

## Miniaturized Tables of Bessel Functions. II\*

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**Abstract.** In a previous study, we discussed the expansion of two-parameter functions in a double series of Chebyshev polynomials, and, in particular, we presented coefficients for the evaluation of the modified Bessel function  $(2z/\pi)^{1/2}e^g K_\nu(z)$  to 20 decimals for all  $z \geq 5$  and all  $\nu, 0 \leq \nu \leq 1$ . In the present study, we give similar coefficients for the evaluation of  $ge^{-g}z^{-\mu}I_\nu(z)$  to at least 20 decimals where  $I_\nu(z)$  is the modified Bessel function of the first kind and  $g$  and  $\mu$  are certain constants which depend on the range of the parameter and variable for four different situations. The ranges are (1)  $0 < z \leq 8, 0 \leq \nu \leq 4$ ; (2)  $0 < z \leq 8, 4 \leq \nu \leq 8$ ; (3)  $z \geq 8, -1 \leq \nu \leq 0$ ; (4)  $z \geq 8, 0 \leq \nu \leq 1$ .

**1. Introduction.** In a previous study [1], we discussed the expansion of two-parameter functions in a double series of Chebyshev polynomials, and, in particular, we presented coefficients for the evaluation of the modified Bessel function  $(2z/\pi)^{1/2} \times e^g K_\nu(z)$  to 20 decimals for all  $z \geq 5$  and all  $\nu, 0 \leq \nu \leq 1$ . Since  $K_\nu(z) = K_{-\nu}(z)$  and  $K_\nu(z)$  satisfies a three-term recurrence formula which is stable in the forward direction, we have in essence coefficients for the evaluation of  $K_\nu(z)$  for all  $z \geq 5$  and all  $\nu \geq 0$ .

In the present study, we give similar coefficients for the evaluation of  $ge^{-g}z^{-\mu}I_\nu(z)$  to at least 20 decimals where  $I_\nu(z)$  is the modified Bessel function of the first kind and  $g$  and  $\mu$  are certain constants which depend on the range of the parameter and variable for four different situations as follows.

	<i>z range</i>	<i>ν range</i>	<i>μ</i>	<i>g</i>
(1)	$0 < z \leq 8$	$0 \leq \nu \leq 4$	$\nu$	1
(2)	$0 < z \leq 8$	$4 \leq \nu \leq 8$	$\nu$	1
(3)	$z \geq 8$	$-1 \leq \nu \leq 0$	$-\frac{1}{2}$	$(2\pi)^{-1/2}$
(4)	$z \geq 8$	$0 \leq \nu \leq 1$	$-\frac{1}{2}$	$(2\pi)^{-1/2}$

The recursion formula for  $I_\nu(z)$  is always stable in the backward direction but only conditionally stable in the forward direction. Thus, even with the coefficients given here, we still lack coefficients to compute  $e^{-g}I_\nu(z)$  for all real  $z$  and for  $\nu$  sufficiently large. A study to correct this deficiency is under way and will be reported at a later date.

**2. Chebyshev Expansions for  $I_\nu(z)$ .** In [2, Vol. 2, pp. 338–340, 359–367], we gave coefficients for the expansion of  $z^{-\nu}I_\nu(z)$  in series of Chebyshev polynomials for

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$0 < z \leq 8$ ,  $\nu = 0, \pm\frac{1}{4}, \pm\frac{1}{3}, \pm\frac{1}{2}, \pm\frac{2}{3}, \pm\frac{3}{4}, 1$ , and, similarly, for the expansion of  $(2\pi z)^{-1/2}e^{-z}I_\nu(z)$  for  $z \geq 8$ ,  $\nu = 0, \frac{1}{4}, \frac{1}{3}, \frac{2}{3}, \frac{3}{4}, 1$ . The coefficients for the range  $0 < z \leq 8$  are based on the  ${}_0F_1$  representation for  $I_\nu(z)$  which does not directly reflect the fact that for fixed  $\nu$ ,  $I_\nu(z)$  grows exponentially with  $z$  as  $z$  increases in the sector  $|\arg z| < \pi/2$ . Now,  $I_\nu(z)$  has a representation in terms of a  ${}_1F_1$  which does reflect this exponential behavior and, in this present paper, development of the desired coefficients is based on this representation for  $0 < z \leq 8$ . The representation used in the cited reference for  $z \geq 8$  is also used in our present study to derive the desired coefficients already noted.

From [2, Vol. 1, p. 213], we have

$$(1) \quad z^{-\nu}e^{-z}I_\nu(z) = [2^\nu\Gamma(\nu + 1)]^{-1} {}_1F_1(a; c; -2z),$$

$$a = c/2 = \frac{1}{2} + \nu.$$

In general, from [2, Vol. 2, p. 35],

$$(2) \quad {}_1F_1(a; c; z) = \sum_{k=0}^{\infty} G_k(a, c, \lambda) T_k^*(z/\lambda),$$

$$(3) \quad G_k(a, c, \lambda) = \frac{\epsilon_k(a)_k \lambda^k}{2^{2k} (c)_k k!} {}_2F_2\left(a + k, \frac{1}{2} + k \mid c + k, 1 + 2k \mid \lambda\right),$$

$$(4) \quad \frac{2G_k(a, c, \lambda)}{\epsilon_k} = \frac{(k+1)}{(k+a)} \left\{ -\frac{(k+3-a)}{(k+2)} + \frac{4(k+c)}{\lambda} \right\} G_{k+1}(a, c, \lambda)$$

$$+ \frac{2}{(k+a)} \left\{ \frac{1}{2}(k+a) + \frac{2(k+1)(k+3-c)}{\lambda} \right\} G_{k+2}(a, c, \lambda)$$

$$+ \frac{(k+1)(k+3-a)}{(k+2)(k+a)} G_{k+3}(a, c, \lambda).$$

In the above,

$$(5) \quad \epsilon_k = 1 \quad \text{if } k = 0, \quad \epsilon_k = 2 \quad \text{if } k > 0.$$

Using [2, Vol. 1, p. 244], we find that for  $a, c$  and  $\lambda$  fixed,

$$(6) \quad G_k(a, c, \lambda) = \frac{\Gamma(c)(\lambda/4)^k k^{a-c}}{\Gamma(a)k!}$$

$$\cdot \left[ 1 + \frac{\lambda^2 - 8\lambda(c-a) - 8(c-a)(c+a-1)}{16k} + O(k^{-2}) \right].$$

Thus, the expansion formula (2) converges and since the  ${}_1F_1$  in (2) is one when  $z = 0$ , it follows that

$$(7) \quad \sum_{k=0}^{\infty} (-)^k G_k(a, c, \lambda) = 1.$$

Further, after the manner of the discussion given in [2, Vol. 2, pp. 159-166], we can show that use of the recursion formula (4) in the backward direction is convergent. Thus, for a fixed  $\lambda$ , we can generate the coefficients  $G_k(a, c, \lambda)$  for given values of  $a$  and  $c$ . Suppose for example that  $c$  is fixed and we permit  $a$  to vary. Then, we can find coefficients  $D_{r,k}(c, \lambda)$  such that

$$(8) \quad G_k(a, c, \lambda) = \sum_{r=0}^{\infty} D_{r,k}(c, \lambda) T_r^*(a/\omega), \quad 0 \leq a \leq \omega,$$

and so achieve a double series of Chebyshev polynomials for the evaluation of  ${}_1F_1(a; c; z)$  for  $c$  fixed, valid for  $0 \leq z \leq \lambda$  and  $0 \leq a \leq \omega$ . The manner of getting  $D_{r,k}(c, \lambda)$  has been given in [1] and we omit further details.

Next, we seek a descending-type expansion in series of Chebyshev polynomials for the evaluation of  $I_\nu(z)$  in the neighborhood of  $z = +\infty$ . To this end, we can write [2, Vol. 1, p. 226, Eq. (9)], [2, Vol. 2, p. 22, Eq. (10)],

$$(9) \quad I_\nu(z) = (2\pi z)^{-1/2} e^z F_\nu(z),$$

$$(10) \quad F_\nu(z) = G_{1,2}^{1,1} \left( 2z \left| \begin{matrix} 1 \\ \frac{1}{2} + \nu, \quad \frac{1}{2} - \nu \end{matrix} \right. \right),$$

$$(11) \quad F_\nu(z) = \sum_{k=0}^{\infty} M_k(\nu, \lambda) T_k^*(\lambda/z), \quad \lambda \text{ fixed, } \lambda/z \leq 1, z > 0,$$

$$(12) \quad M_k(\nu, \lambda) = \pi^{-1/2} \epsilon_k (-)^k G_{2,3}^{2,1} \left( 2\lambda \left| \begin{matrix} 1-k, \quad k+1 \\ \frac{1}{2}, \quad \frac{1}{2} + \nu, \quad \frac{1}{2} - \nu \end{matrix} \right. \right),$$

$$M_k(\nu, \lambda) = \pi^{-1/2} \epsilon_k (-)^k G_{3,2}^{1,2} \left( \frac{1}{2\lambda} \left| \begin{matrix} \frac{1}{2}, \quad \frac{1}{2} - \nu, \quad \frac{1}{2} + \nu \\ k, \quad -k \end{matrix} \right. \right),$$

and from [2, Vol. 2, pp. 153, 154 and Remark 1, p. 155], we have the recursion formula

$$(13) \quad \begin{aligned} \frac{2M_k(\nu, \lambda)}{\epsilon_k} &= 2(k+1) \left\{ 1 - \frac{(2k+3)(k+3/2+\nu)(k+3/2-\nu)}{2(k+2)(k+1/2+\nu)(k+1/2-\nu)} \right. \\ &\quad \left. + \frac{8\lambda}{(k+1/2+\nu)(k+1/2-\nu)} \right\} M_{k+1}(\nu, \lambda) \\ &+ \left\{ 1 - \frac{2(k+1)(2k+3+4\lambda)}{(k+1/2+\nu)(k+1/2-\nu)} \right\} M_{k+2}(\nu, \lambda) \\ &- \frac{(k+1)(k+5/2+\nu)(k+5/2-\nu)}{(k+2)(k+1/2+\nu)(k+1/2-\nu)} M_{k+3}(\nu, \lambda), \quad k \geq 0. \end{aligned}$$

Actually, (13) is not valid if  $\nu$  is half an odd integer unless  $k + \frac{1}{2} - \nu > 0$ . If, for example,  $\nu = \frac{1}{2} + n$ , (13) is only valid for  $k > n$ . However, we can get a further relation if first we multiply through by  $k + \frac{1}{2} - \nu$  and then set  $k + \frac{1}{2} - \nu = 0$  for  $k = n$ . In particular, if  $\nu = \frac{1}{2}$ , we have

$$(14) \quad \begin{aligned} \frac{2M_k(\frac{1}{2}, \lambda)}{\epsilon_k} &= \left[ \frac{8\lambda - 3(k+1)}{k} \right] M_{k+1}(\frac{1}{2}, \lambda) - \left[ \frac{8\lambda + 3(k+2)}{k} \right] M_{k+2}(\frac{1}{2}, \lambda) \\ &- \frac{(k+3)}{k} M_{k+3}(\frac{1}{2}, \lambda), \quad k > 0, \end{aligned}$$

$$(15) \quad (8\lambda - 3)M_1(\frac{1}{2}, \lambda) = (8\lambda + 6)M_2(\frac{1}{2}, \lambda) + 3M_3(\frac{1}{2}, \lambda).$$

It can be shown that

$$(16) \quad M_0(\frac{1}{2}, \lambda) = 2\pi^{-1/2} \operatorname{Erf}(x), \quad x^2 = 2\lambda, \quad \operatorname{Erf}(x) = \int_0^x e^{-t^2} dt,$$

$$(17) \quad M_1(\frac{1}{2}, \lambda) = 8\lambda[1 - M_0(\frac{1}{2}, \lambda)] - 4(2\lambda/\pi)^{1/2}e^{-2\lambda}.$$

With  $z \rightarrow +\infty$ , (11) yields the useful normalization equation

$$(18) \quad \sum_{k=0}^{\infty} (-)^k M_k(\nu, \lambda) = 1.$$

From [2, Vol. 2, pp. 23, 24],

$$(19) \quad M_k(\nu, \lambda) \sim k^{-1}[\mu \exp\{-3(2\lambda k^2 e^{i\pi})^{1/3}\} + \nu \exp\{-3(2\lambda k^2 e^{-i\pi})^{1/3}\}]$$

where  $\mu$  and  $\nu$  are constants. The two other linearly independent solutions of (14) can be taken in a form such that they are

$$(20) \quad O(k^{-1} \exp\{3(2\lambda k^2)^{1/3}\}) \quad \text{and} \quad O(k^{-1} \exp\{-3(2\lambda k^2 e^{i\pi})^{1/3}\}).$$

It follows that the desired solution of (14) is not minimal in the sense of Gautschi [3] or not antidominant in the sense of Wimp [4], and consequently the backward recursion process for the evaluation of  $M_k(\nu, \lambda)$  will fail unless modified. The necessary modification is discussed in [2, Vol. 2, pp. 163-164] and studied further in Wimp [4, Theorem 3]. We now describe this procedure.

Let  $N$  be a large positive integer. Put

$$(21) \quad g_{N+k}^{(N)} = 0, \quad k = 2, 3, \dots, \quad g_{N+1}^{(N)} = 1$$

and compute  $g_n^{(N)}$ ,  $n = N, N - 1, \dots, 0$  from (13) with  $M_k(\nu, \lambda)$  replaced by  $g_k^{(N)}$ . (Here we assume that  $\nu$  is not half an odd integer. The case when  $\nu = \frac{1}{2}$  is treated later.) Put

$$(22) \quad M_n^{(N)}(\nu, \lambda) = \rho^{(N)} g_n^{(N)}, \quad n = 0, 1, \dots, N + 1, \quad \rho^{(N)} = \left( \sum_{n=0}^{N+1} (-)^n g_n^{(N)} \right)^{-1}.$$

Let  $N_1, N_2$  be two different  $N$  values. We can find a number  $\mu$  depending on  $N_1$  and  $N_2$  such that

$$(23) \quad \mu \sum_{k=0}^{N_1+1} M_k^{(N_1)}(\nu, \lambda) + (1 - \mu) \sum_{k=0}^{N_2+1} M_k^{(N_2)}(\nu, \lambda) = (2\pi\lambda)^{1/2} e^{-\lambda} I_\nu(\lambda).$$

Then

$$(24) \quad \lim_{N_1 \rightarrow \infty; N_2 \rightarrow \infty; N_1 \neq N_2} [\mu M_k^{(N_1)}(\nu, \lambda) + (1 - \mu) M_k^{(N_2)}(\nu, \lambda)] = M_k(\nu, \lambda),$$

$k = 0, 1, \dots$

If  $\nu$  is half an odd integer, another technique must be used as the process just described breaks down due to the presence of the product  $(k + \frac{1}{2} + \nu)(k + \frac{1}{2} - \nu)$ . To illustrate, consider the case  $\nu = \frac{1}{2}$ . In this event,

$$(25) \quad F_{1/2}(z) = 1 - e^{-2z}.$$

We have need for the three normalization relations

$$(26) \quad 1 - e^{-2\lambda} = \sum_{k=0}^{\infty} M_k(\frac{1}{2}, \lambda),$$

$$(27) \quad 1 = \sum_{k=0}^{\infty} (-)^k M_k(\frac{1}{2}, \lambda),$$

$$(28) \quad 1 - e^{-4\lambda} = \sum_{k=0}^{\infty} (-)^k M_{2k}(\frac{1}{2}, \lambda),$$

which come from (9)–(11) when  $\nu = \frac{1}{2}$  and  $z = 2\lambda, +\infty$  and  $4\lambda$ , respectively.

Again, let  $N$  be a large positive integer, set

$$(29) \quad g_{N+k}^{(N)} = 0, \quad k = 2, 3, \dots, \quad g_{N+1}^{(N)} = 1,$$

and compute

$$g_n^{(N)}, \quad n = N, N - 1, \dots, 1,$$

from (13) with  $M_k(\frac{1}{2}, \nu)$  replaced by  $g_k^{(N)}$ . Let

$$(30) \quad M_k^{(N)} = \rho^{(N)} g_k^{(N)}, \quad k = 1, 2, \dots, \quad M_0^{(N)} = g_0^{(N)}.$$

Then from (26) and (27), respectively, we have

$$(31) \quad g_0^{(N)} + \rho^{(N)} \sum_{k=1}^{\infty} g_k^{(N)} = 1 - e^{-2\lambda},$$

$$(32) \quad g_0^{(N)} + \rho^{(N)} \sum_{k=1}^{\infty} (-)^k g_k^{(N)} = 1.$$

Thus

$$(33) \quad \rho^{(N)} = \frac{-e^{-2\lambda}}{2 \sum_{k=0}^{\infty} g_{2k+1}^{(N)}}$$

and  $g_0^{(N)}$  can be recovered from either (31) or (32). Let  $N_1, N_2$  be two different  $N$  numbers. We can find a number  $\mu$  depending on  $N_1$  and  $N_2$  such that

$$(34) \quad \mu \sum_{k=0}^{N_1+1} (-)^k M_{2k}^{(N_1)}(\frac{1}{2}, \lambda) + (1 - \mu) \sum_{k=0}^{N_2+1} (-)^k M_{2k}^{(N_2)}(\frac{1}{2}, \lambda) = 1 - e^{-4\lambda}.$$

Then

$$(35) \quad \lim_{N_1 \rightarrow \infty; N_2 \rightarrow \infty; N_1 \neq N_2} [\mu M_k^{(N_1)}(\frac{1}{2}, \nu) + (1 - \mu) M_k^{(N_2)}(\frac{1}{2}, \nu)] = M_k(\frac{1}{2}, \lambda),$$

$k = 0, 1, \dots$

The coefficients can be checked using (16) and (17). Alternatively, we can make use of (17) to find a number  $\mu^*$  such that

$$(36) \quad 8\lambda [\mu^* M_0^{(N_1)}(\frac{1}{2}, \lambda) + (1 - \mu^*) M_0^{(N_2)}(\frac{1}{2}, \lambda)] + \mu^* M_1^{(N_1)}(\frac{1}{2}, \lambda) + (1 - \mu^*) M_1^{(N_2)}(\frac{1}{2}, \lambda) = 8\lambda - 4(2\lambda/\pi)^{1/2} e^{-2\lambda}.$$

Then  $M_k(\frac{1}{2}, \lambda)$  follows as in (35) with  $\mu$  replaced by  $\mu^*$  and (28) can be used as a check.

