

Numerical Investigation of Certain Asymptotic Results in the Theory of Partitions

By M. S. Cheema* and W. E. Conway

Abstract. A numerical investigation of some of the asymptotic formulas in partitions is made. Comparisons with actual computed values show that in certain cases only the relative error tends to zero and the errors are significant. Only in the case of $p_k(n)$ is it found that only a few terms of the asymptotic series are sufficient to obtain the exact value.

1. **Introduction.** Let $p_k(n)$ be defined by

$$\prod_{i=1}^{\infty} (1 - x^i)^{-k} = \sum_{n=0}^{\infty} p_k(n)x^n.$$

In an earlier paper, Cheema and Haskell [2] obtained asymptotic results for $p_k(n)$. These results are useful in obtaining similar results for $u_k(n)$, the number of k -line partitions of n . More recently, Gordon and Houten [5] derived the generating functions for $b_k(n)$, the number of k -line partitions of n , whose nonzero parts decrease strictly along rows, and extended their results to obtain the generating function for $b(n)$, the number of plane partitions of n , whose nonzero parts decrease strictly along rows. They also derived generating functions for the number of k -line and plane partitions under various other restrictions. MacMahon [7] had obtained the generating function for $u_k(n)$ and $u(n)$, the number of k -line and plane partitions of n , and Wright [12] obtained an asymptotic formula for $u(n)$. Following a similar technique, Gordon and Houten [4] have obtained an asymptotic formula for $b(n)$. In this article, we make a numerical investigation of some of these asymptotic results. The asymptotic results for $b(n)$ or $u(n)$, comparable in accuracy to those for $p(n)$ obtained by Hardy-Ramanujan-Rademacher, probably do not exist. Lehmer [8] investigated the asymptotic results for $p(n)$ and showed that $\alpha n^{1/2}$ terms of the series are sufficient so that the error will be less than $\frac{1}{2}$ in absolute value. The generating functions for $b(n)$ or $u(n)$ do not belong to the class of modular functions, thus making it impossible to obtain asymptotic formulas of the same type as that for $p(n)$. Our comparison with actual values of $b(n)$, $b_4(n)$, $p_2(n)$, $p_3(n)$ show certain interesting features. Results indicate that the error terms in the asymptotic formulas for $b_k(n)$ and $b(n)$ are quite significant, and only the relative errors tend to zero as n tends to infinity. Our computations confirm that results similar to those of Lehmer [8] also hold for the series for $p_k(n)$. Lehmer obtained estimates like $A_q(n) = O(q^{1/2+\epsilon})$ for

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$A_q(n)$. Similar estimates have been obtained for $A_q(k, n)$ by Agarwal and Gandhi [1], with the extra condition that $(k, q) = 1$. If one could extend these estimates for all q , then the proof that $\alpha n^{1/2}$ terms of the series for $p_k(n)$ are sufficient so that the error will be less than $\frac{1}{2}$ in absolute value will follow. Actual computations show the number of terms required is even much smaller.

2. Generating Functions. The generating functions for $p_k(n)$, $u(n)$, $u_k(n)$, $b(n)$, and $b_k(n)$ are given by the following:

$$(2.1) \quad \prod_{i=1}^{\infty} (1 - x^i)^{-k} = \sum_{n=0}^{\infty} p_k(n)x^n,$$

$$(2.2) \quad \prod_{i=1}^{\infty} (1 - x^i)^{-1} = \sum_{n=0}^{\infty} u(n)x^n,$$

$$(2.3) \quad \prod_{i=1}^{\infty} (1 - x^i)^{-\min(i, k)} = \sum_{n=0}^{\infty} u_k(n)x^n,$$

$$(2.4) \quad \prod_{i=1}^{\infty} (1 - x^i)^{-\lceil (i+1)/2 \rceil} = \sum_{n=0}^{\infty} b(n)x^n,$$

$$(2.5) \quad P(x)^{\lceil k/2 \rceil} Q(x)^{2\{k/2\}} \prod_{i=1}^{k-2} (1 - x^i)^{\lceil (k-i)/2 \rceil} = \sum_{n=0}^{\infty} b_k(n)x^n,$$

where

$$P(x) = \prod_{i=1}^{\infty} (1 - x^i)^{-1}, \quad Q(x) = \prod_{i=1}^{\infty} (1 - x^{2i-1})^{-1},$$

$[k]$ and $\{k\}$ denote the integer and fractional part of k .

