

respectively. The functions are interpolatable in the regions tabulated, and second central differences are provided.

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31[9].—ALAN FORBES & MOHAN LAL, *Tables of Solutions of the Diophantine Equation  $x^2 + y^2 + z^2 = k^2$* , Memorial University of Newfoundland, St. John's, Newfoundland, Canada, July 1969, x + 200 pp.

Table 2 lists all solutions  $0 < x \leq y \leq z$  for all  $k = 3(2)701$ . Table 1 lists the number of such solutions, for each  $k$ , and the number of primitive solutions. These tables are an extension of an earlier table [1] which went to  $k = 381$ . (See the earlier review for more detail.)

The introduction here reports a few errors in the earlier table [1].

In the earlier review I noted that (empirically) if  $k$  is a prime  $p$ , written as  $8n \pm 1$  or  $8n \pm 5$ , then there are exactly  $n$  solutions here. Here is a proof: By Gauss, (see *History of the Theory of Numbers* by L. E. Dickson, Vol. 2, Chapter VII, Item 20) the number of proper (that is, primitive) solutions of  $m \equiv 1 \pmod{8}$  as

$$m = x^2 + y^2 + z^2,$$

counting all possible permutations and changes of sign, and allowing  $x$ ,  $y$ , or  $z$  to be 0, is

$$3 \cdot 2^{\mu+2} H,$$

where  $m$  is divisible by  $\mu$  primes, and  $H$  is the number of properly primitive classes of binary quadratic forms of determinant  $-m$  that are in the principal genus. For  $m = p^2$ , this becomes

$$(1) \quad 6(p - (-1/p))$$

proper solutions.

Each solution

$$(2) \quad p^2 = 0^2 + x^2 + y^2$$

is counted 24 times by Gauss, but is omitted here. Each solution

$$(3) \quad p^2 = x^2 + x^2 + y^2$$

is counted 24 times by Gauss and once here. Each solution

$$p^2 = x^2 + y^2 + z^2$$

is counted 48 times by Gauss and once here. Now examine

$$p = 8n \pm 1 \quad \text{and} \quad p = 8n \pm 5$$

separately, and allowing for the value of  $(-1/p)$  in (1), and whether representations (2) and (3) do or do not exist, one finds that the  $6(p - (-1/p))$  counts of Gauss become a count of  $n$  here in all four cases. Neat.

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1. MOHAN LAL & JAMES DAWE, *Tables of Solutions of the Diophantine Equation  $x^2 + y^2 + z^2 = k^2$* , Memorial University of Newfoundland, St. John's, Newfoundland, Canada, February 1967. (See *Math. Comp.*, v. 22, 1968, p. 235, RMT 23.)