Another scheme to compute  $M_k(\nu, \lambda)$  for  $\nu$  half an odd integer is to use the procedure described by (21)–(24) to get  $M_k(\nu, \lambda)$  for  $\nu$  in the neighborhood of half an odd integer and then employ the Lagrangian interpolation formula.

**3. Numerical Results.** From (1), (2) and (8), with a slight change of notation, we have

$$(37) \quad \begin{aligned} I_\nu(z) &= z^\nu e^z \sum_{k=0}^{\infty} H_k(\nu) T_k^*(z/8), \quad 0 < z \leq 8, \\ H_k(\nu) &= \sum_{r=0}^{\infty} D_{r,k} T_r^*\left(\frac{\nu - s}{t}\right), \quad s \leq \nu \leq s + t. \end{aligned}$$

In Tables 1 and 2 of the microfiche section we present values of  $D_{r,k}$  which were evaluated by the technique described in [1] for  $s = 0, t = 4$  and  $s = t = 4$ , respectively. To develop the numerics, values of  $\Gamma(\nu + 1)$  were required. These were obtained by use of the schema of my previous paper [5]. Numerous checks were made on the coefficients. In addition to those of the kind discussed in [1], checks were also made using the recurrence formula for  $I_\nu(z)$ , namely

$$(38) \quad I_{\nu+1}(z) + \frac{2\nu}{z} I_\nu(z) - I_{\nu-1}(z) = 0.$$

Further checks were accomplished by comparing values deduced from (37) with those computed from power series, especially when  $\nu$  is half an odd integer, for in this instance

$$(39) \quad \begin{aligned} e^{-s} I_{n+1/2}(z) &= (2\pi z)^{-1/2} [A_n(z) + (-1)^{n+1} e^{-2s} A_n(-z)], \\ A_n(z) &= {}_2F_0\left(-n, n + 1; \frac{1}{2z}\right), \end{aligned}$$

and  $A_n(z)$  is a polynomial in  $z^{-1}$  of degree  $n$ . Wronskian relations were also used to get checks. The computations were designed so that the coefficients for  $0 \leq \nu \leq 4$  are accurate to about 25D while those for  $4 \leq \nu \leq 8$  are accurate to about 27D. To evaluate  $e^{-s} I_\nu(z)$ , we must incorporate the value of  $z'$ . As  $0 < z \leq 8$ , we see that the coefficients are sufficiently accurate to produce  $e^{-s} I_\nu(z)$  to about 20 decimals at least.

From (9)–(11) with a slight change of notation we write

$$(40) \quad I_\nu(z) = (2\pi z)^{-1/2} e^z \sum_{k=0}^{\infty} M_k(\nu) T_k^*(8/z), \quad z \geq 8,$$

$$(41) \quad M_k(\nu) = \sum_{r=0}^{\infty} E_{r,k} T_r^*(\nu), \quad 0 \leq \nu \leq 1,$$

$$(42) \quad M_k(\nu) = \sum_{r=0}^{\infty} F_{r,k} T_r^*(-\nu), \quad -1 \leq \nu \leq 0.$$

In Tables 3 and 4 of the microfiche section we give values of  $E_{r,k}$  and  $F_{r,k}$ , respectively. In the development of these coefficients, the appropriate values of  $(2\pi\lambda)^{1/2} e^{-\lambda} I_\nu(\lambda)$ , as required by (23), were obtained from (37) for  $\nu > 0$  and (38) was used to get the values needed for  $\nu < 0$ . Again, the coefficients were subjected to numerous checks. For example, for  $z = 8$ , we compared values of  $I_\nu(z)$ , as obtained from (40)–(42), with those obtained from (37) and (38) when appropriate. We also used the defining relation for  $K_\nu(z)$  in terms of  $I_\nu(z)$  and  $I_{-\nu}(z)$  to compare values obtained using the coefficients in [1] and the coefficients in the present tables. Further checks were gotten by use of a Wronskian relation. The coefficients are sufficiently accurate to enable the computation of  $e^{-s}(2\pi z)^{1/2} I_\nu(z)$  to about 22 decimals.

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TABLE I

Coefficients in the Expansion of

$$I_\nu(z) = z^\nu e^z \sum_{k=0}^{\infty} B_k(\nu) T_k^*(z/\theta) \quad , 0 < z \leq \theta \quad ,$$

$$B_k(\nu) = \sum_{r=0}^k D_{r,k} T_r^*(\nu/4) \quad , 0 \leq \nu \leq 4 \quad .$$

r	$D_{r,k}, k=0$	r	$D_{r,k}, k=1$
0	0.10000	0	-0.11440
1	-0.15449	1	0.14577
2	0.07049	2	-0.05608
3	-0.01573	3	-0.00571
4	-0.00143	4	0.01561
5	0.00234	5	-0.00007
6	-0.00047	6	0.00215
7	-0.00000	7	-0.00024
8	0.00004	8	-0.00008
9	-0.00002	9	0.00004
10	0.00000	10	-0.00000
11	-0.00000	11	0.00000
12	-0.00000	12	0.00000
13	0.00000	13	-0.00000
14	-0.00000	14	0.00000
15	0.00000	15	-0.00000
16	0.00000	16	-0.00000
17	-0.00000	17	0.00000
18	0.00000	18	-0.00000
19	-0.00000	19	0.00000
20	-0.00000	20	0.00000
21	0.00000	21	-0.00000
22	-0.00000	22	0.00000
23	0.00000	23	-0.00000
24	-0.00000	24	0.00000
25	0.00000	25	-0.00000
26	-0.00000	26	0.00000
27	0.00000	27	-0.00000
28	-0.00000	28	0.00000
29	0.00000	29	-0.00000
30	-0.00000	30	0.00000
31	0.00000	31	-0.00000
32	-0.00000	32	0.00000

  

r	$D_{r,k}, k=2$	r	$D_{r,k}, k=3$
0	0.06537	0	-0.03473
1	-0.09344	1	0.05036
2	0.03043	2	-0.01732
3	0.00470	3	-0.00111
4	-0.00049	4	0.00047
5	0.00018	5	-0.00104
6	-0.00002	6	0.00004
7	-0.00000	7	-0.00001
8	0.00000	8	0.00000
9	-0.00000	9	-0.00000
10	0.00000	10	0.00000
11	-0.00000	11	0.00000
12	-0.00000	12	0.00000
13	0.00000	13	-0.00000
14	-0.00000	14	0.00000
15	0.00000	15	-0.00000
16	-0.00000	16	0.00000
17	0.00000	17	-0.00000
18	-0.00000	18	0.00000
19	0.00000	19	-0.00000
20	-0.00000	20	0.00000
21	0.00000	21	-0.00000
22	-0.00000	22	0.00000
23	0.00000	23	-0.00000
24	-0.00000	24	0.00000
25	0.00000	25	-0.00000
26	-0.00000	26	0.00000
27	0.00000	27	-0.00000
28	-0.00000	28	0.00000
29	0.00000	29	-0.00000
30	-0.00000	30	0.00000
31	0.00000	31	-0.00000
32	-0.00000	32	0.00000



TABLE I

Coefficients in the Expansion of

$$I_0(z) + z^2 \sum_{k=0}^{\infty} H_k(v) T_k^0(z/8), \quad 0 < z \leq 8,$$

$$H_k(v) = \sum_{r=0}^{\infty} D_{r,k} T_r^0(v/4), \quad 0 \leq v \leq 4.$$

r	$D_{r,k}, k=0$					r	$D_{r,k}, k=1$				
0	0.10002	05477	75547	19039	05780	0	-0.11440	51619	17406	17973	75444
1	-0.15549	04347	70450	40904	30657	1	0.16577	56409	40000	85457	00304
2	0.07049	04204	38707	11563	32015	2	-0.05008	41091	30002	85267	64311
3	-0.01573	37961	74031	10004	30031	3	-0.00571	85016	00003	26000	17020
4	-0.00163	44704	50590	41078	33452	4	0.01561	61174	07957	07000	63000
5	0.00234	90709	74779	40520	09003	5	-0.00007	71007	05762	66200	47150
6	-0.00067	24194	44417	00053	30000	6	0.00215	07447	70010	95473	44000
7	-0.00000	19017	71147	77963	54507	7	-0.00024	12050	47610	77034	90007
8	0.00006	04500	51055	63432	10100	8	-0.00008	43011	20354	97999	70114
9	-0.00002	70001	60554	43307	10002	9	0.00004	23000	10050	51004	70250
10	0.00000	56374	00619	23015	41454	10	-0.00000	00125	73259	33425	92475
11	-0.00000	03510	01904	42251	74000	11	0.00000	04305	70000	00072	70300
12	-0.00000	01050	72473	90100	94777	12	0.00000	03147	05262	30004	35243
13	0.00000	00647	00755	70013	71274	13	-0.00000	01150	77002	05701	37790
14	-0.00000	00115	90472	75366	04561	14	0.00000	00207	00000	03000	47001
15	0.00000	00000	02013	00005	02307	15	-0.00000	00013	34311	00757	13017
16	0.00000	00001	70506	63511	02552	16	-0.00000	00003	20055	62374	36512
17	-0.00000	00000	70366	07515	74000	17	0.00000	00001	24770	20000	03200
18	0.00000	00000	12100	00029	90009	18	-0.00000	00000	21101	10300	72507
19	-0.00000	00000	01000	14736	00934	19	0.00000	00000	01721	16013	46000
20	-0.00000	00000	00005	30360	74170	20	0.00000	00000	00123	31900	06991
21	0.00000	00000	00037	51300	31000	21	-0.00000	00000	00006	60335	77152
22	-0.00000	00000	00000	00000	77070	22	0.00000	00000	00011	72053	36037
23	0.00000	00000	00000	05210	10367	23	-0.00000	00000	00001	17757	94124
24	-0.00000	00000	00000	00730	10934	24	0.00000	00000	00000	00005	00035
25	-0.00000	00000	00000	01004	03000	25	0.00000	00000	00000	01791	40100
26	0.00000	00000	00000	00700	53257	26	-0.00000	00000	00000	00307	00030
27	-0.00000	00000	00000	00023	13510	27	0.00000	00000	00000	00000	25030
28	0.00000	00000	00000	00001	10000	28	-0.00000	00000	00000	00002	00001
29	-0.00000	00000	00000	00000	11470	29	-0.00000	00000	00000	00000	20004
30	-0.00000	00000	00000	00000	03004	30	0.00000	00000	00000	00000	00001
31	0.00000	00000	00000	00000	00473	31	-0.00000	00000	00000	00000	00026
32	-0.00000	00000	00000	00000	00036	32	0.00000	00000	00000	00000	00007

r	$D_{r,k}, k=2$					r	$D_{r,k}, k=3$				
0	0.06537	70264	55001	64757	62200	0	-0.03673	87570	75000	20759	70051
1	-0.00304	44313	04000	04539	94495	1	0.05036	46711	69150	01900	23050
2	0.03043	72361	04005	54070	00039	2	-0.01732	40316	19273	40002	04720
3	0.00420	27710	07149	64207	57067	3	-0.00111	15623	00076	47523	30160
4	-0.00000	02201	90030	30073	40000	4	0.00007	26423	75360	00772	70000
5	-0.00010	70519	37157	26000	30000	5	-0.00100	19520	01095	44700	40720
6	-0.00002	00210	64305	00566	09100	6	0.00004	72609	01520	70000	45007
7	-0.00002	02014	00045	50501	40000	7	0.00001	44165	62500	40000	45264
8	0.00000	30164	00101	42305	34530	8	-0.00004	45001	01701	21000	03151
9	-0.00003	44374	47713	43071	74103	9	0.00001	50025	54530	40001	07000
10	0.00000	04025	11514	44006	00030	10	-0.00000	27067	00202	71000	17070
11	-0.00000	02754	19022	25002	90505	11	0.00000	00175	12303	52315	10202
12	-0.00000	02261	70200	51001	40003	12	0.00000	01302	40705	24304	70057
13	0.00000	00700	90447	40007	01000	13	-0.00000	00025	10200	00074	51000
14	-0.00000	00133	04097	40014	60100	14	0.00000	00000	00000	00000	30770
15	0.00000	00007	00016	30010	33323	15	-0.00000	00003	20001	22003	30200
16	0.00000	00002	47106	20072	60012	16	-0.00000	00001	41000	27000	70200
17	-0.00000	00000	00013	37024	20100	17	0.00000	00000	00175	00003	04102
18	0.00000	00000	14100	30100	55100	18	-0.00000	00000	00327	03100	45330
19	-0.00000	00000	01002	77514	49700	19	0.00000	00000	00526	14050	50150
20	-0.00000	00000	00097	40039	51350	20	0.00000	00000	00050	01015	32010
21	0.00000	00000	00004	20070	37377	21	-0.00000	00000	00000	00020	00012
22	-0.00000	00000	00007	01000	47300	22	0.00000	00000	00000	14000	40770
23	0.00000	00000	00000	73077	60002	23	-0.00000	00000	00000	17000	07230
24	-0.00000	00000	00000	00190	67073	24	-0.00000	00000	00000	00110	00270
25	-0.00000	00000	00000	01257	01200	25	0.00000	00000	00000	00000	00002
26	0.00000	00000	00000	00200	30032	26	-0.00000	00000	00000	00130	19241
27	-0.00000	00000	00000	00070	73000	27	0.00000	00000	00000	00013	00200
28	0.00000	00000	00000	00001	25700	28	-0.00000	00000	00000	00000	01001
29	0.00000	00000	00000	00000	15113	29	-0.00000	00000	00000	00000	00020
30	-0.00000	00000	00000	00000	04300	30	0.00000	00000	00000	00000	02311
31	0.00000	00000	00000	00000	00550	31	-0.00000	00000	00000	00000	00271
32	-0.00000	00000	00000	00000	00000	32	0.00000	00000	00000	00000	00021

TABLE 1 (Continued)

Coefficients in the Expansion of

$$I_\nu(z) = z^\nu e^z \sum_{k=0}^{\infty} H_k(\nu) T_k^*(z/\theta), \quad 0 < -z \leq \theta,$$

$$H_k(\nu) = \sum_{r=0}^k D_{r,k} T_r^*(\nu/4), \quad 0 \leq \nu \leq 4.$$

r	$D_{r,k}, k=8$	r	$D_{r,k}, k=9$
0	0.00045 30490 60392 50953 47210	0	-0.00015 27076 71415 16786 53414
1	-0.00072 21120 40652 59362 37235	1	0.00024 65959 47464 37370 27854
2	0.00036 42027 37462 74016 92910	2	-0.00013 05455 25175 88622 00277
3	-0.00010 70561 44261 74306 73015	3	0.00004 34437 16751 47791 67696
4	0.00000 72030 70544 13949 06470	4	-0.00000 64009 32251 36770 70525
5	0.00000 41110 10108 44032 07115	5	-0.00000 16679 99675 11259 40333
6	-0.00000 47742 06354 77454 37905	6	0.00000 13495 64003 28062 02905
7	0.00000 11905 05021 10262 45513	7	-0.00000 04144 74371 77974 87651
8	-0.00000 01063 55473 95302 42405	8	0.00000 00026 60064 56239 16701
9	-0.00000 00360 84901 27020 26314	9	0.00000 00029 76744 37574 40071
10	0.00000 00179 60255 66726 44279	10	-0.00000 00042 74512 36696 62306
11	-0.00000 00037 71543 71607 55520	11	0.00000 00011 73040 43200 26170
12	0.00000 00003 20421 80093 54406	12	-0.00000 00001 41154 80033 96179
13	0.00000 00000 60366 69197 01501	13	-0.00000 00000 00323 24404 02172
14	-0.00000 00000 27894 09395 64049	14	0.00000 00000 05735 40514 30946
15	0.00000 00000 05196 55301 25036	15	-0.00000 00000 01452 53067 51012
16	-0.00000 00000 00437 17834 03500	16	0.00000 00000 00106 49993 07151
17	-0.00000 00000 00042 40649 70056	17	-0.00000 00000 00004 30552 16202
18	0.00000 00000 00021 34103 17056	18	-0.00000 00000 00003 74701 07629
19	-0.00000 00000 00003 75004 04307	19	0.00000 00000 00000 93466 51540
20	0.00000 00000 00000 33064 91625	20	-0.00000 00000 00000 11755 37290
21	0.00000 00000 00000 01050 59200	21	0.00000 00000 00000 00501 90000
22	-0.00000 00000 00000 00003 70399	22	0.00000 00000 00000 00120 33713
23	0.00000 00000 00000 00001 25152	23	-0.00000 00000 00000 00014 63263
24	-0.00000 00000 00000 00014 94527	24	0.00000 00000 00000 00004 43657
25	0.00000 00000 00000 00000 19252	25	-0.00000 00000 00000 00000 26299
26	0.00000 00000 00000 00000 20350	26	-0.00000 00000 00000 00000 02100
27	-0.00000 00000 00000 00000 00003	27	0.00000 00000 00000 00000 00704
28	0.00000 00000 00000 00000 00419	28	-0.00000 00000 00000 00000 00105
29	-0.00000 00000 00000 00000 00010	29	0.00000 00000 00000 00000 00000
30	-0.00000 00000 00000 00000 00002		