3. Asymptotic Results. These are stated by the following:

For $r \leq 12$,

$$(3.1) \quad p_{2r}(n) = \frac{(r/3)^{1/2}}{2^r \pi^{r-1} i} \sum_{q=1}^{\infty} A_q(2r, n) q^{r-1} \frac{d^r}{dn^r} \left[\frac{J_1 \left[\frac{2\pi i \left(\frac{r}{3} \left(n - \frac{r}{12} \right) \right)^{1/2}}{q} \right]}{(n - r/12)^{1/2}} \right].$$

For $2r + 1 < 24$,

$$(3.2) \quad p_{2r+1}(n) = \frac{1}{2^{r+1/2} \pi^{r+1}} \sum_{q=1}^{\infty} A_q(2r + 1, n) q^{r+1/2} \frac{d^{r+1}}{dn^{r+1}} \left[\frac{\sinh \left[\frac{\pi \left(\frac{4r+2}{3} \left(n - \frac{2r+1}{24} \right) \right)^{1/2}}{q} \right]}{\left(n - \frac{2r+1}{24} \right)^{1/2}} \right],$$

where

$$A_q(k, n) = \sum_p (\omega_{p,q})^k \exp(-2\pi ipn/q),$$

$\omega_{p,q}$ being certain $24q$ th roots of unity, summation extends over all p , $1 \leq p \leq q$, such that $(p, q) = 1$.

$$(3.3) \quad b(n) \sim 2^{-3/4} (3\pi\zeta(3))^{-1/2} N^{-49/24} \exp\left\{\frac{3}{2} \zeta(3) N^2 + \frac{\pi^2}{24} N + C - \frac{\pi^4}{3456\zeta(3)}\right\}$$

where

$$N = \left(\frac{n}{\zeta(3)}\right)^{1/3}, \quad C = \int_0^\infty \frac{y \log y}{e^{2\pi y} - 1} dy.$$

A similar result for $u(n)$ is given in Wright [12]. Asymptotic results for $u_k(n)$ and $b_k(n)$ are derived from those of $p_k(n)$. In particular,

$$(3.4) \quad u_k(n) \sim (k-1)! (k-2)! \cdots 2! \frac{d^{k(k-1)/2}}{dn^{k(k-1)/2}} (p_k(n)),$$

$$b_4(n) \sim \frac{1}{i\sqrt{3}} \frac{d^3}{dn^3} \left[\frac{J_1\left(2\pi i \left(\frac{1}{3} \left(n - \frac{1}{12}\right)\right)\right)^{1/2}}{\left(n - \frac{1}{12}\right)^{1/2}} \right],$$

which reduces to

$$(3.5) \quad b_4(n) \sim \frac{\pi^7}{81} \sum_{m=0}^\infty \frac{\pi^{2m} \left(\frac{1}{3} \left(n - \frac{1}{12}\right)\right)^m}{m! (m+4)!}.$$

Similarly, the asymptotic series for $p_2(n)$ and $p_3(n)$ reduce to

$$(3.6) \quad p_2(n) = \frac{1}{18} \sum_{q=1}^\infty A_q(2, n) \sum_{m=0}^\infty \frac{\pi^{2m+3} \left(\frac{1}{3} \left(n - \frac{1}{12}\right)\right)^m}{m! (m+2)! q^{2m+3}},$$

$$(3.7) \quad p_3(n) = \frac{1}{2^{3/2} \pi^2} \sum_{q=1}^\infty A_q(3, n) q^{3/2}$$

$$\cdot \left(\left(\frac{A-B}{2} \right) \exp\left(\frac{\pi}{q} \left(2n - \frac{1}{4}\right)^{1/2}\right) - \left(\frac{A+B}{2} \right) \exp\left(-\frac{\pi}{q} \left(2n - \frac{1}{4}\right)^{1/2}\right) \right)$$

where

$$A = \left(\frac{3}{2} + \frac{\pi^2}{q^2} \left(n - \frac{1}{8}\right)\right) / 2 \left(n - \frac{1}{8}\right)^{5/2}, \quad B = \frac{3\pi}{2\sqrt{2} q \left(n - \frac{1}{8}\right)^2}.$$

The coefficients $A_q(n) = A_q(1, n)$ first occurred in Hardy-Ramanujan's asymptotic series for $p(n)$ and were studied by Lehmer [8], Whiteman [10], [11], Rademacher [9] and others. Selberg discovered the formula

$$(3.8) \quad A_q(n) = \frac{1}{4} \left(\frac{k}{3}\right)^{1/2} \sum (-1)^{|l/6|} \exp\left(\frac{\pi il}{6k}\right).$$

Summation is over integers l such that $(l, 6) = 1$ and $l^2 \equiv 1 - 24n \pmod{24q}$ and $|x|$ denotes the integer nearest to x . Such a formula should also exist for $A_q(k, n)$. Gandhi [3] has studied $A_q(k, n)$ and has obtained some of their properties.

