REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

17[2.05, 2.20, 2.35, 3, 4, 5, 6].—L. COLLATZ, G. MEINARDUS, H. UNGER & H. WERNER, Editors, *Iterationsverfahren, Numerische Mathematik, Approximationtheorie*, Birkhäuser Verlag, Basel, 1970, 257 pp., 25 cm. Price sFr. 36.—.

This book contains the proceedings of three conferences held at the Mathematics Research Institute, Oberwolfach, Germany. The three conferences were:

I. Nonlinear Problems in Numerical Analysis, November 17–23, 1968, organized by L. Collatz and H. Werner,

II. Numerical Methods in Approximation Theory, June 8–14, 1969, organized by L. Collatz and G. Meinardus, and

III. Iterative Methods in Numerical Analysis, November 16–22, 1969, organized by L. Collatz and H. Unger.

A list of the titles of the papers (in translation) and short reviews of each follows.

I. 1. BROSOWSKI, B., HOFMANN, K.-H., SCHAFER, E., and WEBER, H.: *Continuity of metric projections*. Some conditions on a normed linear space X are given to assure that a (set-valued) metric projection P_V onto a subspace V is continuous in an appropriate sense. Also, some conditions are found on X, V and $P_V(f)$ which guarantee that the classical Kolmogorov criterion characterizing best approximations holds for some best approximation.

2. BROSOWSKI, B., HOFMANN, K.-H., SCHAFER, E., and WEBER, H.: *Metric projections on linear subspaces of $C_0[Q, H]$*. Continuity of (set-valued) metric projections of spaces of continuous H-valued functions on a locally compact Hausdorff space is considered (where H is a real pre-Hilbert space). Sets of points in Q where best approximations coincide are involved.

3. FREHSE, J.: *On the convergence of difference and other approximation methods for nonlinear variational problems*. Some methods for approximately minimizing $\int_a^b f(x, u, u')\,dx = 0$ over an appropriate space are considered. In particular, convergence is established for some Ritz, difference, and interval perturbation methods. Variational problems in multidimensional spaces are considered.

4. LAASONEN, P.: *On a method for solving nonlinear systems of equations*. In order to overcome the drawback that a matrix of derivatives is needed in applying Newton's method to a system of nonlinear equations, a scheme is devised which requires no derivatives. It is shown to be quadratically convergent under appropriate assumptions.

5. LANCASTER, P.: *Spectral properties of operator functions*. Results are obtained for the eigenvalues of operators of the form $D(\lambda) = A_0\lambda^l + \cdots + A_{l-1}\lambda + A_l$, where λ is a complex number and $A_0, A_1, \cdots, A_l$ are bounded linear operators mapping a Banach space into itself (and A_0 has a bounded inverse.)

6. MAYER, H.: *Estimates for the defect vector in the solution of linear systems with inaccuracies in the data and their numerical evaluation*. Methods for estimating δ_x in

$(A + R)(x + \delta x) = b + f$ with $|r_{ij}| \leqq \epsilon$, $|f_i| < \delta$ in terms of ϵ, δ, A (or A^{-1}) and b are developed. Numerical methods avoiding the use of A^{-1} are presented.

7. NITSCHE, J.: *Convergence of the Ritz-Galerkin method for nonlinear operator equations.* Error bounds for $x - x_n$ are obtained where $Ax = f$ (A nonlinear on a Hilbert space H) and x_n is obtained by the Ritz-Galerkin method. With appropriate assumptions, it is shown that if x_n is chosen from a subspace H_n then it is almost best in the sense that $||x - x|| \leqq \phi(||f||) \inf ||x - \xi||$.

8. NIXDORFF, K.: *Nonlinear computational methods in course finding.* Two methods for analyzing sounding measurements using several waves of equal frequencies are discussed. The methods are appropriate for use with digital and analog computers.

9. NIXDORFF, K.: *Remarks on the application of the harmonic balance.* This is a short description of the intent of the lectures.

10. REIMER, M.: *Semidefinite Peano kernels of stable difference forms.* The kernel K in difference forms

$$Ly = \sum_{\nu=0}^{k} \sum_{\mu=0}^{m} h^{\mu} a_{\nu}^{(\mu)} y_{\nu}^{(\mu)} = h^{p+1} \int_0^k y^{(p+1)}(x + h + t) K(t)\, dt$$

is investigated.