  

r	$D_{r,k}, k=10$	r	$D_{r,k}, k=11$
0	0.00004 80625 20160 41913 71776	0	-0.00001 41702 42001 50575 19163
1	-0.00007 85631 14425 96071 79229	1	0.00002 34212 90074 41770 43037
2	0.00004 33201 50655 34634 21200	2	-0.00001 33099 17437 05539 62393
3	-0.00001 50257 64304 05546 00304	3	0.00000 52530 07061 24399 47655
4	0.00000 32734 32488 65230 12068	4	-0.00000 13131 34673 11631 90213
5	0.00000 00690 84614 72419 10790	5	0.00000 01164 31034 12632 43166
6	-0.00000 03227 66004 40032 94947	6	0.00000 00007 16427 35571 10416
7	0.00000 01250 90709 62208 60542	7	-0.00000 00327 04034 71775 92722
8	-0.00000 00253 66477 04720 43969	8	0.00000 00003 33024 45019 66040
9	0.00000 00015 90438 10662 69218	9	-0.00000 00010 66513 66222 10422
10	0.00000 00007 60306 52099 56299	10	-0.00000 00000 62967 47031 70604
11	-0.00000 00003 04470 35274 56440	11	0.00000 00000 65573 03327 71306
12	0.00000 00000 57205 70003 93230	12	-0.00000 00000 16330 91000 20053
13	-0.00000 00000 04407 00675 39640	13	0.00000 00000 07110 00301 00600
14	0.00000 00000 00777 00425 00793	14	-0.00000 00000 00018 03160 44173
15	0.00000 00000 00333 10133 00054	15	-0.00000 00000 00058 02971 09092
16	-0.00000 00000 00058 52316 00706	16	0.00000 00000 00014 45069 07490
17	0.00000 00000 00004 44064 00720	17	-0.00000 00000 00001 00012 00227
18	0.00000 00000 00000 34021 00920	18	0.00000 00000 00000 07010 00700
19	-0.00000 00000 00000 18773 00535	19	0.00000 00000 00000 02675 55163
20	0.00000 00000 00000 00230 00705	20	-0.00000 00000 00000 00707 32007
21	-0.00000 00000 00000 00205 00740	21	0.00000 00000 00000 00091 42119
22	0.00000 00000 00000 00001 00320	22	-0.00000 00000 00000 00005 01792
23	0.00000 00000 00000 00005 95420	23	-0.00000 00000 00000 00000 62757
24	-0.00000 00000 00000 00001 05942	24	0.00000 00000 00000 00000 20000
25	0.00000 00000 00000 00000 10003	25	-0.00000 00000 00000 00000 02690
26	-0.00000 00000 00000 00000 00212	26	0.00000 00000 00000 00000 00103
27	0.00000 00000 00000 00000 00109	27	0.00000 00000 00000 00000 00006
28	-0.00000 00000 00000 00000 00022	28	-0.00000 00000 00000 00000 00000
29	-0.00000 00000 00000 00000 00002		

Coefficients in the Expansion of:

$$I_0(z) = z^2 \sum_{k=0}^{\infty} K_k(v) T_k^2(z/8), \quad 0 < z \leq 8,$$

$$K_k(v) = \sum_{r=0}^{\infty} D_{r,k} T_r^2(v/4), \quad 0 \leq v \leq 4.$$

r	$D_{r,k}, k = 12$			
0	0.00000	30267	93741	10205
1	-0.00000	65530	74364	72149
2	0.00000	30535	48454	50216
3	-0.00070	14051	00003	75122
4	0.00000	04554	06053	02976
5	-0.00000	04703	71514	51426
6	-0.00000	00059	08441	80796
7	0.00000	00072	69370	11061
8	-0.00000	00021	20460	16060
9	0.00000	00004	13995	68444
10	-0.00000	00000	23322	50044
11	-0.00000	00000	10362	31632
12	0.00000	00000	03400	20515
13	-0.00000	00000	00684	02730
14	0.00000	00000	00055	91619
15	0.00000	00000	00005	47752
16	-0.00000	00000	00002	92151
17	0.00000	00000	00000	52139
18	-0.00000	00000	00000	04779
19	-0.00000	00000	00000	00104
20	0.00000	00000	00000	00110
21	-0.00000	00000	00000	00021
22	0.00000	00000	00000	00002
23	-0.00000	00000	00000	00000
24	-0.00000	00000	00000	00000
25	0.00000	00000	00000	00000
26	-0.00000	00000	00000	00000
27	0.00000	00000	00000	00000

r	$D_{r,k}, k = 15$			
0	-0.00000	10252	04639	27075
1	0.00000	17254	97231	11263
2	-0.00000	10411	63270	68306
3	0.00000	04551	30320	18292
4	-0.00000	01417	53925	00364
5	0.00000	00202	80707	74777
6	-0.00000	00014	93035	63012
7	-0.00000	00012	06119	11997
8	0.00000	00005	60932	09136
9	-0.00000	00001	26303	02645
10	0.00000	00000	14971	23593
11	0.00000	00000	00023	04470
12	-0.00000	00000	00025	33047
13	0.00000	00000	00174	01794
14	-0.00000	00000	00022	04615
15	0.00000	00000	00000	71902
16	0.00000	00000	00000	42270
17	-0.00000	00000	00000	11368
18	0.00000	00000	00000	01542
19	-0.00000	00000	00000	00059
20	-0.00000	00000	00000	00011
21	0.00000	00000	00000	00004
22	-0.00000	00000	00000	00000
23	0.00000	00000	00000	00000
24	0.00000	00000	00000	00000
25	-0.00000	00000	00000	00000
26	0.00000	00000	00000	00000
27	-0.00000	00000	00000	00000

r	$D_{r,k}, k = 14$			
0	0.00000	02522	92604	40064
1	-0.00000	04204	95302	30596
2	0.00000	02666	07906	00066
3	-0.00000	01204	30302	69020
4	0.00000	00402	91489	78519
5	-0.00000	00093	94706	40560
6	0.00000	00011	50447	20250
7	0.00000	00001	35073	04298
8	-0.00000	00001	14432	44442
9	0.00000	00000	32663	00247
10	-0.00000	00000	04342	00040
11	0.00000	00000	00328	47972
12	0.00000	00000	00096	17073
13	-0.00000	00000	00030	40230
14	0.00000	00000	00006	47678
15	-0.00000	00000	00000	59621
16	-0.00000	00000	00000	02276
17	0.00000	00000	00000	01906
18	-0.00000	00000	00000	00105
19	0.00000	00000	00000	00030
20	-0.00000	00000	00000	00000
21	-0.00000	00000	00000	00000
22	0.00000	00000	00000	00000
23	-0.00000	00000	00000	00000
24	0.00000	00000	00000	00000
25	0.00000	00000	00000	00000
26	-0.00000	00000	00000	00000

r	$D_{r,k}, k = 15$			
0	-0.00000	00590	77099	49470
1	0.00000	01000	60737	80545
2	-0.00000	00304	56065	60609
3	0.00000	00209	95043	20012
4	-0.00000	00105	90163	41992
5	0.00000	00027	40004	02399
6	-0.00000	00004	62239	96032
7	0.00000	00000	10462	40109
8	0.00000	00000	10665	30321
9	-0.00000	00000	07153	03622
10	0.00000	00000	01400	05334
11	-0.00000	00000	00174	00313
12	-0.00000	00000	00071	01300
13	0.00000	00000	00005	93060
14	-0.00000	00000	00001	45069
15	0.00000	00000	00000	10000
16	-0.00000	00000	00000	01091
17	-0.00000	00000	00000	00205
18	0.00000	00000	00000	00007
19	-0.00000	00000	00000	00010
20	0.00000	00000	00000	00000
21	0.00000	00000	00000	00000
22	-0.00000	00000	00000	00000
23	0.00000	00000	00000	00000
24	-0.00000	00000	00000	00000

TABLE 1 (Continued)

Coefficients in the Expansion of

$$I_{\nu}(z) = z^{\nu} \sum_{k=0}^{\infty} H_{\nu}(v) T_k^{\nu}(z/\theta) , \quad 0 < z \leq \theta ,$$

$$H_{\nu}(v) = \sum_{r=0}^{\infty} D_{r,k} T_r^{\nu}(v/4) , \quad 0 \leq v \leq 4 .$$

r	D <sub>r,k</sub> , k = 16				r	D <sub>r,k</sub> , k = 17					
0	0.00000	00130	99479	99372	20572	0	-0.00000	00027	62971	74713	69253
1	-0.00000	00224	97959	91500	61144	1	0.00000	00047	70284	43167	61901
2	0.00000	00144	01044	30525	55039	2	-0.00000	00031	00921	10007	10404
3	-0.00000	00049	00247	42221	37269	3	0.00000	00015	44567	50457	50199
4	0.00000	00025	95465	44615	17847	4	-0.00000	00005	94823	02914	72829
5	-0.00000	00007	29593	82477	50840	5	0.00000	00001	74633	45477	93837
6	0.00000	00001	44049	16429	65871	6	-0.00000	00000	48147	71629	54990
7	-0.00000	00000	15553	39422	06234	7	0.00000	00000	05900	10950	65645
8	0.00000	00000	01823	90929	04667	8	-0.00000	00000	00201	30249	12394
9	0.00000	00000	01332	02250	97336	9	-0.00000	00000	00195	27410	05701
10	-0.00000	00000	00340	93429	00405	10	0.00000	00000	00069	93299	47967
11	0.00000	00000	00055	26000	67426	11	-0.00000	00000	00013	90230	07722
12	-0.00000	00000	00003	97962	34311	12	0.00000	00000	00001	65710	97554
13	0.00000	00000	00000	60901	76239	13	-0.00000	00000	00000	03350	00430
14	0.00000	00000	00000	26493	20205	14	-0.00000	00000	00000	03600	24045
15	-0.00000	00000	00000	04471	72033	15	0.00000	00000	00000	00955	53963
16	0.00000	00000	00000	00506	10240	16	-0.00000	00000	00000	00130	44927
17	-0.00000	00000	00000	00005	99495	17	0.00000	00000	00000	00010	39107
18	0.00000	00000	00000	00009	09766	18	0.00000	00000	00000	00000	53494
19	0.00000	00000	00000	00002	00713	19	-0.00000	00000	00000	00000	30445
20	-0.00000	00000	00000	00000	23970	20	0.00000	00000	00000	00000	05161
21	0.00000	00000	00000	00000	01205	21	-0.00000	00000	00000	00000	00494
22	0.00000	00000	00000	00000	00145	22	0.00000	00000	00000	00000	00013
23	-0.00000	00000	00000	00000	00045	23	0.00000	00000	00000	00000	00005
24	0.00000	00000	00000	00000	00000	24	-0.00000	00000	00000	00000	00001

  

r	D <sub>r,k</sub> , k = 18				r	D <sub>r,k</sub> , k = 19					
0	0.00000	00005	55476	07575	99092	0	-0.00000	00001	06446	27219	93031
1	-0.00000	00009	63597	10954	29575	1	0.00000	00001	05706	06321	47949
2	0.00000	00006	35041	07170	65449	2	-0.00000	00001	23073	03933	11572
3	-0.00000	00003	23652	71409	06475	3	0.00000	00000	54442	70479	90534
4	0.00000	00001	29392	08444	02560	4	-0.00000	00000	26954	45000	73144
5	-0.00000	00000	40755	44474	66195	5	0.00000	00000	00720	15369	53405
6	0.00000	00000	09045	42509	93750	6	-0.00000	00000	02770	07255	00594
7	-0.00000	00000	01778	17266	01521	7	0.00000	00000	00457	10754	20062
8	0.00000	00000	00175	93379	56439	8	-0.00000	00000	00062	70057	52175
9	0.00000	00000	00015	90403	04500	9	0.00000	00000	00002	54000	73203
10	-0.00000	00000	00011	75716	30313	10	0.00000	00000	00001	51375	10044
11	0.00000	00000	00002	95192	50204	11	-0.00000	00000	00000	53403	65932
12	-0.00000	00000	00000	46153	13129	12	0.00000	00000	00000	10404	56424
13	0.00000	00000	00000	03062	13060	13	-0.00000	00000	00000	01203	39222
14	0.00000	00000	00000	00229	55003	14	0.00000	00000	00000	00059	47720
15	-0.00000	00000	00000	00140	41244	15	0.00000	00000	00000	00015	75444
16	0.00000	00000	00000	00029	14204	16	-0.00000	00000	00000	00004	92040
17	-0.00000	00000	00000	00003	37750	17	0.00000	00000	00000	00000	76097
18	0.00000	00000	00000	00000	15537	18	-0.00000	00000	00000	00000	07060
19	0.00000	00000	00000	00000	02942	19	0.00000	00000	00000	00000	00067
20	-0.00000	00000	00000	00000	00051	20	0.00000	00000	00000	00000	00101
21	0.00000	00000	00000	00000	00117	21	-0.00000	00000	00000	00000	00021
22	-0.00000	00000	00000	00000	00000	22	0.00000	00000	00000	00000	00002

TABLE 1 (Continued)

Coefficients in the Expansion of

$$I_\nu(z) = z^\nu e^z \sum_{k=0}^{\infty} i_k(\nu) T_k^*(z/a), \quad 0 < z \leq 8,$$

$$i_k(\nu) = \sum_{r=0}^{\infty} D_{r,k} T_r^*(\nu/a), \quad 0 \leq \nu \leq 4.$$

r	$D_{r,k}, k = 20$			r	$D_{r,k}, k = 21$		
0	0.00000	00000	19507 99318 30362	0	-0.00000	00000	03447 65254 71205
1	-0.00000	00000	34250 44372 62000	1	0.00000	00000	06051 12053 14767
2	0.00000	00000	73116 40820 19643	2	-0.00000	00000	04124 01537 00166
3	-0.00000	00000	12245 00522 38090	3	0.00000	00000	02221 22902 43677
4	0.00000	00000	05175 00462 41770	4	-0.00000	00000	00960 00051 00647
5	-0.00000	00000	01766 21141 26443	5	0.00000	00000	00330 01090 50272
6	0.00000	00000	00405 34037 34406	6	-0.00000	00000	00097 15045 13631
7	-0.00000	00000	00100 02539 10100	7	0.00000	00000	00022 05418 03932
8	0.00000	00000	00017 35732 09010	8	-0.00000	00000	00004 15600 94460
9	-0.00000	00000	00001 00503 00746	9	0.00000	00000	00000 54245 11239
10	0.00000	00000	00000 00663 59210	10	-0.00000	00000	00000 02790 47377
11	-0.00000	00000	00000 00026 10952	11	0.00000	00000	00000 00073 10687
12	0.00000	00000	00000 01992 00727	12	-0.00000	00000	00000 00324 14779
13	-0.00000	00000	00000 00712 04641	13	0.00000	00000	00000 00063 04607
14	0.00000	00000	00000 00029 54640	14	-0.00000	00000	00000 00000 07469
15	-0.00000	00000	00000 00000 03005	15	0.00000	00000	00000 00000 53720
16	0.00000	00000	00000 00000 63103	16	-0.00000	00000	00000 00000 04310
17	-0.00000	00000	00000 00000 13060	17	0.00000	00000	00000 00000 01979
18	0.00000	00000	00000 00000 01770	18	-0.00000	00000	00000 00000 00361
19	-0.00000	00000	00000 00000 00124	19	0.00000	00000	00000 00000 00036
20	0.00000	00000	00000 00000 00004	20	-0.00000	00000	00000 00000 00002
21	-0.00000	00000	00000 00000 00000				

  

r	$D_{r,k}, k = 22$			r	$D_{r,k}, k = 23$		
0	0.00000	00000	00502 40570 00491	0	-0.00000	00000	00094 56572 47209
1	-0.00000	00000	01025 51367 36630	1	0.00000	00000	00167 00771 72700
2	0.00000	00000	00705 56950 60476	2	-0.00000	00000	00115 00542 70004
3	-0.00000	00000	00305 67029 74772	3	0.00000	00000	00064 21259 79427
4	0.00000	00000	00170 36299 33105	4	-0.00000	00000	00020 99007 41900
5	-0.00000	00000	00061 60569 53690	5	0.00000	00000	00010 71201 35027
6	0.00000	00000	00018 37765 03470	6	-0.00000	00000	00003 30021 52475
7	-0.00000	00000	00004 51640 00365	7	0.00000	00000	00000 94710 43125
8	0.00000	00000	00000 00901 40693	8	-0.00000	00000	00000 17003 77169
9	-0.00000	00000	00000 13010 64507	9	0.00000	00000	00000 03071 39034
10	0.00000	00000	00000 01350 02969	10	-0.00000	00000	00000 00300 20000
11	-0.00000	00000	00000 00017 84037	11	0.00000	00000	00000 00024 49619
12	0.00000	00000	00000 00042 65235	12	-0.00000	00000	00000 00003 63103
13	-0.00000	00000	00000 00010 00090	13	0.00000	00000	00000 00001 50700
14	0.00000	00000	00000 00001 72716	14	-0.00000	00000	00000 00000 31115
15	-0.00000	00000	00000 00000 17939	15	0.00000	00000	00000 00000 04144
16	0.00000	00000	00000 00000 00041	16	-0.00000	00000	00000 00000 00339
17	-0.00000	00000	00000 00000 00195	17	0.00000	00000	00000 00000 00001
18	0.00000	00000	00000 00000 00052	18	-0.00000	00000	00000 00000 00000
19	-0.00000	00000	00000 00000 00007	19	0.00000	00000	00000 00000 00001

TABLE 1 (Continued)

Coefficients in the Expansion of

$$I_\nu(z) = z^\nu e^{-z} \sum_{k=0}^{\infty} H_k(\nu) T_k^*(z/\theta) \quad , \quad 0 < z \leq \theta \quad ,$$

$$H_k(\nu) = \sum_{r=0}^k \gamma_{r,k} T_r^*(\nu/4) \quad , \quad 0 \leq \nu \leq 4 \quad .$$

r	$\gamma_{r,k}, k = 24$					r	$\gamma_{r,k}, k = 25$				
0	0.00000	00000	00014	77904	31791	0	-0.00000	00000	00002	22600	22022
1	-0.00000	00000	00026	17101	50594	1	0.00000	00000	00003	95191	33926
2	0.00000	00000	00010	30311	15391	2	-0.00000	00000	00002	78379	30847
3	-0.00000	00000	00018	26930	29147	3	0.00000	00000	00001	57995	21440
4	0.00000	00000	00004	70120	09097	4	-0.00000	00000	00000	73465	01873
5	-0.00000	00000	00001	70152	40437	5	0.00000	00000	00000	20401	53366
6	0.00000	00000	00000	56463	20040	6	-0.00000	00000	00000	09231	13636
7	-0.00000	00000	00000	15041	99761	7	0.00000	00000	00000	02539	51022
8	0.00000	00000	00000	03360	45167	8	-0.00000	00000	00000	00591	07409
9	-0.00000	00000	00000	00620	32045	9	0.00000	00000	00000	00116	06792
10	0.00000	00000	00000	00090	63629	10	-0.00000	00000	00000	00018	09570
11	-0.00000	00000	00000	00009	10496	11	0.00000	00000	00000	00002	31554
12	0.00000	00000	00000	00000	16666	12	-0.00000	00000	00000	00000	17006
13	0.00000	00000	00000	00000	17546	13	-0.00000	00000	00000	00000	00940
14	-0.00000	00000	00000	00000	04770	14	0.00000	00000	00000	00000	00605
15	0.00000	00000	00000	00000	00782	15	-0.00000	00000	00000	00000	00124
16	-0.00000	00000	00000	00000	00000	16	0.00000	00000	00000	00000	00017
17	0.00000	00000	00000	00000	00005	17	-0.00000	00000	00000	00000	00007

