REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

28[2.10, 7].—A. Ghizzetti & A. Ossicini, *Quadrature Formulae*, Academic Press' New York, 1970, 192 pp., 24 cm. Price $10.00.

During the last fifteen years the authors have developed an approach to the construction of quadrature formulae based on earlier work of Von Mises and Radon in 1935. Briefly, given a set of abscissas $\{x_i\}$, a weight function $w(x)$ and a linear differential operator

$$Ef = \sum_{k=0}^{n} a_k(x) f^{(n-k)}(x),$$

one may construct a quadrature formula

$$\int_a^b w(x)f(x)\,dx = \sum_{h=0}^{n-1} \sum_{i=1}^{m} A_{h,i} f^{(h)}(x_i) + Rf$$

having the property that it is exact, i.e., $Rf = 0$ for all functions $f(x)$ satisfying $Ef = 0$. The method of construction involves determining a set of analytic solutions $\phi_j(x)$, $j = 0, 1, \cdots, m$, to the differential equation

$$E^*\phi = w(x),$$

where E^* is the adjoint operator of E. Using the Green-Lagrange identity, a relatively straightforward formula for the weights $A_{h,i}$ ensues which involves linear combinations of the functions $\phi_i(x)$ and their derivatives evaluated at $x = x_i$.

This approach is naturally of theoretical interest and was entirely new to this reviewer. Chapters 1, 2 and 5 describe this theory. These consist essentially of an edited translation of the authors' research papers. They are compactly written, provide little in the way of motivation and are just as difficult to follow as the average research paper. But they are in English (and not Italian) and, in providing the translation, the authors have carried out a very useful service.

The book contains six chapters and a largely disjoint bibliography. Over half the book is contained in Chapters 3 and 4. These are essentially devoted to the individual derivation of virtually every quadrature formula of specified polynomial degree of which this reviewer has ever heard. In Chapter 3, a concise derivation of the required properties of the special functions required for this purpose is given. This includes topics such as the Bernoulli and Jacobi polynomials and the Gamma function. In Chapter 4, a concise derivation of each set of quadrature rules is carried out, one by one. These chapters are also written in the 'research journal' style but are almost devoid of references.

It is only human for a reader who is familiar with this topic to compare this approach with the standard approach. One difficulty which is present in this approach but not present in standard treatments arises in cases in which one is seeking a derivative free quadrature formula. The $m(n - 1)$ arbitrary parameters which occur in the choice of $\phi_i(x)$ have to be chosen to make the weights $A_{h,i} = O_i h \neq 0$. This introduces a linear constraint problem. Once this is solved or circumvented, the derivation of a quadrature formula follows the standard familiar pattern, with occasionally some minor variant.

The only 'new' formulae which I noticed consisted of a set due to Rebolia and Varna which were analogues of up to five-point Gaussian type formulae involving, besides function values, up to four derivatives at each point. Apart from this short collection, the authors apparently have not used their theory to provide any new quadrature formulae or to derive any hitherto undiscovered properties of known quadrature formulae.

To summarize, the part of this book which deals with the theory should be of interest to research workers in the field of numerical quadrature. The remainder of the book consists of a repository of special derivations of quadrature formulae using this new method, and may be of interest to research workers.

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29[2.20, 2.35, 3, 13.35].—E. POLAK, *Computational Methods in Optimization: A Unified Approach*, Academic Press, New York, 1971, xvii + 329 pp., 24 cm. Price $17.50.

The intent of this detailed book is to present in a unified manner almost all of the important algorithms invented to date for solving nonlinear programming, optimal control, root finding, and boundary value problems. The first chapter contains the statements of the problems to be solved, the John, Kuhn-Tucker, and Pontryagin conditions which characterize solutions to these problems, and an exposition of simple prototype models of algorithms for solving these problems. The second chapter deals with methods for finding points in R^n which minimize continuously differentiable functions and then applies these to the problem of finding solutions to unconstrained discrete optimal control and unconstrained continuous optimal control problems. Included are the methods of steepest descent, golden section search (for unconstrained problems in one variable), Newton-Raphson, local variations, conjugate gradients, variable metric (Davidon-Fletcher-Powell), and a modified quasi-Newton method of the author's. Several modifications of the above methods are described. In Chapter 3, the Newton-Raphson method for solving nonlinear equations is used as a basis for algorithms to solve equality constrained optimization problems in R^n, boundary value and discrete optimal control problems, and boundary value and continuous optimal control problems.

Algorithms for solving the general nonlinear programming problem with inequality and equality constraints are described in Chapter 4. Interior and exterior penalty function methods, methods of centers, methods of feasible directions, and gradient projection methods are covered. The use of some of these methods for solving optimal control problems is explained. Chapter 5 deals with discrete optimal control problems