11. WETTERLING, W.: *On minimal conditions and Newton iteration in nonlinear optimization.* Conditions are obtained assuring the existence of a local minimum for a constrained optimization problem: minimize $F(x)$ over $x \in R^n$ such that $f_j(x) \leqq 0$, $j = 1, \cdots, m$. F and f_j are not assumed to possess convexity properties. A result on the applicability of the Newton method is included.

12. ZELLER, K.: *Newton-Chebyshev approximation.* The problem of determining a rational starting function $f_0(x)$ in Heron's method for approximating the function $\sqrt{x}$, so that after n steps $||f_n(x)/\sqrt{x} - 1||$ is minimized, is discussed.

II. 1. ANSELONE, P. M.: *Abstract Riemann integrals, monotone approximations, and generalizations of Korovkin's theorem.* A method of extending the Riemann integral from the continuous functions to a larger class is generalized to a procedure for extending other positive linear functionals or operators from (partially) ordered linear spaces to larger ones. Results on sets of uniform convergence for such functionals and operators are obtained as well as a Korovkin-type theorem for arbitrary partially ordered Banach spaces.

2. CHENEY, E. W., and PRICE, K.: *Minimal interpolating projections.* Let $X = C(T)$ where T is a compact Hausdorff space, and let Y be an n-dimensional subspace. Given $t_1, \cdots, t_m \in T$ and $y_1, \cdots, y_n \in Y$, $Px = \sum x(t_i)y_i$ is called an interpolating projection of X onto Y. A generalized Kolmogorov-criterion for characterizing the minimal interpolating projections from X onto Y (i.e. those with minimal $||P||$) is obtained. As a corollary, it is shown that, if Y is a Haar subspace and P is minimal, then $||P|| = 1$ or $\sum |y_i|$ has $n + 1$ critical points.

3. COLLATZ, L.: *Approximation theory and applications.* Some examples of practical problems involving differential and integral equations are presented to illustrate how many of the standard questions in theoretical approximation theory are too special for applications, while some of the very general investigations seem to have no applicability. For example, the usual nonlinear exponential approximation problem occurs relatively seldom in practice, while a reasonable approximation process for

the boundary-value problem $\Delta u = e^u$ in a square, with $u = c$ on the boundary, leads to a 2-dimensional exponential approximation problem.

4. GILBERT, R. P.: *Integral operator methods for approximating solutions of Dirichlet problems.* Approximate methods for solving the Dirichlet problem associated with $\Delta u(x) - P(r^2)u(x) = 0$ for $x \in D$, an appropriate domain in E^n, are considered. (Here $r = ||x||$ and P is assumed to be nonnegative and in $C'[0, a]$, $a = \sup_{x \in 0} ||x||$.) Several of the classical methods for solving Laplace's equation (the case when $P \equiv 0$) such as the methods of Fredholm integral equations, balayage, Rayleigh-Ritz, and images are extended, leading to natural procedures in approximation theory for estimating solutions. An appendix by K. Atkinson discusses some numerical methods.

5. HAUSSMANN, W.: *Multidimensional Hermite interpolation.* A representation for two-dimensional (0, 1)-Hermite polynomial interpolation is obtained which is used to obtain convergence results. It is shown that for certain grids on $E = [-1, 1] \times [-1, 1]$, called ρ-normal, the Hermite polynomial converges uniformly to any $f \in C(E)$. The rate of convergence of the Fejér operators (Hermite interpolation at the zeros of the Chebyshev polynomial) is determined for $f \in \text{Lip}\ \alpha$.

6. LOCHER, F., and ZELLER, K.: *Approximation on gridpoints.* Many numerical approximation schemes involve fitting a function f on a domain A by functions in a class V by choosing $p \in V$ such that p and f agree on some grid of points in A. Some of the relevant questions, such as form of the grid, representation of the function, *a priori* error estimates, *a posteriori* error estimates and exchange methods for improving the approximation, are discussed.

7. LUPAS, A.: *On the approximation by linear positive operators.* The Szasz-Mirakyan operator is a positive linear operator defined on a class of functions W with domain $[0, \infty)$. It is shown that these operators preserve the convexity (or nonconvexity) of any $f \in W$, and that if f is a polynomial of degree m, then so is sf. Moreover, a kind of r-order variation diminishing property is defined, and the general Baskakov operators are shown to possess it.