  

r	$\gamma_{r,k}, k = 23$					r	$\gamma_{r,k}, k = 27$				
0	0.00000	00000	00000	32351	94786	0	-0.00000	00000	00000	04542	22679
1	-0.00000	00000	00000	57509	99010	1	0.00000	00000	00000	00100	40767
2	0.00000	00000	00000	40025	60210	2	-0.00000	00000	00000	05780	24604
3	-0.00000	00000	00000	23410	12771	3	0.00000	00000	00000	03340	43435
4	0.00000	00000	00000	11046	30992	4	-0.00000	00000	00000	01600	40209
5	-0.00000	00000	00000	04340	00451	5	0.00000	00000	00000	00640	67108
6	0.00000	00000	00000	01445	70909	6	-0.00000	00000	00000	00217	40155
7	-0.00000	00000	00000	00409	14266	7	0.00000	00000	00000	00063	00904
8	0.00000	00000	00000	00090	09109	8	-0.00000	00000	00000	00015	73653
9	-0.00000	00000	00000	00020	35909	9	0.00000	00000	00000	00003	37475
10	0.00000	00000	00000	00003	52438	10	-0.00000	00000	00000	00000	61809
11	-0.00000	00000	00000	00000	49527	11	0.00000	00000	00000	00000	09480
12	0.00000	00000	00000	00000	05093	12	-0.00000	00000	00000	00000	01157
13	-0.00000	00000	00000	00000	00214	13	0.00000	00000	00000	00000	00095
14	0.00000	00000	00000	00000	00054	14	0.00000	00000	00000	00000	00000
15	0.00000	00000	00000	00000	00017	15	-0.00000	00000	00000	00000	00002
16	-0.00000	00000	00000	00000	00003						

  

r	$\gamma_{r,k}, k = 23$					r	$\gamma_{r,k}, k = 29$				
0	0.00000	00000	00000	00616	73717	0	-0.00000	00000	00000	00001	06573
1	-0.00000	00000	00000	01102	00004	1	0.00000	00000	00000	00145	13559
2	0.00000	00000	00000	00790	99624	2	-0.00000	00000	00000	00104	73356
3	-0.00000	00000	00000	00462	43308	3	0.00000	00000	00000	00061	75823
4	0.00000	00000	00000	00223	75940	4	-0.00000	00000	00000	00030	22436
5	-0.00000	00000	00000	00090	05440	5	0.00000	00000	00000	00012	46113
6	0.00000	00000	00000	00031	44546	6	-0.00000	00000	00000	00004	30257
7	-0.00000	00000	00000	00009	33427	7	0.00000	00000	00000	00001	32789
8	0.00000	00000	00000	00002	39372	8	-0.00000	00000	00000	00000	34906
9	-0.00000	00000	00000	00000	53157	9	0.00000	00000	00000	00000	07991
10	0.00000	00000	00000	00000	10195	10	-0.00000	00000	00000	00000	01593
11	-0.00000	00000	00000	00000	01670	11	0.00000	00000	00000	00000	00275
12	0.00000	00000	00000	00000	00227	12	-0.00000	00000	00000	00000	00040
13	-0.00000	00000	00000	00000	00024	13	0.00000	00000	00000	00000	00005
14	0.00000	00000	00000	00000	00001						

TABLE 1 (Continued)  
Coefficients in the Expansion of

$$I_{\nu}(z) = z^{\nu} \sum_{k=0}^{\infty} i_{\nu,k} \gamma_k^{\nu}(z/\theta), \quad 0 < z \leq \theta,$$

$$H_k(\nu) = \sum_{r=0}^{\infty} D_{r,k} \gamma_r^{\nu}(\nu/4), \quad 0 \leq \nu \leq 4.$$

r	$D_{r,k}, k = 30$					r	$D_{r,k}, k = 31$				
0	0.00000	00000	00000	00010	32521	0	-0.00000	00000	00000	00001	27450
1	-0.00000	00000	00000	00010	51842	1	0.00000	00000	00000	00002	29143
2	0.00000	00000	00000	00013	43140	2	-0.00000	00000	00000	00001	66992
3	-0.00000	00000	00000	00007	90333	3	0.00000	00000	00000	00001	00004
4	0.00000	00000	00000	00003	94901	4	-0.00000	00000	00000	00000	49963
5	-0.00000	00000	00000	00001	64971	5	0.00000	00000	00000	00000	21130
6	0.00000	00000	00000	00000	50944	6	-0.00000	00000	00000	00000	07661
7	-0.00000	00000	00000	00000	10197	7	0.00000	00000	00000	00000	02406
8	0.00000	00000	00000	00000	04091	8	-0.00000	00000	00000	00000	00660
9	-0.00000	00000	00000	00000	01150	9	0.00000	00000	00000	00000	00159
10	0.00000	00000	00000	00000	00237	10	-0.00000	00000	00000	00000	00034
11	-0.00000	00000	00000	00000	00043	11	0.00000	00000	00000	00000	00005
12	0.00000	00000	00000	00000	00007	12	-0.00000	00000	00000	00000	00001

  

r	$D_{r,k}, k = 32$					r	$D_{r,k}, k = 33$				
0	0.00000	00000	00000	00000	15295	0	-0.00000	00000	00000	00000	01702
1	-0.00000	00000	00000	00000	27521	1	0.00000	00000	00000	00000	03211
2	0.00000	00000	00000	00000	20147	2	-0.00000	00000	00000	00000	02361
3	-0.00000	00000	00000	00000	12150	3	0.00000	00000	00000	00000	01433
4	0.00000	00000	00000	00000	06127	4	-0.00000	00000	00000	00000	00729
5	-0.00000	00000	00000	00000	02621	5	0.00000	00000	00000	00000	00315
6	0.00000	00000	00000	00000	00963	6	-0.00000	00000	00000	00000	00117
7	-0.00000	00000	00000	00000	00307	7	0.00000	00000	00000	00000	00034
8	0.00000	00000	00000	00000	00086	8	-0.00000	00000	00000	00000	00011
9	-0.00000	00000	00000	00000	00021	9	0.00000	00000	00000	00000	00003
10	0.00000	00000	00000	00000	00005						

  

r	$D_{r,k}, k = 34$					r	$D_{r,k}, k = 35$				
0	0.00000	00000	00000	00000	00202	0	-0.00000	00000	00000	00000	00022
1	-0.00000	00000	00000	00000	00364	1	0.00000	00000	00000	00000	00040
2	0.00000	00000	00000	00000	00269	2	-0.00000	00000	00000	00000	00030
3	-0.00000	00000	00000	00000	00164	3	0.00000	00000	00000	00000	00014
4	0.00000	00000	00000	00000	00004	4	-0.00000	00000	00000	00000	00009
5	-0.00000	00000	00000	00000	00037	5	0.00000	00000	00000	00000	00004
6	0.00000	00000	00000	00000	00014	6	-0.00000	00000	00000	00000	00002
7	-0.00000	00000	00000	00000	00005						
8	0.00000	00000	00000	00000	00001						

  

r	$D_{r,k}, k = 36$				
0	0.00000	00000	00000	00000	00002
1	-0.00000	00000	00000	00000	00004
2	0.00000	00000	00000	00000	00003
3	-0.00000	00000	00000	00000	00002
4	0.00000	00000	00000	00000	00001

TABLE 2

Coefficients in the Expansion of

$$I_{\nu}(z) = z^{\nu} e^z \sum_{k=0}^{\infty} H_k(\nu) T_k^{\nu}(z/B), \quad 0 < z \leq B,$$

$$H_k(\nu) = \sum_{r=0}^k D_{r,k} T_r^{\nu}\left(\frac{z}{B}\right), \quad k \geq \nu \geq 0.$$

r	$D_{r,k}, k=0$					r	$D_{r,k}, k=1$					
0	0.00011	07064	64036	21261	20906	36	-0.00018	16009	96106	15785	86566	64
1	-0.00019	52739	30678	96655	52411	68	0.00032	15104	27002	35305	26666	82
2	0.00013	65407	76672	70971	01776	67	-0.00022	65159	26060	63012	62662	55
3	-0.00007	65604	16577	77266	67618	96	0.00012	56597	26701	93717	39165	53
4	0.00003	60104	59905	10192	30526	73	-0.00005	60632	66300	72754	69060	01
5	-0.00001	31074	53200	42766	65312	10	0.00002	12378	70215	70921	58784	71
6	0.00000	40401	32936	67806	65663	13	-0.00000	65672	65207	66162	12650	71
7	-0.00000	10520	60660	51967	09200	00	0.00000	16633	82523	36611	32625	30
8	0.00000	02215	50352	66305	11101	00	-0.00000	03615	96660	66600	21612	50
9	-0.00000	00366	42707	10030	00200	61	0.00000	00533	76626	62252	30663	25
10	0.00000	00000	17755	51656	04763	29	-0.00000	00069	66095	70351	18700	14
11	-0.00000	00000	62400	16267	20916	57	0.00000	00002	66097	56097	09026	50
12	0.00000	00001	80076	36037	65600	30	0.00000	00007	60703	26086	30360	36
13	-0.00000	00000	31496	66369	15390	17	-0.00000	00000	61083	76136	62266	66
14	0.00000	00000	05611	66655	26565	96	0.00000	00000	10197	12275	61525	35
15	-0.00000	00000	06635	35616	73305	60	-0.00000	00000	01162	77777	79673	20
16	0.00000	00000	00020	00036	15763	50	0.00000	00000	00051	36676	11012	13
17	-0.00000	00000	00005	66606	00327	67	0.00000	00000	00011	66610	00007	65
18	0.00000	00000	00002	11007	21563	63	-0.00000	00000	00003	17577	21916	65
19	-0.00000	00000	00000	36366	39625	17	0.00000	00000	00000	50095	26607	72
20	0.00000	00000	00000	03627	07010	30	-0.00000	00000	00000	06230	26732	66
21	-0.00000	00000	00000	00140	56633	20	0.00000	00000	00000	00293	23066	02
22	0.00000	00000	00000	00020	63962	21	0.00000	00000	00000	00030	62625	26
23	-0.00000	00000	00000	00006	70562	06	-0.00000	00000	00000	00011	62666	66
24	0.00000	00000	00000	00000	00152	67	0.00000	00000	00000	00001	71765	62
25	-0.00000	00000	00000	00000	00162	66	-0.00000	00000	00000	00000	15766	50
26	0.00000	00000	00000	00000	00069	66	0.00000	00000	00000	00000	00656	63
27	-0.00000	00000	00000	00000	00060	62	0.00000	00000	00000	00000	00675	65
28	0.00000	00000	00000	00000	00011	65	-0.00000	00000	00000	00000	00020	65
29	-0.00000	00000	00000	00000	00001	50	0.00000	00000	00000	00000	00002	63
30	0.00000	00000	00000	00000	00000	12	-0.00000	00000	00000	00000	00000	21
31	-0.00000	00000	00000	00000	00000	01	0.00000	00000	00000	00000	00000	01

r	$D_{r,k}, k=2$					r	$D_{r,k}, k=3$					
0	0.00011	16001	23696	55506	32560	60	-0.00005	70393	95099	66595	62665	61
1	-0.00020	06719	66366	50701	22016	20	0.00010	25160	10663	72209	76611	37
2	0.00013	99224	65490	32212	62959	71	-0.00007	16666	20366	66375	00716	67
3	-0.00007	79951	65126	66316	65061	11	0.00003	98225	00022	17006	67966	50
4	0.00003	52739	06700	67025	30333	66	-0.00001	00022	05670	67095	61073	61
5	-0.00001	30604	32610	70378	03961	60	0.00000	66776	16609	06612	35166	61
6	0.00000	40077	30000	66267	50106	27	-0.00000	20632	07504	70606	65606	62
7	-0.00000	10074	22396	76017	55066	73	0.00000	05135	16666	57617	26106	66
8	0.00000	02635	17666	60076	65662	72	-0.00000	01036	66206	60360	71626	66
9	-0.00000	00307	60733	66033	06093	22	0.00000	00156	65377	50733	50062	66
10	0.00000	00025	51015	62636	25007	67	-0.00000	00013	06017	02273	03021	66
11	-0.00000	00002	60620	59966	67502	65	0.00000	00001	33393	57236	06774	60
12	0.00000	00001	63976	67659	20672	70	0.00000	00000	02121	00054	32001	66
13	-0.00000	00000	30153	00622	30662	32	-0.00000	00000	19566	57731	20501	72
14	0.00000	00000	06756	33612	70600	66	0.00000	00000	67106	03707	01669	16
15	-0.00000	00000	00652	56639	75122	92	-0.00000	00000	00310	66690	70370	66
16	0.00000	00000	00017	00000	52376	35	0.00000	00000	00007	22561	77566	16
17	-0.00000	00000	00009	70901	76007	16	0.00000	00000	00005	16660	63626	10
18	0.00000	00000	70002	62633	51361	70	-0.00000	00000	00001	33061	90014	02
19	-0.00000	00000	00000	19006	75062	23	0.00000	00000	00000	20030	09757	36
20	0.00000	00000	00000	03956	35560	00	-0.00000	00000	00000	01020	63770	66
21	-0.00000	00000	00000	00157	61069	00	0.00000	00000	00000	00006	52703	10
22	0.00000	00000	00000	00030	30275	23	0.00000	00000	00000	00017	00661	16
23	-0.00000	00000	00000	00000	27010	61	-0.00000	00000	00000	00006	35560	66
24	0.00000	00000	00000	00001	16665	66	0.00000	00000	00000	00000	50036	66
25	-0.00000	00000	00000	00000	10165	37	-0.00000	00000	00000	00000	05665	66
26	0.00000	00000	00000	00000	00376	01	0.00000	00000	00000	00000	00166	70
27	-0.00000	00000	00000	00000	00057	60	0.00000	00000	00000	00000	00032	22
28	0.00000	00000	00000	00000	00016	22	-0.00000	00000	00000	00000	00007	52
29	-0.00000	00000	00000	00000	00001	75	0.00000	00000	00000	00000	00001	61
30	0.00000	00000	00000	00000	00000	16	-0.00000	00000	00000	00000	00000	01



TABLE 2 (Continued)

Coefficients in the Expansion of

$$I_{\nu}(x) = x^{\nu} \sum_{n=0}^{\infty} D_{\nu,k}^{(n)} x^n / \Gamma(\frac{n+1}{2}), \quad 0 < x \leq 8, \quad ,$$

$$D_{\nu,k}^{(n)} = \sum_{r=0}^n D_{\nu,k}^{(r)} \binom{n}{r}, \quad 0 \leq r \leq n.$$

$D_{r,k}^{(n)}, k=4$		$D_{r,k}^{(n)}, k=5$	
0	0.00002 53634 40155 04194 68305 6A	0	-0.00000 47954 74449 52094 01995 05
1	-0.00004 40921 70442 44458 07772 06	1	0.00001 73507 01005 53295 00041 00
2	0.00003 13201 49061 84503 00601 04	2	-0.00001 21330 27610 36009 09052 92
3	-0.00001 74877 09355 04531 70504 68	3	0.00000 67944 32021 23200 34409 64
4	0.00000 70250 70974 50419 94669 47	4	-0.00000 30057 73709 08209 00904 7A
5	-0.00000 29505 06139 03109 43909 53	5	0.00000 11000 95556 53240 20416 9A
6	0.00000 09074 69441 40822 30954 61	6	-0.00000 03001 54002 01707 50100 07
7	-0.00000 07200 48595 59329 13301 04	7	0.00000 00924 70470 04915 52979 29
8	0.00000 00470 07209 61917 44579 40	8	-0.00000 00193 76394 04169 10003 50
9	-0.00000 00072 97021 25702 04444 07	9	0.00000 00431 20731 02414 20231 24
10	0.00000 00006 77353 36106 07943 05	10	-0.00000 00003 20257 65017 00242 4A
11	0.00000 00009 39394 08722 70176 4A	11	-0.00000 00000 02764 07579 50174 40
12	-0.00000 00000 33057 07766 33014 64	12	0.00000 00000 11010 52417 93703 55
13	0.00000 00000 00250 79779 71192 54	13	-0.00000 00000 03029 33762 16427 14
14	-0.00000 00000 01340 53073 45509 5A	14	0.00000 00000 00524 70113 39443 20
15	0.00000 00000 00144 41019 32597 95	15	-0.00000 00000 00000 96003 06553 20
16	-0.00000 00000 00004 75022 54172 64	16	0.00000 00000 00003 11303 68434 6A
17	0.00000 00000 00001 96435 53505 10	17	0.00000 00000 00000 55247 51048 8A
18	-0.00000 00000 00000 04493 26608 7A	18	-0.00000 00000 00000 10753 47197 0A
19	0.00000 00000 00000 08416 07574 72	19	0.00000 00000 00000 03000 07064 01
20	-0.00000 00000 00000 00834 02675 29	20	-0.00000 00000 00000 00000 40621 6A
21	0.00000 00000 00000 00032 77471 80	21	0.00000 00000 00000 00017 17530 97
22	-0.00000 00000 00000 00000 57261 04	22	0.00000 00000 00000 00001 73044 73
23	0.00000 00000 00000 00001 77678 94	23	-0.00000 00000 00000 00000 59192 93
24	-0.00000 00000 00000 00000 24534 46	24	0.00000 00000 00000 00000 00765 43
25	0.00000 00000 00000 00000 02160 3A	25	-0.00000 00000 00000 00000 00024 13
26	-0.00000 00000 00000 00000 00076 57	26	0.00000 00000 00000 00000 00037 14
27	0.00000 00000 00000 00000 00012 04	27	0.00000 00000 00000 00000 00003 09
28	-0.00000 00000 00000 00000 00003 10	28	-0.00000 00000 00000 00000 00001 47
29	0.00000 00000 00000 00000 00000 30	29	0.00000 00000 00000 00000 00000 17
30	0.00000 00000 00000 00000 00000 03	30	-0.00000 00000 00000 00000 00000 01

