", to each section. This auxiliary material gives pertinent literature citations, and valuable extensions of the text which give the reader a feeling for the state of our current knowledge in this field. They supply a two page annotated list of basic reference texts and a thirty-five page comprehensive bibliography.

The authors deal almost exclusively with methods that involve, at most, first-order derivatives. They divide the book into five parts, with each part containing two or more sections.

Part I—Background Material—contains:

Section 1—Sample Problems—an interesting collection of problems from ordinary