

ori transformations". The last term denotes efforts to facilitate the computation of a neighboring solution of a given (or computed) solution. The techniques range from old classical approaches to the use of local or moving coordinate systems which combine analytic and computational methods. The authors state no theorems nor do they give rigorous derivations; instead, they demonstrate the rationale of the various lines of attack and try to illuminate them by well-chosen examples. Naturally, they refer to the relevant literature for a more thorough treatment.

The third part of the book is preceded by a section on basic facts about differential equations (selected so as to prepare for the later discussion) and a section on numerical methods which, very briefly, gives some of the principal concepts. In this section, a variety of classes of numerical methods is discussed (including some not so well-known ones). The authors make an interesting attempt at an evaluation of their relative merit. Again, the reader has to resort to the quoted literature for any hard-core facts, if he does not already know them. Throughout the book, the main emphasis rests on initial-value problems. In the first two parts of the book, some attention is given to boundary-value problems.

While these parts may serve as a welcome guide to the literature, it is the third part on transformations which makes the book a very valuable contribution to numerical analysis as well as to scientists who have to solve differential equations. Though many of the techniques described will need some further development and computational experience (as the authors freely admit), it is hoped that their exhibition will stimulate worthwhile efforts in that direction.

H. J. S.

42[7].—JACQUES DUTKA, *The Square Root of 2 to 1,000,000 Decimals*, ms. of 200 computer sheets + 1 supplementary page, deposited in the UMT file.

The decimal value of the square root of 2 to one million places is herein presented on 200 computer sheets, each containing 5000 decimal digits. A supplementary page gives the succeeding 82 decimal places.

This carefully checked calculation required a total of about 47.5 hours of computer time on the IBM 360/91 system at Columbia University.

Further details and pertinent background information have been given by the author in a paper [1] appearing elsewhere in this journal.

J. W. W.

1. JACQUES DUTKA, "The square root of 2 to 1,000,000 decimals," *Math. Comp.*, v. 25, 1971, pp. 927–930.

43[8].—H. LEON HARTER & D. B. OWEN, Editors, *Selected Tables in Mathematical Statistics*, Markham Publishing Co., Chicago, 1970, vii + 405 pp., 29 cm. Price \$5.80 cloth.

This book is Volume I of a projected series of volumes of mathematical tables prepared under the aegis of the Institute of Mathematical Statistics. A list of the tables and their authors follows:

1. Cumulative Noncentral Chi-Square Distribution—Haynam, Govindarajulu, and Leone.
2. Exact Sampling Distribution of the Two-Sample Kolmogorov-Smirnov Criterion  $D_{mn}$ —Kim and Jennrich.
3. Critical Values and Probability Levels for the Wilcoxon Rank Sum Test and the Wilcoxon Signed Rank Test—Wilcoxon, Katti, and Wilcox.
4. Null Distribution of the First Three Product-Moment Statistics for Exponential, Half-Gamma, and Normal Scores—Lewis and Goodman.
5. Tables to Facilitate the Use of Orthogonal Polynomials for Two Types of Error Structures—Stewart.

The first table is misnamed, as it consists of two tables, one giving the power of the chi-square test for levels of significance .001, .005, .01, .025, .05, and .1, for degrees of freedom  $\nu = 1(1)30(2)50(5)100$ , and for noncentrality parameter  $\lambda = 0(.1)1(.2)3(.5)5(1)40(2)50(5)100$ , and the other giving the noncentrality parameter for the aforementioned degrees of freedom and levels of significance and for cumulative noncentral chi-square distribution levels of .1(.02).7(.01).99. Thus, the table should be renamed "Tables related to the cumulative noncentral chi-square distribution."

Table 2 is in two parts. Part 1 gives initial values  $C$  for the Kolmogorov-Smirnov test for  $m \leq n = 1(1)25$ , corresponding to all levels of significance ranging from .001 to .14, and includes the largest  $C$  corresponding to a level of significance below .001 and the smallest  $C$  corresponding to a level of significance above .1. Part 2 gives critical values for  $\alpha = .001, .005, .01, .025, .05, .1$ .

Part 1 of Table 3 gives the critical values (and exact level of significance) which have smallest significance level greater than and largest significance level less than the one-tail nominal levels .005, .01, .025, and .05 for all combinations of  $m$  and  $n$  ranging from 3 to 50 for the Wilcoxon rank sum test. Part 2 gives exact probability levels for all possible rank totals which have probability between .0001 and .5 for sample sizes  $n = 5(1)50$ .