  

$D_{r,k}^{(n)}, k=6$		$D_{r,k}^{(n)}, k=7$	
0	0.00000 34037 55297 43570 75759 38	0	-0.00000 10709 27419 06137 40554 01
1	-0.00000 60346 34317 91670 07509 71	1	0.00000 19150 06737 04329 70254 6A
2	0.00000 42320 44260 60908 30977 90	2	-0.00000 13475 90009 41301 37793 50
3	-0.00000 23015 47624 42040 94063 78	3	0.00000 07025 63041 52032 96435 09
4	0.00000 10910 07626 27648 90290 10	4	-0.00000 03523 44054 37592 47416 07
5	-0.00000 04129 19303 23553 37691 63	5	0.00000 01344 39063 16420 05023 10
6	0.00000 01297 03089 54196 05353 70	6	-0.00000 04429 00011 01008 20778 50
7	-0.00000 00334 95366 42066 70961 4A	7	0.00000 00114 23724 03130 36532 0A
8	0.00000 00072 73519 05450 77727 04	8	-0.00000 00025 21601 02069 43109 47
9	-0.00000 00012 34762 42001 79104 00	9	0.00000 00004 40973 02495 61244 51
10	0.00000 00001 46660 07707 41046 4A	10	-0.00000 00000 50341 00248 00044 97
11	-0.00000 00000 00054 10004 23972 26	11	0.00000 00000 03939 54911 95700 64
12	0.00000 00000 02091 00592 30107 49	12	-0.00000 00000 00043 74130 13004 03
13	-0.00000 00000 00977 50500 35018 27	13	0.00000 00000 00275 03901 76206 42
14	0.00000 00000 00103 95507 47715 97	14	-0.00000 00000 00050 04001 70430 07
15	-0.00000 00000 00023 57000 42478 41	15	0.00000 00000 00000 20762 00307 9A
16	0.00000 00000 00001 07147 07266 87	16	-0.00000 00000 00000 74257 41997 47
17	-0.00000 00000 00000 09677 62543 05	17	0.00000 00000 00000 00515 17460 70
18	0.00000 00000 00000 05439 51400 51	18	-0.00000 00000 00000 01290 71046 7A
19	-0.00000 00000 00000 01010 94434 13	19	0.00000 00000 00000 00290 96707 91
20	0.00000 00000 00000 00119 90370 53	20	-0.00000 00000 00000 00039 05046 77
21	-0.00000 00000 00000 00000 11204 92	21	0.00000 00000 00000 00003 20919 74
22	0.00000 00000 00000 00000 23732 70	22	-0.00000 00000 00000 00000 05001 6A
23	-0.00000 00000 00000 00000 16307 05	23	0.00000 00000 00000 00000 03504 0A
24	0.00000 00000 00000 00000 02741 03	24	-0.00000 00000 00000 00000 00747 0A
25	-0.00000 00000 00000 00000 00200 77	25	0.00000 00000 00000 00000 00000 30
26	0.00000 00000 00000 00000 00016 70	26	-0.00000 00000 00000 00000 00000 47
27	-0.00000 00000 00000 00000 00000 51	27	0.00000 00000 00000 00000 00000 0A
28	0.00000 00000 00000 00000 00000 20	28	-0.00000 00000 00000 00000 00000 0A
29	-0.00000 00000 00000 00000 00000 04	29	0.00000 00000 00000 00000 00000 01

TABLE (Continued)

Coefficients in the expansion of

$$I_0(z) = z^{\nu} \sum_{k=0}^{\infty} h_k(\nu) x_k^{\nu}(z/\theta), \quad 0 < z \leq \theta,$$

$$h_k(\nu) = \sum_{r=0}^{\infty} D_{r,k} x_r^{\nu}\left(\frac{z}{\theta}\right), \quad 0 \leq \nu \leq 8.$$

r	$D_{r,k}, k = 0$				
0	0.00000	03151	97421	31286	80956 29
1	-0.00000	05601	90660	17353	64470 55
2	0.00000	03956	79032	50031	51525 34
3	-0.00000	02257	73643	54717	76100 79
4	0.00000	01049	70009	51137	66015 67
5	-0.00000	00405	64576	48297	23172 27
6	0.00000	00131	70077	54454	32954 74
7	-0.00000	00035	64656	96514	66060 52
8	0.00000	00000	09445	01294	17677 05
9	-0.00000	00001	50772	45295	30409 39
10	0.00000	00000	21783	54302	90939 61
11	-0.00000	00000	01990	80964	40691 65
12	0.00000	00000	00968	67700	67610 22
13	0.00000	00000	00065	46702	55709 99
14	-0.00000	00000	00016	41030	43461 65
15	0.00000	00000	00002	61261	40007 39
16	-0.00000	00000	00000	20117	75427 81
17	0.00000	00000	00000	01317	50082 13
18	0.00000	00000	00000	00230	60218 36
19	-0.00000	00000	00000	00072	70511 04
20	0.00000	00000	00000	00011	33520 27
21	-0.00000	00000	00000	00001	14769 36
22	0.00000	00000	00000	00000	05670 27
23	0.00000	00000	00000	00000	00534 31
24	-0.00000	00000	00000	00000	00174 46
25	0.00000	00000	00000	00000	00024 67
26	-0.00000	00000	00000	00000	00002 21
27	0.00000	00000	00000	00000	00000 09
28	0.00000	00000	00000	00000	00000 01

r	$D_{r,k}, k = 9$				
0	-0.00000	00055	32642	67270	05076 13
1	0.00000	01522	25933	11405	50547 77
2	-0.00000	01079	51740	53629	43496 54
3	0.00000	00619	50950	94964	45027 50
4	-0.00000	00290	85326	91665	85057 50
5	0.00000	00113	77761	61404	19252 76
6	-0.00000	00037	34413	30328	14970 81
7	0.00000	00010	14926	60902	57675 65
8	-0.00000	00007	41540	11445	40369 54
9	0.00000	00000	46923	43007	57214 37
10	-0.00000	00000	00000	07299	42194 12
11	0.00000	00000	00000	00000	71947 43257 47
12	-0.00000	00000	00029	60174	39977 47
13	0.00000	00000	00012	92950	02717 24
14	0.00000	00000	00004	11391	76764 37
15	-0.00000	00000	00000	74318	20407 82
16	0.00000	00000	00000	09280	70900 06
17	-0.00000	00000	00000	04667	62677 78
18	0.00000	00000	00000	00015	70137 41
19	-0.00000	00000	00000	00015	27010 16
20	0.00000	00000	00000	00007	00265 43
21	-0.00000	00000	00000	00000	34754 10
22	0.00000	00000	00000	00000	02542 47
23	0.00000	00000	00000	00000	00001 43
24	-0.00000	00000	00000	00000	00003 01
25	0.00000	00000	00000	00000	00005 92
26	-0.00000	00000	00000	00000	00000 64
27	0.00000	00000	00000	00000	00000 04

r	$D_{r,k}, k = 10$				
0	0.00000	00216	95285	06645	19010 03
1	-0.00000	00306	47040	00055	92627 90
2	0.00000	00275	34721	37374	85196 61
3	-0.00000	00150	82923	06211	97474 33
4	0.00000	00075	36513	60612	51977 07
5	-0.00000	00029	82054	33636	00571 85
6	0.00000	00009	94189	87993	10546 10
7	-0.00000	00007	00990	40367	40704 52
8	0.00000	00000	67361	57079	51935 49
9	-0.00000	00001	13500	71017	90551 05
10	0.00000	00000	02240	01083	60485 10
11	-0.00000	00000	00284	65484	77249 08
12	0.00000	00000	00020	66342	13445 22
13	-0.00000	00000	00001	55479	94004 05
14	0.00000	00000	00000	09307	43550 64
15	-0.00000	00000	00000	10961	92145 62
16	0.00000	00000	00000	02708	57975 72
17	-0.00000	00000	00000	00260	77948 51
18	0.00000	00000	00000	00009	10557 60
19	-0.00000	00000	00000	00002	42607 00
20	0.00000	00000	00000	00000	64439 01
21	-0.00000	00000	00000	00000	09191 52
22	0.00000	00000	00000	00000	00059 21
23	-0.00000	00000	00000	00000	00037 14
24	0.00000	00000	00000	00000	00004 26
25	-0.00000	00000	00000	00000	00001 21
26	0.00000	00000	00000	00000	00000 14
27	-0.00000	00000	00000	00000	00000 01

r	$D_{r,k}, k = 11$				
0	-0.00000	00051	69961	00995	90230 04
1	0.00000	00092	27607	43614	07045 29
2	-0.00000	00065	00132	13455	26523 74
3	0.00000	00038	31001	60567	60896 14
4	-0.00000	00018	34957	64393	42054 90
5	0.00000	00007	34684	15759	70516 16
6	-0.00000	00002	40422	60423	20204 41
7	0.00000	00000	71617	69747	52010 17
8	-0.00000	00000	17601	72714	93790 33
9	0.00000	00000	03676	65240	29066 66
10	-0.00000	00000	00641	02800	64674 57
11	0.00000	00000	00009	00120	95414 11
12	-0.00000	00000	00000	05174	39154 11
13	0.00000	00000	00000	15390	42915 15
14	-0.00000	00000	00000	15485	12672 25
15	0.00000	00000	00000	04294	51477 73
16	-0.00000	00000	00000	00704	20093 50
17	0.00000	00000	00000	00001	10092 51
18	-0.00000	00000	00000	00005	51944 34
19	0.00000	00000	00000	00000	16000 13
20	-0.00000	00000	00000	00000	11912 33
21	0.00000	00000	00000	00000	02117 52
22	-0.00000	00000	00000	00000	00240 29
23	0.00000	00000	00000	00000	00014 01
24	-0.00000	00000	00000	00000	00000 04
25	0.00000	00000	00000	00000	00000 25
26	-0.00000	00000	00000	00000	00000 11

TABLE 2 (Continued)

Coefficients in the Expansion of

$$I_\nu(z) = z^\nu e^z \sum_{k=0}^{\infty} K_k(\nu) I_k^0(z/8), \quad 0 < z \leq 8, \quad ,$$

$$K_k(\nu) = \sum_{r=0}^{\infty} D_{r,k} \tau_r^{\nu} \left(\frac{\nu-4}{4}\right), \quad 4 \leq \nu \leq 8, \quad .$$

$D_{r,k}, k = 12$					$D_{r,k}, k = 13$							
r	0	1	2	3	r	0	1	2	3			
0	0.00000	00011	62350	42951	71351	30	-0.00000	00002	47444	41001	97550	84
1	-0.00000	00020	77414	00096	27248	64	0.00000	00004	42919	64964	30839	43
2	0.00000	00014	91834	99319	63225	72	-0.00000	00003	19353	51908	41626	31
3	-0.00000	00008	72145	49300	23265	44	0.00000	00001	87931	60487	46549	50
4	0.00000	00004	21442	44664	37639	71	-0.00000	00000	91631	82095	53033	49
5	-0.00000	00001	70717	82020	25978	77	0.00000	00000	37539	47876	21036	65
6	0.00000	00000	50418	01560	64823	85	-0.00000	00000	13071	94448	67482	34
7	-0.00000	00000	17196	00993	13293	87	0.00000	00000	03901	00975	49290	36
8	0.00000	00000	04326	22214	18196	88	-0.00000	00000	01003	37168	61841	35
9	-0.00000	00000	00931	95646	63689	84	0.00000	00000	00222	36065	07750	80
10	0.00000	00000	00170	06796	67565	85	-0.00000	00000	00042	20032	50702	40
11	-0.00000	00000	00025	54463	39742	14	0.00000	00000	00006	73065	49951	83
12	0.00000	00000	00002	92544	70799	69	-0.00000	00000	00000	06333	44500	37
13	-0.00000	00000	00000	18100	51276	86	0.00000	00000	00000	07632	20620	58
14	0.00000	00000	00000	01062	70029	54	-0.00000	00000	00000	00006	50954	05
15	0.00000	00000	00000	00840	37026	88	-0.00000	00000	00000	00134	37990	35
16	-0.00000	00000	00000	00163	22234	36	0.00000	00000	00000	00033	45143	34
17	0.00000	00000	00000	00021	85027	27	-0.00000	00000	00000	00005	19634	99
18	-0.00000	00000	00000	00001	99200	06	0.00000	00000	00000	00000	57393	50
19	0.00000	00000	00000	00000	06004	04	-0.00000	00000	00000	00000	03000	37
20	0.00000	00000	00000	00000	01621	19	-0.00000	00000	00000	00000	00005	29
21	-0.00000	00000	00000	00000	00418	09	0.00000	00000	00000	00000	00007	30
22	0.00000	00000	00000	00000	00057	75	-0.00000	00000	00000	00000	00011	92
23	-0.00000	00000	00000	00000	00005	30	0.00000	00000	00000	00000	00001	35
24	0.00000	00000	00000	00000	00004	24	-0.00000	00000	00000	00000	00000	10
25	0.00000	00000	00000	00000	00000	07						
26	-0.00000	00000	00000	00000	00000	01						

$D_{r,k}, k = 14$					$D_{r,k}, k = 15$							
r	0	1	2	3	r	0	1	2	3			
0	0.00000	00000	50033	17274	22377	30	-0.00000	00000	00635	14420	12364	84
1	-0.00000	00000	89683	77772	85554	87	0.00000	00000	17294	52415	05150	63
2	0.00000	00000	64926	03060	33702	87	-0.00000	00000	12569	06718	41151	15
3	-0.00000	00000	34454	60930	26735	83	0.00000	00000	07491	90073	30510	55
4	0.00000	00000	10913	78015	24049	27	-0.00000	00000	03716	17106	60517	62
5	-0.00000	00000	07834	19507	29453	19	0.00000	00000	01555	69488	06915	22
6	0.00000	00000	02765	11405	51059	20	-0.00000	00000	00554	24703	64460	04
7	-0.00000	00000	00039	14142	60107	21	0.00000	00000	00171	47470	04310	71
8	0.00000	00000	00220	23623	60473	42	-0.00000	00000	00045	47317	11932	50
9	-0.00000	00000	00050	09180	44407	21	0.00000	00000	00010	60508	13222	55
10	0.00000	00000	00009	04306	08266	24	-0.00000	00000	00002	16527	18160	15
11	-0.00000	00000	00001	65104	04906	10	0.00000	00000	00000	37903	76457	25
12	0.00000	00000	00000	22991	06551	41	-0.00000	00000	00000	95627	04957	07
13	-0.00000	00000	00000	07451	04966	77	0.00000	00000	00000	00676	60677	30
14	0.00000	00000	00000	00142	12551	75	-0.00000	00000	00000	00057	07742	95
15	0.00000	00000	00000	00013	81965	42	0.00000	00000	00000	00000	06826	83
16	-0.00000	00000	00000	00005	09062	15	0.00000	00000	00000	00000	83499	75
17	0.00000	00000	00000	00001	09504	09	-0.00000	00000	00000	00000	20324	47
18	-0.00000	00000	00000	00000	14161	61	0.00000	00000	00000	00000	03000	97
19	0.00000	00000	00000	00000	01276	99	-0.00000	00000	00000	00000	00334	30
20	-0.00000	00000	00000	00000	00052	17	0.00000	00000	00000	00000	00021	65
21	0.00000	00000	00000	00000	00007	37	-0.00000	00000	00000	00000	00000	10
22	-0.00000	00000	00000	00000	00002	07	0.00000	00000	00000	00000	00000	20
23	0.00000	00000	00000	00000	00000	29	-0.00000	00000	00000	00000	00000	04
24	-0.00000	00000	00000	00000	00000	03	0.00000	00000	00000	00000	00000	01

Coefficients in the Expansion of

$$I_\nu(z) = z^\nu c^z \sum_{k=0}^{\infty} H_k(\nu) T_k^*(z/\theta), \quad 0 < \nu \leq 8,$$

$$H_k(\nu) = \sum_{r=0}^k D_{r,k} T_r^*\left(\frac{\nu-4}{4}\right), \quad i \leq \nu \leq 8.$$

$D_{r,k}, k = 15$					$D_{r,k}, k = 17$									
0	0.00000	00000	01771	42270	67220	30	0	-0.00000	00000	00311	58951	38395	04	
1	-0.00000	00000	03183	83909	27507	19	1	0.00000	00000	00560	75462	70119	70	
2	0.00000	00000	02322	96879	96424	91	2	-0.00000	00000	00410	66242	94770	88	
3	-0.00000	00000	01393	82133	18132	62	3	0.00000	00000	00247	72627	61067	23	
4	0.00000	00000	00496	65504	50190	21	4	-0.00000	00000	00124	47191	16762	27	
5	-0.00000	00000	00294	63959	94774	77	5	0.00000	00000	00053	33493	53182	92	
6	0.00000	00000	00106	46417	27482	60	6	-0.00000	00000	00019	53025	30888	83	
7	-0.00000	00000	00033	37390	50566	07	7	0.00000	00000	00006	20009	65088	59	
8	0.00000	00000	00009	80950	10935	53	8	-0.00000	00000	00001	71723	02374	82	
9	-0.00000	00000	00002	16411	40087	50	9	0.00000	00000	00000	41721	34424	71	
10	0.00000	00000	00000	45001	23104	13	10	-0.00000	00000	00000	08910	90501	28	
11	-0.00000	00000	00000	00105	95120	20	11	0.00000	00000	00000	01670	44033	47	
12	0.00000	00000	00000	01200	17577	94	12	-0.00000	00000	00000	00272	04917	40	
13	-0.00000	00000	00000	00167	70650	07	13	0.00000	00000	00000	00030	16764	25	
14	0.00000	00000	00000	00017	09325	19	14	-0.00000	00000	00000	00004	38208	94	
15	-0.00000	00000	00000	00001	00646	66	15	0.00000	00000	00000	00000	36451	80	
16	0.00000	00000	00000	00000	00795	03	16	-0.00000	00000	00000	00000	00000	90915	11
17	0.00000	00000	00000	00000	03210	01	17	-0.00000	00000	00000	00000	00309	70	
18	-0.00000	00000	00000	00000	00507	79	18	0.00000	00000	00000	00000	00009	18	
19	0.00000	00000	00000	00000	00074	99	19	-0.00000	00000	00000	00000	00014	78	
20	-0.00000	00000	00000	00000	00006	06	20	0.00000	00000	00000	00000	00001	62	
21	0.00000	00000	00000	00000	00000	34	21	-0.00000	00000	00000	00000	00000	17	
22	0.00000	00000	00000	00000	00000	02								
23	-0.00000	00000	00000	00000	00000	01								

