

For parabolic and hyperbolic problems, the question of stability is studied using the von Neumann method, which is based on a harmonic decomposition of the error. Other methods used for stability analysis include a matrix method for parabolic problems and a method due to Courant, Friedrichs, and Lewy for hyperbolic problems.

The author states that the book is aimed at science and engineering students in the second or third year of their undergraduate studies. No specialized knowledge of mathematics is assumed beyond undergraduate courses in calculus and in matrix theory. In particular, a knowledge of the theory of partial differential equations is not assumed. It seems, however, that in this country the book would be more appropriate for a student at a more advanced level with a good first course in numerical analysis.

The book is clearly written and provides a good introduction to the subject. However, probably because of the attempt to treat a large amount of material in a short space, certain criticisms are perhaps inevitable. First, there are so many methods presented that the reader is likely to become confused. It might have been better to have presented fewer methods and given more comparative evaluations and numerical results. More discussion of iterative methods for solving large linear systems would have been helpful—for instance, semi-iterative methods. Also, the treatment of the S.O.R. method for elliptic problems is very brief; in particular, there is very little discussion on the use of the method in practical cases. Only a very limited class of hyperbolic equations are considered. These involve regions with boundaries parallel to a coordinate axis. Also, a brief summary of the properties of characteristics would have been helpful. For elliptic equations some mention of the existing knowledge of discretization errors (e.g., Gershogrin's results and more recent work) would seem appropriate. Finally, it should be mentioned that the chapter on linear algebra has some errors (for example, it is stated that if two matrices have the same eigenvalues then they are similar).

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9 [7].—L. S. BARK, N. I. DMITRIEVA, L. N. ZAKHAR'EV & A. A. LEMANSKII, *Tablitsy Sobstvennykh Znachenii Uravneniia Mat'e (Tables of Characteristic Values of the Mathieu Equation)*, Computing Center, Acad. Sci. USSR, Moscow, 1970, xi + 151 pp., 27 cm. Price 1.81 rubles.

For the Mathieu equation in the canonical form  $d^2y/dx^2 + (p - 2q \cos 2x)y = 0$ , these tables give to 7S (in floating-point form) the characteristic values for both the even and odd periodic solutions, for a more extensive range of the parameter  $q$  than heretofore.

More explicitly, the first table (pp. 3–86) contains the characteristic values  $a_{2n}$ ,  $a_{2n+1}$  (associated with the even periodic solution) for  $n = 0(1)15$  and  $q = 0.1(0.1)100$ . A continuation of this table (pp. 87–111) gives these characteristic

values for  $n = 16(1)50$  and  $q = 1(1)100$ .

The characteristic values  $b_{2n}$ ,  $b_{2n+1}$  (associated with the odd periodic solution) are tabulated on pp. 112–150 for  $q = 0.1(0.1)100$  and  $n = 0(1)m$ , where  $m$  increases from 1 to 9 for  $b_{2n}$  and from 0 to 8 for  $b_{2n+1}$ , with increasing values of  $q$ .

Immediate comparison of these tables is possible with the considerably abridged 8D table (Table 20.1) in the NBS *Handbook* [1]. Such a comparison has revealed to this reviewer that the Russian values were simply chopped at seven significant figures, thereby resulting in last-figure tabular errors approaching a unit. It might be pointed out here that greater precision (over the more restricted ranges of  $n \leq 7$  and  $q \leq 25$ ) can also be obtained from another NBS publication [2], in conjunction with the relations  $a_n = be_n - 2q$ ,  $b_n = bo_{n+1} - 2q$ , and  $s = 4q$ .

An introduction to the present tables describes their contents and preparation and includes three illustrative examples of the application of appropriate interpolative procedures. The appended list of seven references does not include any of the pertinent NBS publications, which contain extensive bibliographies relating to Mathieu functions.

Despite such defects, these tables contain much new numerical information, constituting a valuable addition to that available in previous tables of the characteristic values of the Mathieu equation.

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1. MILTON ABRAMOWITZ & IRENE STEGUN, Editors, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, National Bureau of Standards, Applied Mathematics Series, No. 55, U. S. Government Printing Office, Washington, D. C., 1964.
2. *Tables Relating to Mathieu Functions*, National Bureau of Standards, Applied Mathematics Series, No. 59, U. S. Government Printing Office, Washington, D. C., 1967.

10 [7].—YUDELL L. LUKE, *The Special Functions and Their Approximations*, Academic Press, New York, 1969, vol. I, xx + 349 pp., vol. II, xx + 485 pp., 23 cm. Price \$19.50 per volume.

The special functions of mathematical physics are, simply, those functions which arise most frequently in the classical problems of applied mathematics and physics. Setting aside the elementary functions (logarithmic, exponential, circular and hyperbolic), there remain a number of well-known, named functions which are of very wide application. They are usually functions of more than one variable, satisfying fairly simple linear differential equations from which many basic properties may be derived. Such properties have been explored by many authors; Y. L. Luke's contribution has been the development of a unified basis for these special functions with special attention to methods for their computation. They are all treated as special cases of the hypergeometric functions.

Treatment of the Gaussian hypergeometric function  ${}_2F_1$  embraces the Legendre functions, the incomplete beta function, the complete elliptic functions of the first and second kinds, and the familiar systems of orthogonal polynomials. The confluent hypergeometric function  ${}_1F_1$  includes the Bessel functions and their relatives, and the incomplete gamma function. Connections with a still wider class of functions are demonstrated in a discussion of Meijer's  $G$ -function, a generalization of the hypergeometric function. It is shown that each of the most commonly used functions