Table 4 contains the results of a Monte Carlo investigation to obtain the sampling distribution of the product-moment statistic for samples drawn from the unit normal, mean 1 exponential, and half-gamma (mean 1 and shape parameter .5) distribution. They are tabulated for probability levels .001, .002, .005, .01, .02, .025, .05, .1, .9, .95, .975, .98, .99, .995, .998, .999, for lags 1, 2, and 3, for sample sizes 11(1)40 for the normal and half-gamma data and 11(1)174(2)250(10)500, 900, 2500, 5000, 9000 and for exponential data.

Table 5 is in two parts. For  $n = 3(1)20$ , where  $n$  is the number of data points, Part 1 gives the orthonormalizing factor and transformation matrix  $T$  for transforming from an orthogonal polynomial fit of degree  $k$ ,  $k = 1(1) \min(n, 4)$ , to a fit for the "natural" independent variables of the problem. Also tabulated are  $TT'$  and standard deviations and upper 97.5% confidence limit for (1) an estimated dependent variable from the set of data used to produce the fit, (2) an estimated new random observation of a dependent variable for a given set of independent variables, and (3) the residual between observed and estimated dependent variable.

Part 2 considers a nonstandard error model, wherein the error in an observation is the sum of the errors of its predecessors plus a contribution for error in this observation also. As an aid in using orthogonal polynomials to fit polynomials to such data, the following are tabulated: (1) the transformation matrix  $T$  as before, (2) two alterna-

tive transformation matrices  $S$  and  $M$ , used to determine the “natural” parameters directly from the data without using an orthogonal polynomial fit first, and (3) tables to aid in determining the predicted value of the independent variable and its variance. These tables also cover the range  $n = 3(1)20$ ,  $k = 1(1) \min(n, 4)$ .

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44[8].—CHARLES E. LAND, *Tables of Critical Values for Testing Hypotheses about Linear Functions of the Normal Mean and Variance*, Department of Statistics, Oregon State University, Corvallis, Oregon 97331, ms. of 4 typewritten pp. and 59 computer sheets deposited in the UMT file.

These unpublished tables constitute an appendix to a paper that is published elsewhere [1].

The critical values define the uniformly most powerful unbiased level  $\alpha$  tests of hypotheses  $H_\theta: \mu + \lambda\sigma^2 = \theta$ , where  $\lambda$  and  $\theta$  are arbitrary numbers and  $\mu$  and  $\sigma^2$  are the mean and variance of a normal distribution, against one-sided (Table 1) and two-sided (Table 2) alternatives.

The critical value  $t(\nu, \xi, \alpha)$  given in Table 1 is the  $\alpha$ th quantile of the distribution with density proportional to

$$f_\nu(t | \xi) = (\nu + t^2)^{-(\nu+1)/2} \exp\{(\nu + 1)\xi t / (\nu + t^2)^{1/2}\}.$$

The tabulation is to 3D for  $\alpha = .0025, .005, .01, .025, .05, .1, .25, .50, .75, .90, .95, .975, .99, .995$ , and  $.9975$ ;  $\xi = 0(.1).5(.25)2(.5)5(1)10(2)20(5)50(10)100$ ; and  $\nu = 2(1)10(2)20(5)50(10)100(20)200(50)500(100)1000$ .

The two-sided critical values  $t_1(\nu, \xi, \alpha)$  and  $t_2(\nu, \xi, \alpha)$  satisfy the two equations

$$\int_{t_1}^{t_2} f_\nu(t | \xi) dt = (1 - \alpha) \int_{-\infty}^{\infty} f_\nu(t | \xi) dt,$$

$$\int_{t_1}^{t_2} t(\nu + t^2)^{-1/2} f_\nu(t | \xi) dt = (1 - \alpha) \int_{-\infty}^{\infty} t(\nu + t^2)^{-1/2} f_\nu(t | \xi) dt.$$

They are given to 3D in Table 2 for  $\alpha = .005, .01, .02, .05, .1, .2$ , and  $.5$ ;  $\nu = 2(2) 20(10)100$ ; and the same range of  $\xi$  as in Table 1 except for increasing omissions of small positive  $\xi$ , beginning at  $\nu = 6$ .

Details of the methods used in calculating these tables on a CDC 3300 system are supplied by the author in a four-page introduction.

J. W. W.

1. C. E. LAND, “Confidence intervals for linear functions of the normal mean and variance,” *Ann. Math. Statist.*, v. 42, 1971. (To appear.)