$D_{r,k}, k = 13$					$D_{r,k}, k = 13$								
0	0.00000	00000	00052	53082	27450	27	0	-0.00000	00000	00000	50701	41137	71
1	-0.00000	00000	00094	67023	99279	93	1	0.00000	00000	00015	34740	95225	70
2	0.00000	00000	00069	50140	50131	13	2	-0.00000	00000	00011	31955	88068	53
3	-0.00000	00000	00042	21434	97273	36	3	0.00000	00000	00006	90547	60895	57
4	0.00000	00000	00021	44220	67972	08	4	-0.00000	00000	00003	53341	10088	17
5	-0.00000	00000	00009	24532	75406	17	5	0.00000	00000	00001	53740	57496	08
6	0.00000	00000	00003	42541	85967	54	6	-0.00000	00000	00000	57508	81203	41
7	-0.00000	00000	00001	10162	76501	54	7	0.00000	00000	00000	18754	81332	22
8	0.00000	00000	00000	30996	80494	55	8	-0.00000	00000	00000	45354	07404	63
9	-0.00000	00000	00000	07673	67408	95	9	0.00000	00000	00000	81349	34343	14
10	0.00000	00000	00000	01676	67055	71	10	-0.00000	00000	00000	00301	04324	94
11	-0.00000	00000	00000	00323	37542	92	11	0.00000	00000	00000	00059	56042	54
12	0.00000	00000	00000	00054	82633	67	12	-0.00000	00000	00000	00010	47982	42
13	-0.00000	00000	00000	00000	00074	26	13	0.00000	00000	00000	00001	00539	17
14	0.00000	00000	00000	00001	00924	50	14	-0.00000	00000	00000	00000	21375	20
15	-0.00000	00000	00000	00000	10017	60	15	0.00000	00000	00000	00000	02373	22
16	0.00000	00000	00000	00000	00625	64	16	-0.00000	00000	00000	00000	00194	44
17	0.00000	00000	00000	00000	00019	82	17	0.00000	00000	00000	00000	00007	31
18	-0.00000	00000	00000	00000	00013	68	18	0.00000	00000	00000	00000	00001	35
19	0.00000	00000	00000	00000	00002	55	19	-0.00000	00000	00000	00000	00000	32
20	-0.00000	00000	00000	00000	00000	33	20	0.00000	00000	00000	00000	00000	08
21	0.00000	00000	00000	00000	00000	03	21	-0.00000	00000	00000	00000	00000	01

TABLE 2 (Continued)

Coefficients in the Expansion of

$$I_\nu(x) = r^\nu c^\nu \sum_{k=0}^{\infty} h_k(\nu) r_k^\nu (x/8), \quad 0 < x \leq 1,$$

$$h_k(\nu) = \sum_{r=0}^{\infty} D_{r,k} r^\nu \left(\frac{\nu-4}{4}\right), \quad 4 \leq \nu \leq 8.$$

$D_{r,k}, k = 20$						$D_{r,k}, k = 21$					
0	0.00000	00000	00001	32405	79325	39					
1	-0.00000	00000	00002	39295	85973	63					
2	0.00000	00000	00001	77008	30857	65					
3	-0.00000	00000	00001	00009	00006	58					
4	0.00000	00000	00000	55967	83362	88					
5	-0.00000	00000	00000	24564	90062	63					
6	0.00000	00000	00000	89295	94544	68					
7	-0.00000	00000	00000	83064	55301	72					
8	0.00000	00000	00000	00007	53249	13					
9	-0.00000	00000	00000	00227	27360	50					
10	0.00000	00000	00000	00051	60997	33					
11	-0.00000	00000	00000	00010	44552	96					
12	0.00000	00000	00000	00001	00549	51					
13	-0.00000	00000	00000	00000	30115	10					
14	0.00000	00000	00000	00000	00219	79					
15	-0.00000	00000	00000	00000	00507	35					
16	0.00000	00000	00000	00000	00049	63					
17	-0.00000	00000	00000	00000	00003	32					
18	-0.00000	00000	00000	00000	00000	00					
19	0.00000	00000	00000	00000	00000	05					
20	-0.00000	00000	00000	00000	00000	01					

  

$D_{r,k}, k = 22$						$D_{r,k}, k = 23$					
0	0.00000	00000	00000	02075	71112	23					
1	-0.00000	00000	00000	05205	57765	45					
2	0.00000	00000	00000	00076	90002	29					
3	-0.00000	00000	00000	02401	84710	29					
4	0.00000	00000	00000	01254	34530	61					
5	-0.00000	00000	00000	00559	63767	30					
6	0.00000	00000	00000	00215	91423	49					
7	-0.00000	00000	00000	00077	79162	13					
8	0.00000	00000	00000	00021	63147	28					
9	-0.00000	00000	00000	00005	70596	40					
10	0.00000	00000	00000	00001	34312	07					
11	-0.00000	00000	00000	00000	20315	29					
12	0.00000	00000	00000	00000	05355	46					
13	-0.00000	00000	00000	00000	00908	25					
14	0.00000	00000	00000	00000	00137	68					
15	-0.00000	00000	00000	00000	00018	46					
16	0.00000	00000	00000	00000	00002	15					
17	-0.00000	00000	00000	00000	00000	21					
18	0.00000	00000	00000	00000	00000	01					

  

$D_{r,k}, k = 24$						$D_{r,k}, k = 25$					
0	0.00000	00000	00000	00054	20473	57					
1	-0.00000	00000	00000	00090	40547	30					
2	0.00000	00000	00000	00073	70536	75					
3	-0.00000	00000	00000	00044	14039	08					
4	0.00000	00000	00000	00024	39616	08					
5	-0.00000	00000	00000	00011	05014	44					
6	0.00000	00000	00000	00004	33943	84					
7	-0.00000	00000	00000	00001	49303	05					
8	0.00000	00000	00000	00000	45408	19					
9	-0.00000	00000	00000	00000	12297	05					
10	0.00000	00000	00000	00000	02902	70					
11	-0.00000	00000	00000	00000	00050	04					
12	0.00000	00000	00000	00000	00120	14					
13	-0.00000	00000	00000	00000	00022	70					
14	0.00000	00000	00000	00000	00003	68					
15	-0.00000	00000	00000	00000	00000	53					
16	0.00000	00000	00000	00000	00000	07					
17	-0.00000	00000	00000	00000	00000	01					

  

$D_{r,k}, k = 26$						$D_{r,k}, k = 27$					
0	-0.00000	00000	00000	00001	10068	44					
1	0.00000	00000	00000	00720	10373	20					
2	-0.00000	00000	00000	00543	97204	04					
3	0.00000	00000	00000	00330	60061	00					
4	-0.00000	00000	00000	00177	95206	07					
5	0.00000	00000	00000	00000	00070	75					
6	-0.00000	00000	00000	00001	14071	14					
7	0.00000	00000	00000	00010	61114	37					
8	-0.00000	00000	00000	00003	19099	01					
9	0.00000	00000	00000	00000	05310	08					
10	-0.00000	00000	00000	00000	20304	39					
11	0.00000	00000	00000	00000	04377	48					
12	-0.00000	00000	00000	00000	00045	45					
13	0.00000	00000	00000	00000	00147	03					
14	-0.00000	00000	00000	00000	00022	98					
15	0.00000	00000	00000	00000	00003	21					
16	-0.00000	00000	00000	00000	00000	08					
17	0.00000	00000	00000	00000	00000	04					

  

$D_{r,k}, k = 28$						$D_{r,k}, k = 29$					
0	-0.00000	00000	00000	00007	10068	44					
1	0.00000	00000	00000	00012	09266	14					
2	-0.00000	00000	00000	00009	00613	78					
3	0.00000	00000	00000	00006	00300	01					
4	-0.00000	00000	00000	00003	23500	63					
5	0.00000	00000	00000	00001	47000	78					
6	-0.00000	00000	00000	00000	50446	03					
7	0.00000	00000	00000	00000	20701	19					
8	-0.00000	00000	00000	00000	06200	09					
9	0.00000	00000	00000	00000	01710	08					
10	-0.00000	00000	00000	00000	00420	62					
11	0.00000	00000	00000	00000	00093	21					
12	-0.00000	00000	00000	00000	00018	68					
13	0.00000	00000	00000	00000	00003	70					
14	-0.00000	00000	00000	00000	00000	58					
15	0.00000	00000	00000	00000	00000	08					
16	-0.00000	00000	00000	00000	00000	01					

TABLE . (Concluded)

Coefficients in the Expansion of

$$I_\nu(z) - z^\nu e^z \sum_{k=0}^{\infty} K_k(\nu) T_k^\nu(z/\theta), \quad 0 < z \leq \theta,$$

$$K_k(\nu) = \sum_{r=0}^{\infty} D_{r,k} T_r^\nu\left(\frac{\nu-1}{4}\right), \quad 4 \leq \nu \leq 3.$$

$D_{r,k}, k = 26$						$D_{r,k}, k = 27$					
0	0.00000	00000	00000	00000	89977 33	0	-0.00000	00000	00000	00000	11057 88
1	-0.00000	00000	00000	00000	63521 41	1	0.00000	00000	00000	00000	20114 24
2	0.00000	00000	00000	00001	23186 24	2	-0.00000	00000	00000	00000	15192 24
3	-0.00000	00000	00000	00000	77703 0A	3	0.00000	00000	00000	00000	09622 4A
4	0.00000	00000	00000	00000	41550 26	4	-0.00000	00000	00000	00000	05174 30
5	-0.00000	00000	00000	00000	19087 00	5	0.00000	00000	03000	00000	02392 25
6	0.00000	00000	00000	00000	07610 87	6	-0.00000	00000	00000	00000	00962 10
7	-0.00000	00000	00000	00000	02670 51	7	0.00000	00000	07000	00000	00340 21
8	0.00000	00000	00000	00000	00829 4A	8	-0.00000	00000	00000	00000	00104 70
9	-0.00000	00000	00000	00000	00229 9A	9	0.00000	00000	00000	00000	00029 91
10	0.00000	00000	00000	00000	00057 2A	10	-0.00000	00000	00000	00000	00007 5A
11	-0.00000	00000	00000	00000	00012 8A	11	0.00000	00000	00000	00000	00001 72
12	0.00000	00000	00000	00000	00002 62	12	-0.00000	00000	00000	00000	00000 3A
13	-0.00000	00000	00000	00000	00000 40	13	0.00000	00000	00000	00000	00000 07
14	0.00000	00000	00000	00000	00000 00	14	-0.00000	00000	00000	00000	00000 01
15	-0.00000	00000	00000	00000	00000 01						

  

$D_{r,k}, k = 28$						$D_{r,k}, k = 29$					
0	0.00000	00000	00000	00000	01319 1A	0	-0.00000	00000	00000	00000	00152 90
1	-0.00000	00000	00000	00000	02401 67	1	0.00000	00000	00000	00000	00270 59
2	0.00000	00000	00000	00000	01010 5A	2	-0.00000	00000	00000	00000	00211 4A
3	-0.00000	00000	00000	00000	01156 39	3	0.00000	00000	00000	00000	00136 40
4	0.00000	00000	00000	00000	00625 0A	4	-0.00000	00000	00000	00000	00073 33
5	-0.00000	00000	00000	00000	00290 03	5	0.00000	00000	00000	00000	00036 33
6	0.00000	00000	00000	00000	00117 04	6	-0.00000	00000	00000	00000	00014 01
7	-0.00000	00000	00000	00000	00042 01	7	0.00000	00000	00000	00000	00005 01
8	0.00000	00000	00000	00000	00013 30	8	-0.00000	00000	00000	00000	00001 61
9	-0.00000	00000	00000	00000	00003 77	9	0.00000	00000	00000	00000	00000 4A
10	0.00000	00000	00000	00000	00000 9A	10	-0.00000	00000	00000	00000	00000 12
11	-0.00000	00000	00000	00000	00000 22	11	0.00000	00000	00000	00000	00000 07
12	0.00000	00000	00000	00000	00000 05	12	-0.00000	00000	00000	00000	00000 01
13	-0.00000	00000	00000	00000	00000 01						

  

$D_{r,k}, k = 30$						$D_{r,k}, k = 31$					
0	0.00000	00000	00000	00000	00017 23	0	-0.00000	00000	00000	00000	00001 40
1	-0.00000	00000	00000	00000	00031 42	1	0.00000	00000	00000	00000	00003 4A
2	0.00000	00000	00000	00000	00023 91	2	-0.00000	00000	00000	00000	00002 47
3	-0.00000	00000	00000	00000	00015 32	3	0.00000	00000	00000	00000	00001 64
4	0.00000	00000	00000	00000	00000 36	4	-0.00000	00000	00000	00000	00000 97
5	-0.00000	00000	00000	00000	00003 9A	5	0.00000	00000	00000	00000	00000 4A
6	0.00000	00000	00000	00000	00001 62	6	-0.00000	00000	00000	00000	00000 10
7	-0.00000	00000	00000	00000	00000 59	7	0.00000	00000	00000	00000	00000 07
8	0.00000	00000	00000	00000	00000 10	8	-0.00000	00000	00000	00000	00000 02
9	-0.00000	00000	00000	00000	00000 04	9	0.00000	00000	00000	00000	00000 01
10	0.00000	00000	00000	00000	00000 01						

  

$D_{r,k}, k = 3$						$D_{r,k}, k = 4$					
0	0.00000	00000	00000	00000	00000 20	0	-0.00000	00000	00000	00000	00000 07
1	-0.00000	00000	00000	00000	00000 37	1	0.00000	00000	00000	00000	00000 04
2	0.00000	00000	00000	00000	00000 20	2	-0.00000	00000	00000	00000	00000 07
3	-0.00000	00000	00000	00000	00000 10	3	0.00000	00000	00000	00000	00000 02
4	0.00000	00000	00000	00000	00000 10	4	-0.00000	00000	00000	00000	00000 01
5	-0.00000	00000	00000	00000	00000 04	5	0.00000	00000	00000	00000	00000 01
6	0.00000	00000	00000	00000	00000 02						
7	-0.00000	00000	00000	00000	00000 01						

TABLE 5

Coefficients in the Expansion of

$$I_\nu(z) = (2\pi z)^{-\frac{1}{2}} e^z \sum_{k=0}^{\infty} M_k(\nu) T_k^*(z), \quad z \in \mathbb{C},$$

$$M_k(\nu) = \sum_{r=0}^{\infty} E_{r,k} T_r^*(\nu), \quad 0 \leq \nu \leq 1.$$

r	$E_{r,k}, k=0$					r	$E_{r,k}, k=1$				
0	0.99601	36885	05338	32062	28	0	-0.00401	40331	65105	02968	37
1	-0.01629	18723	83299	78423	34	1	-0.01652	76673	31909	09023	13
2	-0.00398	00326	49903	99358	09	2	-0.00400	55537	88259	83470	52
3	0.00005	26713	08143	15867	04	3	0.00007	15305	84267	23656	84
4	0.00000	62557	24366	64173	90	4	0.00000	84304	90481	04021	30
5	-0.00000	01006	36595	12001	05	5	-0.00000	01542	85341	01459	00
6	-0.00000	00076	95123	50578	52	6	-0.00000	00117	19102	90032	59
7	0.00000	00001	45705	44979	69	7	0.00000	00002	39194	71285	79
8	0.00000	00000	07922	57176	97	8	0.00000	00000	17901	04629	29
9	-0.00000	00000	00180	95644	18	9	-0.00000	00000	00310	87745	11
10	-0.00000	00000	00007	06098	39	10	-0.00000	00000	00011	97168	51
11	0.00000	00000	00000	20817	71	11	0.00000	00000	00000	36923	95
12	0.00000	00000	00000	00523	57	12	0.00000	00000	00000	00905	40
13	-0.00000	00000	00000	00022	42	13	-0.00000	00000	00000	00040	58
14	-0.00000	00000	00000	00000	25	14	-0.00000	00000	00000	00000	43
15	0.00000	00000	00000	00000	02	15	0.00000	00000	00000	00000	04

  

r	$E_{r,k}, k=2$					r	$E_{r,k}, k=3$				
0	-0.00002	83035	61655	55270	96	0	-0.00000	06955	69590	22719	23
1	-0.00024	46146	55718	44712	36	1	-0.00000	93976	31721	38630	48
2	-0.00002	61088	96649	30277	62	2	-0.00000	06108	79764	78893	94
3	0.00001	97288	39252	72507	42	3	0.00000	09226	90028	28737	34
4	0.00000	22450	65467	74325	00	4	0.00000	00651	86021	31753	10
5	-0.00000	00673	10178	71437	91	5	-0.00000	00149	81940	84090	02
6	-0.00000	00049	79011	31089	05	6	-0.00000	00010	16748	15913	45
7	0.00000	00001	31070	19933	23	7	0.00000	00000	46811	33701	05
8	0.00000	00000	06482	59728	70	8	0.00000	00000	02270	84124	62
9	-0.00000	00000	00196	71869	04	9	-0.00000	00000	00091	52809	65
10	-0.00000	00000	00007	23110	98	10	-0.00000	00000	00003	01669	80
11	0.00000	00000	00000	29759	45	11	0.00000	00000	00000	14155	50
12	0.00000	00000	00000	00578	63	12	0.00000	00000	00000	00260	67
13	-0.00000	00000	00000	00030	06	13	-0.00000	00000	00000	00018	17
14	-0.00000	00000	00000	00000	25	14	-0.00000	00000	00000	00000	08
15	0.00000	00000	00000	00000	03	15	0.00000	00000	00000	00000	02

