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21[3].—B. N. Pshenichnyi, *Necessary Conditions for an Extremum*, Marcel Dekker, Inc., New York, 1971, xviii + 230 pp., 24 cm. Price $11.50.

Dr. B. N. Pshenichnyi is one of the most prolific contributors to the literature on optimization. He has written highly regarded papers on optimality conditions, on optimization algorithms, on optimal control problems, on minimax problems and on games. The breadth of his research experience has contributed substantially to the well-balanced perspective and maturity of this beautifully executed monograph on optimality conditions. Dr. Pshenichnyi avoids cumbersome results. Because of this, his monograph does not include an exhaustive study of constraint qualifications, nor the most general discrete maximum principle, nor the Pontryagin maximum principle for the general case, nor the most general possible optimality conditions for constrained optimization problems. Instead, by masterfully introducing a few simplifying, but not particularly constraining, assumptions, Dr. Pshenichnyi manages to present in an elegant fashion most of the important optimality conditions without getting bogged down in the very messy analysis which is required to treat the most general case. The result is an excellent and most readable middle-level text. The quality of the translation, carried out by Dr. K. Makowski under the supervision of the translation editor, Dr. L. W. Neustadt, both of the University of Southern California, is impeccable, and they deserve a commendation.

As to the actual contents of the monograph, which consists of an introduction and five chapters, Dr. Pshenichnyi starts out in the introduction with a few basic concepts of functional analysis and with some properties of convex sets and of convex functionals. Chapter I continues with more advanced properties of convex functionals defined on Banach spaces, and, in particular, it presents their directional derivatives.

Chapter II derives optimality conditions for convex programming problems in Banach space, with and without differentiability assumptions. In particular, the Kuhn-Tucker conditions are obtained.

Chapter III is primarily concerned with formulas for the directional derivatives of functions of the form $\mu(x) = \max_{\alpha \in Z} \varphi(x, \alpha)$, with x and α elements of Banach spaces, which occur in minimax problems.

Chapter IV obtains a necessary condition of optimality for general mathematical programming problems in Banach space. This condition is quite similar to, though not as general as, the condition in [1]. It is then related to a very general optimality condition due to Dubovitskii and Milyutin [2].

Finally, Chapter V applies the general optimality condition obtained in Chapter IV to a number of specific problems to obtain specialized optimality conditions. These problems include the classical mathematical programming problem, mathematical programming problems with a continuum of constraints, minimax problems, Chebyshev approximation problems, linear optimal control problems with state space constraints, problems in convex inequalities, the moment problem and a discrete maximum principle.

The book concludes with an annotated bibliography.

E. POLAK

Department of Electrical Engineering and Computer Sciences
University of California
Berkeley, California 94720

1. H. HALKIN & L. W. NEUSTADT, "General necessary conditions for optimization problems," *Proc. Nat. Acad. Sci. U.S.A.*, v. 56, 1966, pp. 1066–1071.
2. A. Y. DUBOVITSKII & A. A. MILYUTIN, "Extremum problems with constraints," *Soviet Math. Dokl.*, v. 4, no. 2, 1963, pp. 452–455.

22[4].—LEON LAPIDUS & JOHN H. SEINFELD, *Numerical Solution of Ordinary Differential Equations*, Academic Press, New York, 1971, xii + 299 pp., 24 cm. Price $16.50.

This book provides an excellent survey of the most important methods for the numerical solution of initial value problems for ordinary differential equations.

After an introductory chapter, a chapter is devoted to each of the following topics: Runge-Kutta and allied single-step methods; stability of multi-step and Runge-Kutta methods; predictor-corrector methods; extrapolation methods; methods for stiff equations. The treatment of each topic is up-to-date (the references run through 1969). For example, the hybrid methods due to Butcher (1965), Danchick (1968), and Kohfeld and Thompson (1967) are all described in detail.

Much of the recent work on numerical methods for ordinary differential equations has been evolutionary in the sense that existing methods have been mutated, usually by the introduction of additional parameters, so as to improve certain desirable characteristics such as stability. This evolutionary process is brought out very clearly in the present book where the different mutations are compared and the more successful ones are noted as worthy of further development.

The authors pay great attention to the question of which methods are best computationally. The numerical results in the literature are discussed. In addition, almost all the methods described were used to solve test problems and the results are summarized. On the basis of all this information, the authors recommend highly the fourth order Adams-Bashforth-Adams-Moulton predictor-corrector, the third order semi-implicit Runge-Kutta method of Rosenbrock, and a fifth order Runge-Kutta method of Butcher.