33[6].—PHILIP M. ANSELONE, *Collectively Compact Operator Approximation Theory and Applications to Integral Equations*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1971, xiii + 138 pp., 24 cm. Price $12.50.

One widely used numerical method for solving a linear Fredholm integral equation of the second kind,

$$x(s) - \int_0^1 k(s, t)x(t)\,dt = y(s) \qquad (0 \leqq s \leqq 1), \tag{1}$$

is to approximate the integral by an appropriate quadrature formula. This leads to a system of linear equations by collocation. An approximation to the exact solution is obtained in the form

$$x_n(s) = K_n x_n(s) + y(s), \tag{2}$$

where K_n is the operator defined by

$$K_n x(s) = \sum_{j=1}^{n} w_j k(s, t_j)x(t_j), \tag{3}$$

where the w_j's and the t_j's are weights and nodes for the quadrature rule used for the integral in (1). The operator K_n may be regarded as an approximation to the exact integral operator K.

In this way, one obtains a collection of operators $\{K_n\}_{n=1}^{\infty}$. In a typical situation, where $k(s, t)$ in (1) is continuous, the K_n's may be regarded as operators on $C[0, 1]$ into itself. Moreover, in such cases, the collection $\{K_n\}$ has an additional property called *collective compactness*, that is, the unit ball of $C[0, 1]$ is mapped by the K_n's into *a fixed compact* set.

This book presents a systematic study of collectively compact operators by an author who is directly responsible for the development of this subject matter. The Galerkin method or the projection method for Eq. (1) is a special case of collectively compact operator approximations. Also, this method may be applicable to not only Eq. (1) but also to eigenvalue problems of linear integral operators. The reviewer feels that this book is a "must" for those who wish to engage in a serious study of numerical techniques for linear and nonlinear integral equations.

The organization of the book is excellent. Theorems are clearly stated and proofs are carried out in full. The book is essentially self-contained.

Chapter 1 of this book is devoted to that part of the general theory of linear operators needed in the subsequent development and to a quick development of approximation theory for collectively compact operators. In Chapter 2 and Chapter 3, applications to integral equations are discussed in length. As an example of an application, the approximate solution of the transport equations is included. A brief section in Chapter 3 directs the reader to references on applications of this method to boundary value problems of partial differential equations.

Chapter 4 of this book discusses spectral approximation by collectively compact operators. This chapter should be very valuable because many useful theorems (e.g., Theorems 4.7 and 4.8) are proved in full.

Chapter 5 may be regarded as one in functional analysis in which collectively compact sets of operators are compared with bounded sets and with totally bounded sets.

The last chapter (Chapter 6) is mainly devoted to applications to nonlinear integral equations via linearization by Newton's method.

There are two appendices. The first one by the author summarizes basic properties of compact operators and provides a convenient reference for reading this book. The second appendix, by Joel Davis, discusses practical implementation of some of the methods in this book and provides numerical examples (Table 1 to Table 6). There is a bibliography at the end of the book which cites 99 references.

As a whole, this book gives, in a relatively few pages (136 pages), a first-hand account of essential parts of collectively compact operator theory which have been developed in the past decade, mainly by the author and his colleagues.

This book may be used in an advanced graduate course or in a research seminar. In the former, the instructor may feel the need to supplement his lecture with appropriate exercises since no exercises are provided in this book.

YASUHIKO IKEBE

Center for Numerical Analysis
The University of Texas
Austin, Texas 78712

34[7].—P. F. BYRD & M. D. FRIEDMAN, *Handbook of Elliptic Integrals for Engineers and Scientists,* Springer-Verlag, New York, 1971, xvi + 358 pp., 24 cm. Price $18.50.

The first edition of this volume appeared in 1954 and, to my surprise, I find that it was never reviewed in these annals. This valuable and useful tome I am sure is quite well known. Nonetheless, some descriptive remarks are in order. Its title is a good summary of its contents. It contains over 3000 important formulas to facilitate the reduction and evaluation of elliptic integrals. Also included are short tables of the elliptic integrals of the first and second kind, Jacobi's q-function, Heuman's Λ_0-function and Jacobi's (K-multiplied) zeta-function.

The second edition is unfortunately not much different from the first. The authors recognize that since 1954 numerous computational methods for efficient calculation of elliptic integrals and Jacobian elliptic functions have been published. Further, an abundance of new tabular material has appeared. To improve the first edition by augmenting its contents to include this material would, in the view of the authors, necessitate another volume and must be deferred. Thus, the first edition is reproduced essentially without change, except for a supplementary bibliography and corrections. Unfortunately, the bibliography is incomplete, especially with regard to tabular material, and not all errors noted in the first edition have been corrected.

Errata to the first edition have been recorded in these annals [1], [2], [3], [4]. We have examined these data against the new edition and find some errors still persist. For example, consider the evaluation of

$$\Pi(\phi, \alpha^2, 1) = \int_0^{\varphi} \frac{\alpha\theta}{(1 - \alpha^2 \sin^2 \theta) \cos \theta}.$$