  

r	$E_{r,k}, k=4$					r	$E_{r,k}, k=5$				
0	-0.00000	00381	75275	85093	63	0	-0.00000	00059	25360	97692	36
1	-0.00000	06379	69369	39048	23	1	-0.00000	00688	62950	04823	26
2	-0.00000	00228	37493	95620	65	2	0.00000	00012	68210	16741	60
3	0.00000	00677	89040	26021	41	3	0.00000	00076	65226	96602	42
4	0.00000	00030	09633	14738	60	4	0.00000	00000	88362	19310	74
5	-0.00000	00014	86421	86403	41	5	-0.00000	00001	93188	29938	77
6	-0.00000	00000	65538	08421	60	6	-0.00000	00000	03329	64095	74
7	0.00000	00000	10799	78762	11	7	0.00000	00000	01865	05438	56
8	0.00000	00000	00419	65913	59	8	0.00000	00000	00030	16772	40
9	-0.00000	00000	00031	61364	69	9	-0.00000	00000	00008	47512	47
10	-0.00000	00000	00000	79674	76	10	-0.00000	00000	00000	08430	46
11	0.00000	00000	00000	06150	19	11	0.00000	00000	00000	02118	99
12	0.00000	00000	00000	00069	23	12	-0.00000	00000	00000	00002	16
13	-0.00000	00000	00000	00008	09	13	-0.00000	00000	00000	00003	44
14	0.00000	00000	00000	00000	02	14	0.00000	00000	00000	00000	05
15	0.00000	00000	00000	00000	01						

TABLE 3 (Continued)

Coefficients in the Expansion of

$$I_\nu(z) = (2\pi z)^{-\frac{1}{2}} e^{-z} \sum_{k=0}^{\infty} M_k(\nu) T_k^2(z/z), \quad z \geq 8, \quad ,$$

$$M_k(\nu) = \sum_{r=0}^{\infty} E_{r,k} T_r^2(\nu), \quad 0 \leq \nu \leq 1. \quad .$$

r	$E_{r,k}, k = 6$			
0	-0.00000	00019	78212	12066 64
1	-0.00000	00102	00031	71716 72
2	0.00000	00014	05119	69317 49
3	0.00000	00011	76123	57379 91
4	-0.00000	00000	58099	50802 29
5	-0.00000	00000	31921	89916 04
6	0.00000	00000	00007	24683 92
7	0.00000	00000	00349	06250 02
8	-0.00000	00000	00006	10226 41
9	-0.00000	00000	00001	09474 45
10	0.00000	00000	00000	03278 06
11	0.00000	00000	00000	00564 71
12	-0.00000	00000	00000	00012 91
13	-0.00000	00000	00000	00000 97
14	0.00000	00000	00000	00000 03

r	$E_{r,k}, k = 7$			
0	-0.00000	00007	30866	32786 40
1	-0.00000	00014	93852	77992 09
2	0.00000	00006	01464	43819 15
3	0.00000	00001	79069	10742 40
4	-0.00000	00000	32505	70263 18
5	-0.00000	00000	05057	01336 39
6	0.00000	00000	00554	50949 13
7	0.00000	00000	00057	87771 33
8	-0.00000	00000	00004	70904 90
9	-0.00000	00000	00000	72532 31
10	0.00000	00000	00000	02342 44
11	0.00000	00000	00000	00095 65
12	-0.00000	00000	00000	00007 45
13	-0.00000	00000	00000	00000 13
14	0.00000	00000	00000	00000 02

r	$E_{r,k}, k = 8$			
0	-0.00000	00002	21600	60720 02
1	-0.00000	00000	46346	09513 21
2	0.00000	00002	23022	07317 47
3	0.00000	00000	07097	35545 06
4	-0.00000	00000	11219	20950 25
5	-0.00000	00004	00235	05707 69
6	0.00000	00000	00198	70308 17
7	0.00000	00000	00002	18008 18
8	-0.00000	00000	00001	71492 40
9	-0.00000	00000	00000	00308 26
10	0.00000	00000	00000	00035 55
11	-0.00000	00000	00000	00006 31
12	-0.00000	00000	00000	00002 48
13	0.00000	00000	00000	00000 04

r	$E_{r,k}, k = 9$			
0	-0.00000	00000	41900	55790 09
1	0.00000	00000	09743	74528 40
2	0.00000	00000	45566	98357 54
3	-0.00000	00000	09775	02430 10
4	-0.00000	00000	02340	53868 72
5	0.00000	00000	00279	36021 34
6	0.00000	00000	00041	44833 81
7	-0.00000	00000	00003	62585 15
8	-0.00000	00000	00000	34547 15
9	0.00000	00000	00000	02647 09
10	0.00000	00000	00000	00153 59
11	-0.00000	00000	00000	00012 07
12	-0.00000	00000	00000	00000 37
13	0.00000	00000	00000	00000 04

r	$E_{r,k}, k = 10$			
0	0.00000	00000	01769	02144 80
1	0.00000	00000	38545	06038 43
2	-0.00000	00000	00663	29497 76
3	-0.00000	00000	04402	50830 16
4	0.00000	00000	00025	69456 29
5	0.00000	00000	00126	65203 78
6	-0.00000	00000	00000	92934 20
7	-0.00000	00000	00001	50474 08
8	0.00000	00000	00000	01804 50
9	0.00000	00000	00000	01063 97
10	-0.00000	00000	00000	00019 24
11	-0.00000	00000	00000	00004 25
12	0.00000	00000	00000	00000 11
13	0.00000	00000	00000	00000 01

r	$E_{r,k}, k = 11$			
0	0.00000	00000	04375	77462 10
1	0.00000	00000	05431	20942 97
2	-0.00000	00000	04280	74095 01
3	-0.00000	00000	00649	41703 33
4	0.00000	00000	00219	60452 22
5	0.00000	00000	00010	36096 61
6	-0.00000	00000	00004	14500 00
7	-0.00000	00000	00000	21035 45
8	0.00000	00000	00000	03974 60
9	0.00000	00000	00000	00114 58
10	-0.00000	00000	00000	00022 40
11	-0.00000	00000	00000	00000 27
12	0.00000	00000	00000	00000 08



TABLE 3 (Continued)

Coefficients in the Expansion of

$$I_{\nu}(z) = (2\pi z)^{-\frac{1}{2}} e^z \sum_{k=0}^{\infty} M_k(\nu) T_k^*(z/z), \quad z \geq 0,$$

$$M_k(\nu) = \sum_{r=0}^{\infty} E_{r,k} T_r^*(\nu), \quad 0 \leq \nu \leq 1.$$

r	$E_{r,k}, \gamma = 12$					r	$E_{r,k}, \gamma = 13$				
0	0.00000	00000	01202	47474	53	0	-0.00000	00000	00116	71293	14
1	-0.00000	00000	02020	10004	98	1	-0.00000	00000	01120	18598	69
2	-0.00000	00000	01269	13315	16	2	0.00000	00000	00092	51424	40
3	0.00000	00000	00224	65057	78	3	0.00000	00000	00129	25357	13
4	0.00000	00000	00065	53275	44	4	-0.00000	00000	00004	83236	87
5	-0.00000	00000	00006	74293	67	5	-0.00000	00000	00003	78732	17
6	-0.00000	00000	00001	19434	53	6	0.00000	00000	00000	10936	36
7	0.00000	00000	00000	09413	78	7	0.00000	00000	00000	04867	92
8	0.00000	00000	00000	01052	55	8	-0.00000	00000	00000	00139	04
9	-0.00000	00000	00000	00075	22	9	-0.00000	00000	00000	00033	85
10	-0.00000	00000	00000	00005	13	10	0.00000	00000	00000	00001	08
11	0.00000	00000	00000	00000	38	11	0.00000	00000	00000	00000	14
12	0.00000	00000	00000	00000	01	12	-0.00000	00000	00000	00000	01

  

r	$E_{r,k}, \gamma = 14$					r	$E_{r,k}, \gamma = 15$				
0	-0.00000	00000	00160	74526	03	0	-0.00000	00000	00021	02319	23
1	-0.00000	00000	00052	14605	52	1	0.00000	00000	00118	79215	58
2	0.00000	00000	00161	99991	75	2	0.00000	00000	00023	75611	36
3	0.00000	00000	00006	69416	97	3	-0.00000	00000	00013	62902	06
4	-0.00000	00000	00000	43079	67	4	-0.00000	00000	00001	22660	07
5	-0.00000	00000	00000	17039	12	5	0.00000	00000	00000	40915	14
6	0.00000	00000	00000	16088	04	6	0.00000	00000	00000	02134	75
7	0.00000	00000	00000	10123	39	7	-0.00000	00000	00000	00555	34
8	-0.00000	00000	00000	11155	23	8	-0.00000	00000	00000	00016	64
9	0.00000	00000	00000	00000	36	9	0.00000	00000	00000	00004	22
10	0.00000	00000	00000	00000	88	10	0.00000	00000	00000	00000	06
11	-0.00000	00000	00000	00000	01	11	-0.00000	00000	00000	00000	02

  

r	$E_{r,k}, k = 16$					r	$E_{r,k}, k = 17$				
0	0.00000	00000	00015	71000	80	0	0.00000	00000	00005	16156	99
1	0.00000	00000	00020	24021	77	1	-0.00000	00000	00010	75033	89
2	-0.00000	00000	00015	42055	63	2	-0.00000	00000	00005	44377	21
3	-0.00000	00000	00003	32009	33	3	0.00000	00000	00001	22978	90
4	0.00000	00000	00000	11110	74	4	0.00000	00000	00000	20402	63
5	0.00000	00000	00000	09615	41	5	-0.00000	00000	00000	03706	52
6	-0.00000	00000	00000	01606	73	6	-0.00000	00000	00000	00528	39
7	-0.00000	00000	00000	00118	09	7	0.00000	00000	00000	00054	09
8	0.00000	00000	00000	00016	53	8	0.00000	00000	00000	00004	81
9	0.00000	00000	00000	00000	74	9	-0.00000	00000	00000	00000	44
10	-0.00000	00000	00000	00000	10	10	-0.00000	00000	00000	00000	02

TABLE 3 (Concluded)

Coefficients in the Expansion of

$$I_\nu(z) = (2\pi z)^{-\frac{1}{2}} e^z \sum_{k=0}^{\infty} M_k(\nu) T_k^*(z/z), \quad z \geq 0,$$

$$M_k(\nu) = \sum_{r=0}^{\infty} E_{r,k} T_r^*(\nu), \quad 0 \leq \nu \leq 1.$$

r	$E_{r,k}, k = 18$				
0	-0.00000	00000	00001	41637	33
1	-0.00000	00000	00005	16350	90
2	0.00000	00000	00001	35193	84
3	0.00000	00000	00000	60207	46
4	-0.00000	00000	00000	07203	72
5	-0.00000	00000	00000	01784	52
6	0.00000	00000	00000	00148	78
7	0.00000	00000	00000	00023	13
8	-0.00000	00000	00000	00001	63
9	-0.00000	00000	00000	00000	16
10	0.00000	00000	00000	00000	01

r	$E_{r,k}, k = 19$				
0	-0.00000	00000	00000	86469	63
1	0.00000	00000	00000	94070	21
2	0.00000	00000	00000	89618	69
3	-0.00000	00000	00000	10014	39
4	-0.00000	00000	00000	04707	93
5	0.00000	00000	00000	00347	60
6	0.00000	00000	00000	00089	79
7	-0.00000	00000	00000	00005	21
8	-0.00000	00000	00000	00000	86
9	0.00000	00000	00000	00000	05

r	$E_{r,k}, k = 20$				
0	0.00000	00000	00000	13296	15
1	0.00000	00000	00000	81839	70
2	-0.00000	00000	00000	12253	81
3	-0.00000	00000	00000	09552	00
4	0.00000	00000	00000	00663	96
5	0.00000	00000	00000	00206	14
6	-0.00000	00000	00000	00014	42
7	-0.00000	00000	00000	00003	80
8	0.00000	00000	00000	00000	17
9	0.00000	00000	00000	00000	03

r	$E_{r,k}, k = 21$				
0	0.00000	00000	00000	13621	70
1	-0.00000	00000	00000	09393	20
2	-0.00000	00000	00000	14033	00
3	0.00000	00000	00000	01067	32
4	0.00000	00000	00000	00740	94
5	-0.00000	00000	00000	00035	14

TABLE 4

Coefficients in the Expansion of

$$I_{\nu}(z) (2mz)^{-\frac{1}{2}} e^{-z} \sum_{k=0}^m M_k(\nu) \mathcal{H}_k^{\nu}(0/z), \quad z \geq 0,$$

$$M_k(\nu) = \sum_{r=0}^k F_{r,k} \mathcal{H}_r^{\nu}(-\nu), \quad -1 \leq \nu \leq 0.$$

r	$F_{r,k}, k=0$				r	$F_{r,k}, k=1$				
0	0.99601	37031	10062	92849	90	-0.00001	40055	47007	09157	09
1	-0.01629	10721	84954	64801	36	-0.01652	76669	55031	29084	46
2	-0.00398	00480	51814	44059	19	-0.00400	55829	13349	37099	53
3	0.00005	26710	90531	02006	32	0.00007	15301	86829	45908	26
4	0.00000	62365	36754	49192	46	0.00000	84400	26456	50079	56
5	-0.00000	01006	25100	61567	94	-0.00000	01542	63565	00820	78
6	-0.00000	00077	10550	16600	93	-0.00000	00117	40360	08027	44
7	0.00000	00001	45403	83309	60	0.00000	00002	30774	69314	73
8	0.00000	00000	00068	01127	46	0.00000	00000	13175	70100	52
9	-0.00000	00000	00178	83779	20	-0.00000	00000	00306	86391	79
10	-0.00000	00000	00007	85314	20	-0.00000	00000	00013	46683	83
11	0.00000	00000	00000	19644	35	0.00000	00000	00000	34702	63
12	0.00000	00000	00000	00794	93	0.00000	00000	00000	01416	96
13	-0.00000	00000	00000	00018	30	-0.00000	00000	00000	00032	79
14	-0.00000	00000	00000	00000	87	-0.00000	00000	00000	00001	59
15	0.00000	00000	00000	00000	01	0.00000	00000	00000	00000	02

r	$F_{r,k}, k=2$				r	$F_{r,k}, k=3$				
0	-0.00002	83602	44301	73586	45	-0.00000	06780	57418	27298	08
1	-0.00024	46143	35595	14034	16	-0.00000	93973	87831	89049	08
2	-0.00002	61334	85720	32833	32	-0.00000	06293	46140	41325	32
3	0.00001	97205	00955	60704	30	0.00000	09224	40303	93086	01
4	0.00000	22463	61540	01929	21	0.00000	00661	58458	85473	96
5	-0.00000	00672	91651	32205	60	-0.00000	00149	67037	16843	57
6	-0.00000	00050	03500	39201	15	-0.00000	00010	35153	70318	13
7	0.00000	00001	30713	15072	78	0.00000	00000	46539	95615	60
8	0.00000	00000	07093	70172	95	0.00000	00000	02443	44770	40
9	-0.00000	00000	00193	31160	75	-0.00000	00000	00008	94404	07
10	-0.00000	00000	00008	40602	94	-0.00000	00000	00000	94936	62
11	0.00000	00000	00000	23877	67	0.00000	00000	00000	12733	93
12	0.00000	00000	00000	01006	38	0.00000	00000	00000	00576	44
13	-0.00000	00000	09000	00023	48	-0.00000	00000	00000	00013	23
14	-0.00000	00000	00000	00001	22	-0.00000	00000	00000	00000	79
15	0.00000	00000	00000	00000	02	0.00000	00000	00000	00000	01

r	$F_{r,k}, k=4$				r	$F_{r,k}, k=5$				
0	-0.00000	00265	57352	50002	07	0.00000	00008	04148	50337	71
1	-0.00000	06378	04026	84799	49	-0.00000	00607	64141	60307	30
2	-0.00000	00350	87526	82507	90	-0.00000	00058	26643	12163	46
3	0.00000	00676	14329	85938	00	0.00000	00075	60030	24413	79
4	0.00000	00036	53706	62145	91	0.00000	00004	60636	93762	90
5	-0.00000	00014	76072	41543	17	-0.00000	00001	87491	98689	65
6	-0.00000	00000	77694	01234	17	-0.00000	00000	10325	06609	39
7	0.00000	00000	10616	44039	97	0.00000	00000	01756	07641	42
8	0.00000	00000	00533	12746	75	0.00000	00000	00095	00093	61
9	-0.00000	00000	00029	87520	08	-0.00000	00000	00007	44735	92
10	-0.00000	00000	00001	40549	72	-0.00000	00000	00000	42034	34
11	0.00000	00000	00000	05190	08	0.00000	00000	00000	01561	22
12	0.00000	00000	00000	00273	15	0.00000	00000	00000	00111	22
13	-0.00000	00000	00000	00005	62	-0.00000	00000	00000	00001	54
14	-0.00000	00000	00000	00000	43	-0.00000	00000	00000	00000	20

TABLE 4 (Continued)

