REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

37[2.10].—FRANK STENGER, *Tabulation of Certain Fully Symmetric Numerical Integration Formulas of Degree* 7, 9 *and* 11. Paperbound xeroxed report having 8 typewritten pages and 45 pages of tables prepared from computer output; see microfiche card, this issue.

There are three regions considered, these being (i) the hypercube $|x_i| < 1$, (ii) the hypersphere $\sum x_i^2 < 1$, and (iii) the integral over all space with weight function $\exp[-\sum x_i^2]$, respectively. For each (d, n), $d = 7, 9, 11$, $n = 1, 2, \cdots, 20$, there are listed the weights and abscissas of two n-dimensional rules of degree d for each region. Sixteen-significant-decimal figures are listed and the author claims fourteen-place accuracy. In addition, simple explicit expressions are given for the coefficients and evaluation points of the formulas of degree $d = 3$ and 5.

The underlying theory corresponding to these rules is given in [1] and the place of that theory in the current state of the art of multidimensional quadrature is discussed in Haber [2].

The rules are fully symmetric. That is, the same weight is attached to the function value $f(x_1, x_2, \cdots, x_n)$ as is attached to all distinct function values of the form $f(\pm x_{i_1}, \pm x_{i_2}, \cdots, \pm x_{i_n})$ where $i_1, i_2, \cdots, i_n$ is a permutation of $1, 2, \cdots, n$, any combination of signs being allowed.

For $d = 7, 9$ (or 11), the values of x_i are restricted to five (or seven) values 0, $\pm u$, $\pm v$ (and $\pm w$). In the cases of the hypercube and the infinite integral these include the Gaussian abscissas of that degree. Thus, the degree 9 rules involve only a subset of the points which would be required by the product Gaussian rule of the same degree. For the hypersphere u, v (and w) depend also on n.

These tables fill a gap in the literature and Dr. Stenger has performed a useful service in preparing them.

W. G.

1. J. MCNAMEE & F. STENGER, "Construction of fully symmetric numerical integration formulas," *Numer. Math.*, v. 10, 1967, pp. 327–344.
2. S. HABER, "Numerical evaluation of multiple integrals," *SIAM Rev.*, v. 12, 1970, pp. 481–526.

38[2.10, 2.25].—P. C. CHAKRAVARTI, *Integrals and Sums, Some New Formulae for their Numerical Evaluation*, The Athlone Press, University of London, 1970, x + 89 pp., 24 cm. Price $7.25.

This monograph reports the results of the author's research over a period of ten years.

The author's primary interest seems to be in constructing quadrature formulas which use finite-difference expansions. A prototype is Gregory's quadrature formula.

He has constructed a large number of variants which deal with different situations. These include those in which there is a weighting function e^{px}, those in which endpoints do not coincide with grid points and those in which endpoint connections at these inconvenient points are expressed in terms of derivatives. The weighting function $(x - x_0)^\alpha$ is also treated but on a less ambitious scale. The coefficients in this case involve Riemann zeta functions and related functions. These are tabulated and in general the user has to resort to numerical interpolation to calculate the coefficients. The finite-difference enthusiast will surely find many new and complicated expansions here.

As is conventional in this field there is no discussion about the convergence of these expansions. Numerical examples are chosen from the rather limited subset of functions for which the expansions happen to converge.

This reviewer found one part of the book to be of more general interest. The author derives a generalization of the Euler-Maclaurin summation formula. When $F(x) = e^{px}f(x)$, the asymptotic expansion on the right-hand side in the conventional form

$$h \sum_{r=0}^{n-1} F(x_r + sh) - \int_{x_0}^{x_n} F(t)\,dt \simeq \sum_{r=0}^{N} h^{r+1} \frac{B_{r+1}(s)}{(r+1)!} \{F^{(r)}(x_n) - F^{(r)}(x_0)\} + O(h^{N+2})$$

may be transformed into

$$\sum_{r=0}^{N} h^{r+1} \frac{e^{sph}\ D_r(s, ph)}{r!} \{e^{px_n}f^{(r)}(x_n) - e^{px_0}f^{(r)}(x_0)\} + O(h^{N+2}),$$

where the function $D_r(s, q)$ has many interesting elementary properties, some of which are quite tricky. Some of these are generalizations of analogous properties of the Bernoulli functions and the Euler functions. The author presents an excellent account of this function. In an appendix the author establishes the close connection between this function and the higher transcendental function

$$\phi(z, -r, s) = \sum_{j=0}^{\infty} (s + j)^r z^j .$$

The section about the D-functions would have certainly reached a wider and responsive audience if it had been published as a paper in a journal. However, the bulk of the book serves mainly as a repository for finite-difference expansions.

J. N. L.

39[2.20].—J. H. Garcia Rodriguez & F. Reveron Osio, *Tablas para la Resolucion de Ecuaciones de Tercero y Quinto Grado* (Tables for the Solution of Equations of the Third and Fifth Degree), Universidad de los Andes, Facultad de Ingenieria, Escuela de Ingenieria Electrica, Merida, Venezuela, 1968, 201 pp.

For solving the general cubic $Ay^3 + By^2 + Cy + D = 0$, after it is reduced to

(1) $$X^3 + pX + q = 0$$

by the transformation $y = X - B/3A$, the authors give, on pp. 17–176, tables for the roots of (1) for $p = -100(1)100$, $q = 0(1)100$, to 5D (6S for about 70 percent of the