Chapter IV obtains a necessary condition of optimality for general mathematical programming problems in Banach space. This condition is quite similar to, though not as general as, the condition in [1]. It is then related to a very general optimality condition due to Dubovitskii and Milyutin [2].

Finally, Chapter V applies the general optimality condition obtained in Chapter IV to a number of specific problems to obtain specialized optimality conditions. These problems include the classical mathematical programming problem, mathematical programming problems with a continuum of constraints, minimax problems, Chebyshev approximation problems, linear optimal control problems with state space constraints, problems in convex inequalities, the moment problem and a discrete maximum principle.

The book concludes with an annotated bibliography.

E. POLAK

Department of Electrical Engineering and Computer Sciences
University of California
Berkeley, California 94720

1. H. HALKIN & L. W. NEUSTADT, "General necessary conditions for optimization problems," *Proc. Nat. Acad. Sci. U.S.A.*, v. 56, 1966, pp. 1066–1071.
2. A. Y. DUBOVITSKII & A. A. MILYUTIN, "Extremum problems with constraints," *Soviet Math. Dokl.*, v. 4, no. 2, 1963, pp. 452–455.

22[4].—LEON LAPIDUS & JOHN H. SEINFELD, *Numerical Solution of Ordinary Differential Equations*, Academic Press, New York, 1971, xii + 299 pp., 24 cm. Price $16.50.

This book provides an excellent survey of the most important methods for the numerical solution of initial value problems for ordinary differential equations.

After an introductory chapter, a chapter is devoted to each of the following topics: Runge-Kutta and allied single-step methods; stability of multi-step and Runge-Kutta methods; predictor-corrector methods; extrapolation methods; methods for stiff equations. The treatment of each topic is up-to-date (the references run through 1969). For example, the hybrid methods due to Butcher (1965), Danchick (1968), and Kohfeld and Thompson (1967) are all described in detail.

Much of the recent work on numerical methods for ordinary differential equations has been evolutionary in the sense that existing methods have been mutated, usually by the introduction of additional parameters, so as to improve certain desirable characteristics such as stability. This evolutionary process is brought out very clearly in the present book where the different mutations are compared and the more successful ones are noted as worthy of further development.

The authors pay great attention to the question of which methods are best computationally. The numerical results in the literature are discussed. In addition, almost all the methods described were used to solve test problems and the results are summarized. On the basis of all this information, the authors recommend highly the fourth order Adams-Bashforth-Adams-Moulton predictor-corrector, the third order semi-implicit Runge-Kutta method of Rosenbrock, and a fifth order Runge-Kutta method of Butcher.

There are some errors and omissions. On page 2, it is stated that every system of m first-order differential equations is equivalent to one mth order equation, which is not true. In the discussion of Runge-Kutta methods, only scalar equations are treated in detail and the impression is given that the results can always be extended to systems, although Butcher has given an example where this is not the case. The coverage of the theoretical aspects of stability is a little too superficial, and the definition of A-stability is not quite right. There seems to be no mention in the text of the work of Butcher proving that there is no explicit n-step Runge-Kutta method of order n if $n \geqq 5$. Finally, there is no mention of the following topics: statistical estimates for round-off error; rigorous error bounds using interval arithmetic; methods using Chebyshev series; methods using splines; methods using Lie series; and methods for differential equations with right-hand sides which have singularities or discontinuities. However, these are minor complaints about an otherwise excellent book.

COLIN W. CRYER

Computer Science Department
University of Wisconsin
Madison, Wisconsin 53706

23[5,6].—V. S. VLADIMIROV, *Equations of Mathematical Physics*, Marcel Dekker, Inc., New York, 1971, vi + 418 pp., 24 cm. Price $19.75.

This book is a translation into English of a textbook which first appeared in Russian in 1967 and which is being used at Moscow University. It contains a comprehensive treatment of the standard boundary value problems for second order partial differential equations. Its most distinguishing feature is its consistent use of distribution theory. The presentation is elegant, thorough and yet easily accessible. The author has succeeded in integrating the distribution theory into the analysis of the boundary value problems of mathematical physics in a natural and coherent way.

The book consists of six chapters. The first chapter introduces the necessary background material from analysis, including a brief presentation, partly without proofs, of the basic facts of Lebesgue integration and of operator theory in the Hilbert space L_2. It then describes the physical interpretation of the common second order partial differential equations and discusses their classification. The second chapter presents distribution theory, including Fourier transformation of tempered distributions. A number of specific concrete distributions which are needed in the remainder of the text are analyzed here in detail. The third chapter treats the concept of a fundamental solution in distribution terminology, with special application to the initial value problem for hyperbolic and parabolic equations. The fourth chapter develops the theory of integral equations, in particular, the Fredholm theorems and the Hilbert-Schmidt theory. The fifth chapter, which is the longest, deals with elliptic equations. Among the topics covered are eigenvalue problems and expansion theorems, the Sturm-Liouville problem and its reduction via Green's function to an integral equation, harmonic functions with the mean value property and the maximum principle, and special functions occurring in connection with special domains. The final chapter on mixed problems for hyperbolic and parabolic equations covers the method of