Coefficients in the Expansion of

$$I_\nu(z) = (2\pi z)^{-\frac{1}{2}} e^z \sum_{k=0}^{\infty} M_k(\nu) T_k^{\nu}(z/z), \quad z \geq 0,$$

$$M_k(\nu) = \sum_{r=0}^{\infty} F_{r,k} T_r^{\nu}(-\nu), \quad -1 \leq \nu \leq 0.$$

r	$F_{r,k}, k = 0$				r	$F_{r,k}, k = 7$					
0	0.00000	00013	59960	11622	06	0	0.00000	00006	39301	09412	38
1	-0.00000	00101	56762	43650	01	1	-0.00000	00014	71331	92232	90
2	-0.00000	00020	33689	14806	15	2	-0.00000	00007	62543	32163	88
3	0.00000	00011	21962	86038	42	3	0.00000	00001	95262	72913	79
4	0.00000	00001	25941	45836	72	4	0.00000	00000	42629	18632	84
5	-0.00000	00000	28974	07413	86	5	-0.00000	00000	03767	35385	82
6	-0.00000	00000	82628	54141	32	6	-0.00000	00000	00033	37800	66
7	0.00000	00000	00293	74811	61	7	0.00000	00000	00033	51503	26
8	0.00000	00000	00025	39700	84	8	0.00000	00000	00007	79054	32
9	-0.00000	00000	00001	37102	21	9	-0.00000	00000	00000	10027	47
10	-0.00000	00000	00000	13160	71	10	-0.00000	00000	00000	03997	38
11	0.00000	00000	00000	00203	33	11	-0.00000	00000	00000	00022	64
12	0.00000	00000	00000	00039	87	12	0.00000	00000	00000	00011	99
13	-0.00000	00000	00000	00000	03	13	0.00000	00000	00000	00000	25
14	-0.00000	00000	00000	00000	07	14	-0.00000	00000	00000	00000	02

r	$F_{r,k}, k = 6$				r	$F_{r,k}, k = 9$					
0	0.00000	00002	11714	43771	52	0	0.00000	00000	43443	91217	12
1	-0.00000	00000	38359	03807	00	1	0.00000	00000	91790	59079	11
2	-0.00000	00002	32674	54544	51	2	-0.00000	00000	44286	12399	43
3	-0.00000	00000	00536	43795	88	3	-0.00000	00000	11935	36055	00
4	0.00000	00000	12304	43927	06	4	0.00000	00000	02164	37310	08
5	0.00000	00000	00219	27562	54	5	0.00000	00000	00394	12088	59
6	-0.00000	00000	00227	05695	14	6	-0.00000	00000	00035	31739	54
7	-0.00000	00000	00006	29124	77	7	-0.00000	00000	00005	70557	37
8	0.00000	00000	00001	98760	23	8	0.00000	00000	00000	24809	02
9	0.00000	00000	00000	07348	00	9	0.00000	00000	00000	04434	53
10	-0.00000	00000	00000	00933	58	10	-0.00000	00000	00000	00068	15
11	-0.00000	00000	00000	00045	17	11	-0.00000	00000	00000	00020	39
12	0.00000	00000	00000	00002	39	12	-0.00000	00000	00000	00000	08
13	0.00000	00000	00000	00000	16	13	0.00000	00000	00000	00000	06

r	$F_{r,k}, k = 10$				r	$F_{r,k}, k = 11$					
0	-0.00000	00000	00480	69909	40	0	-0.00000	00000	04034	60168	90
1	0.00000	00000	38779	22005	68	1	0.00000	00000	05352	09581	43
2	0.00000	00000	01789	27353	85	2	0.00000	00000	04590	23127	67
3	-0.00000	00000	04648	71626	60	3	-0.00000	00000	00565	54033	91
4	-0.00000	00000	00173	44411	40	4	-0.00000	00000	00288	15103	69
5	0.00000	00000	00138	89705	20	5	0.00000	00000	00013	50242	17
6	0.00000	00000	00005	46060	16	6	0.00000	00000	00005	26730	32
7	-0.00000	00000	00001	77459	21	7	-0.00000	00000	00000	10656	29
8	-0.00000	00000	00000	07930	11	8	-0.00000	00000	00000	05344	87
9	0.00000	00000	00000	01176	97	9	0.00000	00000	00000	00001	25
10	0.00000	00000	00000	00062	77	10	0.00000	00000	00000	00031	02
11	-0.00000	00000	00000	00004	34	11	0.00000	00000	00000	00000	46
12	-0.00000	00000	00000	00000	30	12	-0.00000	00000	00000	00000	11
13	0.00000	00000	00000	00000	01						

TABLE 4 (Continued)

Coefficients in the Expansion of

$$I_\nu(z) = (2\pi z)^{-\frac{1}{2}} e^z \sum_{k=0}^{\infty} M_k(\nu) T_k^*(z/z), \quad z \geq 0,$$

$$M_k(\nu) = \sum_{r=0}^{\infty} F_{r,k} T_r^*(-\nu), \quad -1 \leq \nu \leq 0.$$

r	$F_{r,k}, k = 12$			
0	-0.00000	00000	01205	37348 52
1	-0.00000	00000	02065	54339 77
2	0.00000	00000	01266	86429 37
3	0.00000	00000	00272	67155 31
4	-0.00000	00000	00064	94458 64
5	-0.00000	00000	00009	37139 50
6	0.00000	00000	00001	15369 35
7	0.00000	00000	00000	14464 29
8	-0.00000	00000	00000	00947 64
9	-0.00000	00000	00000	00122 89
10	0.00000	00000	00000	00003 83
11	0.00000	00000	00000	00000 64
12	-0.00000	00000	00000	00000 01

r	$F_{r,k}, k = 13$			
0	0.00000	00000	00004	00047 99
1	-0.00000	00000	01125	60726 55
2	-0.00000	00000	00121	31000 94
3	0.00000	00000	00135	04014 57
4	0.00000	00000	00008	61210 07
5	-0.00000	00000	00004	07714 02
6	-0.00000	00000	00000	22705 81
7	0.00000	00000	00000	05322 91
8	0.00000	00000	00000	00302 97
9	-0.00000	00000	00000	00036 58
10	-0.00000	00000	00000	00002 33
11	0.00000	00000	00000	00000 14
12	0.00000	00000	00000	00000 01

r	$F_{r,k}, k = 14$			
0	0.00000	00000	00152	83527 72
1	-0.00000	00000	00048	99666 40
2	-0.00000	00000	00169	01001 04
3	0.00000	00000	00003	13641 66
4	0.00000	00000	00009	32366 51
5	0.00000	00000	00000	02194 84
6	-0.00000	00000	00000	18678 83
7	-0.00000	00000	00000	00270 66
8	0.00000	00000	00000	00106 43
9	0.00000	00000	00000	00004 66
10	-0.00000	00000	00000	00001 07
11	-0.00000	00000	00000	00000 04

r	$F_{r,k}, k = 15$			
0	0.00000	00000	00022	75729 95
1	0.00000	00000	00120	07774 39
2	-0.00000	00000	00022	23571 40
3	-0.00000	00000	00014	90697 95
4	0.00000	00000	00001	02010 53
5	0.00000	00000	00000	08209 57
6	-0.00000	00000	00000	01432 57
7	-0.00000	00000	00000	00694 77
8	0.00000	00000	00000	00005 69
9	0.00000	00000	00000	00005 50
10	0.00000	00000	00000	00000 04
11	-0.00000	00000	00000	00000 03

r	$F_{r,k}, k = 16$			
0	-0.00000	00000	00014	40479 02
1	0.00000	00000	00028	14931 16
2	0.00000	00000	00016	50090 38
3	-0.00000	00000	00003	20176 11
4	-0.00000	00000	00000	95161 17
5	0.00000	00000	00000	00854 05
5	0.00000	00000	00000	02037 54
7	-0.00000	00000	00000	00099 11
8	-0.00000	00000	00000	00022 27
9	0.00000	00000	00000	00000 50
10	0.00000	00000	00000	00000 14

r	$F_{r,k}, k = 17$			
0	-0.00000	00000	00005	16646 94
1	-0.00000	00000	00010	92090 25
2	0.00000	00000	00005	44048 82
3	0.00000	00000	00001	41019 76
4	-0.00000	00000	00000	20254 25
5	-0.00000	00000	00000	04790 75
6	0.00000	00000	00000	00515 59
7	0.00000	00000	00000	00074 02
8	-0.00000	00000	00000	00004 44
9	-0.00000	00000	00000	00000 64
10	0.00000	00000	00000	00000 02

Coefficients in the Expansion of

$$I_\nu(z) = (2\pi z)^{-\frac{1}{2}} e^z \sum_{k=0}^{\infty} M_k(\nu) I_k^*(z/x), \quad z \geq \varepsilon,$$

$$M_k(\nu) = \sum_{r=0}^{\infty} F_{r,k} I_r^*(-\nu), \quad -1 \leq \nu \leq 0.$$

$F_{r,k}, k = 18$					$F_{r,k}, k = 19$				
r					r				
0	0.00000	00000	00001	26357 60	0	0.00000	00000	00000	84820 02
1	-0.00000	00000	00005	17316 42	1	0.00000	00000	00000	96972 95
2	-0.00000	00000	00001	48638 16	2	-0.00000	00000	00000	91083 96
3	0.00000	00000	00000	61296 30	3	-0.00000	00000	00000	13040 30
4	0.00000	00000	00000	08972 61	4	0.00000	00000	00000	04887 57
5	-0.00000	00000	00000	01828 24	5	0.00000	00000	00000	00469 39
6	-0.00000	00000	00000	00294 51	6	-0.00000	00000	00000	00094 44
7	0.00000	00000	00000	00023 53	7	-0.00000	00000	00000	00007 78
8	0.00000	00000	00000	00002 41	8	0.00000	00000	00000	00000 90
9	-0.00000	00000	00000	00000 16	9	0.00000	00000	00000	00000 07
10	-0.00000	00000	00000	00000 02					

  

$F_{r,k}, k = 20$					$F_{r,k}, k = 21$				
r					r				
0	-0.00000	00000	00000	11300 91	0	-0.00000	00000	00000	13266 50
1	0.00000	00000	00000	02155 07	1	-0.00000	00000	00000	09667 03
2	0.00000	00000	00000	13938 70	2	0.00000	00000	00000	14347 32
3	-0.00000	00000	00000	09084 17	3	0.00000	00000	00000	01357 20
4	-0.00000	00000	00000	00887 05	4	-0.00000	00000	00000	00780 67
5	0.00000	00000	00000	00303 22	5	-0.00000	00000	00000	00051 69
6	0.00000	00000	00000	00021 50	6	0.00000	00000	00000	00015 45
7	-0.00000	00000	00000	00004 09	7	0.00000	00000	00000	00000 91
8	-0.00000	00000	00000	00000 27	8	-0.00000	00000	00000	00000 15
9	0.00000	00000	00000	00000 03	9	-0.00000	00000	00000	00000 01

  

$F_{r,k}, k = 22$					$F_{r,k}, k = 23$				
r					r				
0	0.00000	00000	00000	01284 32	0	0.00000	00000	00000	02115 61
1	-0.00000	00000	00000	12943 73	1	0.00000	00000	00000	01341 97
2	-0.00000	00000	00000	01620 24	2	-0.00000	00000	00000	02290 52
3	0.00000	00000	00000	01566 59	3	-0.00000	00000	00000	00190 55
4	0.00000	00000	00000	00107 15	4	0.00000	00000	00000	00125 24
5	-0.00000	00000	00000	00048 64	5	0.00000	00000	00000	00007 30
6	-0.00000	00000	00000	00002 72	6	-0.00000	00000	00000	00002 50
7	0.00000	00000	00000	00000 67	7	-0.00000	00000	00000	00000 13
8	0.00000	00000	00000	00000 04	8	0.00000	00000	00000	00000 03
9	-0.00000	00000	00000	00000 01					

  

$F_{r,k}, k = 24$					$F_{r,k}, k = 25$				
r					r				
0	-0.00000	00000	00000	00223 04	0	-0.00000	00000	00000	00347 31
1	0.00000	00000	00000	02094 53	1	-0.00000	00000	00000	00277 79
2	0.00000	00000	00000	00276 20	2	0.00000	00000	00000	00374 99
3	-0.00000	00000	00000	00253 62	3	0.00000	00000	00000	00038 10
4	-0.00000	00000	00000	00018 00	4	-0.00000	00000	00000	00020 47
5	0.00000	00000	00000	00007 90	5	-0.00000	00000	00000	00001 42
6	0.00000	00000	00000	00000 45	6	0.00000	00000	00000	00000 41
7	-0.00000	00000	00000	00000 11	7	0.00000	00000	00000	00000 02
8	-0.00000	00000	00000	00000 01					

Coefficients in the Expansion of

$$I_\nu(z) = (2\pi z)^{-\frac{1}{2}} e^z \sum_{k=0}^{\infty} M_k(\nu) T_k^*(z/z), \quad z \geq 0,$$

$$M_k(\nu) = \sum_{r=0}^k F_{r,k} T_r^*(-\nu), \quad -1 \leq \nu \leq 0.$$

r	F <sub>r,k</sub> , k = 26				
0	0.00000	00000	00000	00053	88
1	-0.00000	00000	00000	00345	72
2	-0.00000	00000	00000	00063	84
3	0.00000	00000	00000	00041	71
4	0.00000	00000	00000	00003	97
5	-0.00000	00000	00000	00001	29
6	-0.00000	00000	00000	00000	09
7	0.00000	00000	00000	00000	02

r	F <sub>r,k</sub> , k = 27				
0	0.00000	00000	00000	00057	44
1	0.00000	00000	00000	00070	39
2	-0.00000	00000	00000	00061	65
3	-0.00000	00000	00000	00009	26
4	0.00000	00000	00000	00003	34
5	0.00000	00000	00000	00000	33
6	-0.00000	00000	00000	00000	07
7	-0.00000	00000	00000	00000	01

r	F <sub>r,k</sub> , k = 28				
0	-0.00000	00000	00000	00014	31
1	0.00000	00000	00000	00056	22
2	0.00000	00000	00000	00016	37
3	-0.00000	00000	00000	00006	73
4	-0.00000	00000	00000	00000	98
5	0.00000	00000	00000	00000	21
6	0.00000	00000	00000	00000	02

r	F <sub>r,k</sub> , k = 29				
0	-0.00000	00000	00000	00009	11
1	-0.00000	00000	00000	00018	36
2	0.00000	00000	00000	00009	60
3	0.00000	00000	00000	00002	35
4	-0.00000	00000	00000	00000	52
5	-0.00000	00000	00000	00000	08
6	0.00000	00000	00000	00000	01

r	F <sub>r,k</sub> , k = 30				
0	0.00000	00000	00000	00003	72
1	-0.00000	00000	00000	00008	48
2	-0.00000	00000	00000	00004	16
3	0.00000	00000	00000	00000	99
4	0.00000	00000	00000	00000	24
5	-0.00000	00000	00000	00000	03
6	-0.00000	00000	00000	00000	81

r	F <sub>r,k</sub> , k = 31				
0	0.00000	00000	00000	00001	25
1	0.00000	00000	00000	00004	58
2	-0.00000	00000	00000	00001	30
3	-0.00000	00000	00000	00000	57
4	0.00000	00000	00000	00000	07
5	0.00000	00000	00000	00000	02

r	F <sub>r,k</sub> , k = 32				
0	-0.00000	00000	00000	00000	96
1	0.00000	00000	00000	00000	96
2	0.00000	00000	00000	00000	99
3	-0.00000	00000	00000	00000	11
4	-0.00000	00000	00000	00000	06

r	F <sub>r,k</sub> , k = 33				
0	-0.00000	00000	00000	00000	10
1	-0.00000	00000	00000	00001	06
2	0.00000	00000	00000	00000	10
3	0.00000	00000	00000	00000	13

TABLE 4 (Concluded)

Coefficients in the Expansion of

$$I_\nu(z) = (2\pi z)^{-\frac{1}{2}} e^z \sum_{k=0}^{\infty} M_k(\nu) T_k^*(z/z), \quad z > 8, \quad ,$$

$$M_k(\nu) = \sum_{r=0}^{\infty} F_{r,k} T_r^*(-\nu), \quad -1 \leq \nu \leq 0. \quad .$$

r	F <sub>r,k</sub> , k = 34				
0	0.00000	00000	00000	00000	20
1	-0.00000	00000	00000	00000	01
2	-0.00000	00000	00000	00000	22
3	-0.00000	00000	00000	00000	00
4	0.00000	00000	00000	00000	01

r	F <sub>r,k</sub> , k = 35				
0	-0.00000	00000	00000	00000	02
1	0.00000	00000	00000	00000	22
2	0.00000	00000	00000	00000	02
3	-0.00000	00000	00000	00000	03

r	F <sub>r,k</sub> , k = 36				
0	-0.00000	00000	00000	00000	04
1	-0.00000	00000	00000	00000	04
2	0.00000	00000	00000	00000	04
3	0.00000	00000	00000	00000	01

r	F <sub>r,k</sub> , k = 37				
0	0.00000	00000	00000	00000	01
1	-0.00000	00000	00000	00000	04
2	-0.00000	00000	00000	00000	01

r	F <sub>r,k</sub> , k = 38				
0	0.00000	00000	00000	00000	01
1	0.00000	00000	00000	00000	02
2	-0.00000	00000	00000	00000	01

F<sub>r,k</sub>, k = 39  
 The coefficients for k = 39  
 are all zero to 22 decimals.

r	F <sub>r,k</sub> , k = 40				
0	-0.00000	00000	00000	00000	00
1	-0.00000	00000	00000	00000	01



TABULATION OF CERTAIN FULLY SYMMETRIC  
NUMERICAL INTEGRATION FORMULAS OF DEGREE

7, 9 AND 11

by

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# TABULATION OF CERTAIN FULLY SYMMETRIC

## NUMERICAL INTEGRATION FORMULAS OF

### DEGREE 7, 9 and 11

In this paper we tabulate some fully symmetric quadrature formulas which are computed by use of the algorithm described in [1]. See also Lyness [3] for a development of a related theory, and Haber [2] for a review of this theory. The formulas can be used to approximate three different types of integrals which take the form

$$(1) \quad I(f) = \int_{R^n} w(\underline{x}) f(\underline{x}) dV(\underline{x})$$

where  $R^n$  is a fully symmetric region in Euclidean  $n$ -space  $E^n$ , that is, whenever  $\underline{x} = (x^1, \dots, x^n) \in R^n$  then  $\underline{y} \in R^n$  where  $\underline{y}$  is any point obtainable from  $\underline{x}$  by interchanging the coordinates of  $\underline{x}$  or changing the sign of the coordinates of  $\underline{x}$ . The function  $w(\underline{x})$  is a fully symmetric weight function, that is,  $w(\underline{x}) = w(\underline{y})$ , which is positive in  $R^n$ , and  $dV(\underline{x}) = dx^1 \dots dx^n$ . The three different types of integrals  $I(f)$  for which we tabulate formulas are described by

(a)  $n$ -cube,  $R^n = \{(x^1, \dots, x^n) : -1 \leq x^i \leq 1, i = 1, 2, \dots, n\}$   
 $w(\underline{x}) = 1$ ;

(b)  $n$ -sphere,  $R^n = \{(x^1, \dots, x^n) : \sum_{i=1}^n (x^i)^2 \leq 1\}$ ,  
 $w(\underline{x}) = 1$ ; and

(c) infinite  $n$ -space,  $R^n = \{(x^1, \dots, x^n) : \sum_{i=1}^n (x^i)^2 < \infty\}$   
 $w(\underline{x}) = \exp \{-\sum_{i=1}^n (x^i)^2\}$ .