8. SMITH, L. B.: *Using interactive graphical computer systems on approximation problems.* A review of on-line graphical computer systems for mathematical problems and a discussion of their advantages for approximation theory is given. A least squares system is discussed in some detail, and some areas where on-line systems would be useful are suggested.

III. 1. DÖRING, B.: *A theorem on a class of iterative methods considered by Grebenjuk.* A simple general principle for deriving several classes of higher order methods for the solution of nonlinear operator equations in Banach spaces is presented. A class of methods studied by Grebenjuk using majorants is examined in a new way to produce existence, uniqueness and convergence theorems under simpler conditions and with much improved error estimates.

2. GEKELER, E.: *Relaxation methods for a class of nonlinear systems.* Two theorems are established on the convergence of relaxation methods applied to the nonlinear systems arising in the numerical solution of Hammerstein integral equations. The theorems apply, for example, to the systems generated by the method of Theodorsen and Wittich applied to a certain conformal mapping problem.

3. NIETHAMMER, W.: *Acceleration of the convergence of one-step iterative methods by summation.* A k-step iterative method for solving a system of linear equations is

obtained from a given one-step method by averaging the last k-iterates. Results on the domains of convergence and acceleration are obtained, both of which may be much larger than the domain of convergence of the original method.

4. WACKER, H. J.: *A method for nonlinear boundary value problems.* To solve the operator equation $T(y) = 0$, the problem is embedded in a family $T(s, y) = 0$ with $0 \leqq s \leqq 1$, and such that $T(0, y) = 0$ is easily solvable, and such that $T(1, y) = T(y)$. For a sequence of s's, the solution for s_i can be used as a starting value for the computation at s_{i+1}.

LARRY L. SCHUMAKER

Department of Mathematics
University of Texas
Austin, Texas 78712

18[2.10].—V. I. KRYLOV & A. A. PAL'TSEV, *Tables for Numerical Integration of Functions with Logarithmic and Power Singularities*, translated from Russian, Israel Program for Scientific Translations, Jerusalem, 1971, iv + 172 pp., 25 cm. Price $10.—.

The original edition of these tables was published in 1967 by the "Nauka i Tekhnika" Publishing House in Minsk.

Herein are tabulated the elements of four Gaussian quadrature formulas involving the respective weight functions $x^\alpha \ln(e/x)$, $x^\beta \ln(e/x) \ln[e/(1 - x)]$, $\ln(1/x)$, and $x^\beta e^{-x} \ln(1 + x^{-1})$. The range of integration for the first three is the interval (0, 1), while that for the fourth is $(0, \infty)$. The tabular points (nodes) and corresponding weight coefficients are uniformly presented to 15S in floating-point format, and the number of points extends from 1 to 10, inclusive. In Table 1 the exponent α assumes the values $-0.9(0.01)0(0.1)5$, while in Tables 2 and 4 the exponent β assumes the values 0(1)5.

Only the material in Table 3 appears to have been published elsewhere. An 8S table was given by Anderson [1] and an extensive 30S table appears in the book of Stroud & Secrest [2], which confirms the accuracy of Table 3.

Two examples of the application of Table 1 are presented, and interpolation with respect to α in that table is discussed in detail.

A bibliography of six items contains a reference to the paper of Anderson but not to the work of Stroud & Secrest, which presumably was not available to the authors.

J. W. W.

1. D. G. ANDERSON, "Gaussian quadrature formulae for $\int_0^1 - \ln x f(x)\, dx$," *Math. Comp.*, v. 19, 1965, pp. 477–481.
2. A. H. STROUD & DON SECREST, *Gaussian Quadrature Formulas*, Prentice-Hall, Englewood Cliffs, N.J., 1966. (See *Math. Comp.*, v. 21, 1967, pp. 125–126, RMT **14**.)

19[2.20].—B. DEJON & P. HENRICI, Editors, *Constructive Aspects of the Fundamental Theorem of Algebra*, John Wiley & Sons, New York, 1969, vii + 337 pp., 23 cm. Price $9.95.

These papers are the published proceedings of a symposium held on June 5–7,