A table of $A_q(n)$ for $q \leq 18$ was given by Hardy and Ramanujan [6]. It is easy to obtain a similar table for $A_q(k, n)$. For example,

$$\begin{aligned}
 A_1(k, n) &= 1, \\
 A_2(k, n) &= (-1)^n, \\
 A_3(k, n) &= 2 \cos\left(\frac{2n\pi}{3} - \frac{k\pi}{18}\right), \\
 A_4(k, n) &= 2 \cos\left(\frac{n\pi}{2} - \frac{k\pi}{8}\right), \\
 A_5(k, n) &= 2 \cos\left(\frac{2n\pi}{5} - \frac{k\pi}{5}\right) + 2 \cos\left(\frac{4n\pi}{5}\right).
 \end{aligned}
 \tag{3.9}$$

For $k = 1$, these reduce to the formulas obtained by Hardy and Ramanujan.

The value of C in (3.3) was obtained by numerical integration of

$$\int_0^\infty \frac{y \log(y)}{e^{2\pi y} - 1} dy = \int_0^1 \frac{y \log y}{e^{2\pi y} - 1} dy + \int_1^\infty \frac{y \log(y)}{e^{2\pi y} - 1} dy.$$

The second integral is easy to evaluate. The substitution $2\pi y = u$ reduces the first integral to $(1/4\pi^2) \int_0^{2\pi} (u \log(u/2\pi)/(e^u - 1)) du$ which is written as

$$\frac{1}{4\pi^2} \int_0^{2\pi} \left(\frac{u}{e^u - 1} - 1\right) \log\left(\frac{u}{2\pi}\right) du - \frac{1}{2\pi}$$

or as

$$\frac{1}{4\pi^2} \int_0^\epsilon \sum_{k=1}^\infty \frac{u^k}{k!} B_k \log\left(\frac{u}{2\pi}\right) du + \frac{1}{4\pi^2} \int_\epsilon^{2\pi} \left(\frac{u}{e^u - 1} - 1\right) \log\left(\frac{u}{2\pi}\right) du - \frac{1}{2\pi}$$

where B_k are the well-known Bernoulli numbers. Taking $\epsilon = .1$ in these integrals and adding the second integral, we obtain $C = -.08271057185022965322$.

In the asymptotic calculations for $b(n)$, $\zeta(3) = 1.20205690315959428540$ and $\pi = 3.14159265358979323846$ were used.

NUMERICAL RESULTS

$b_4(n)$

n	Exact Values	Asymptotic values
50	7861151	10769606
100	81121690641	102674670000
150	122723468142192	149502323956804
200	65197255804176049	77556919390336123
250	17410617101196225420	2036877853915727707
299	2563183245773772069521	2961963804368820491881

$b(n)$		
n	Exact Values	Asymptotic values
50	134035641	131450402
100	40271460800459	39515125499612
150	1695070342263449292	1664761688894903178
200	23318651621796353657014	22920072294220827550007
250	150270591063287164944741750	147801858831116594095994117
299	470334179683225117053159476103	462868009280107428736730772768

4. Comments on Numerical Results. Comparisons of exact values with those given by asymptotic formulas for $b_4(n)$ and $b(n)$ indicate that the error in the asymptotic results are quite significant. However, in the case of $p_2(n)$ and $p_3(n)$, the asymptotic results are excellent. We were able to calculate the exact values of $p_2(50)$, $p_2(100)$, $p_3(100)$, and $p_3(200)$ by the asymptotic formula. In fact, three terms of the asymptotic series gave the exact value of $p_2(50)$, while four terms were needed for $p_2(100)$. For $p_3(100)$, four terms of the series were needed, while for $p_3(200)$ it took six terms. The double precision accuracy of twenty eight digits for the CDC 6400 was exceeded by $p_3(299)$. However, the asymptotic series after four terms gave $p_3(299) = 37464893330090346069484200521$ which is larger than the exact value by only 14. This value was not improved by using additional terms up to and including eighteen, which is not surprising as we think the error is in the first four terms and is due to the limit on the accuracy of double precision arithmetic on the CDC 6400.

The first six terms of the series for $p_3(200)$ are listed as are the first four terms of the series for $p_2(100)$, together with their exact values in the following table. In the appendix, we give the exact values of $b_4(n)$, $b(n)$, $p_2(n)$ and $p_3(n)$ for n ranging from one to fifty only. The complete table up to two hundred and ninety nine has been published on the microfiche card that accompanies this issue of the journal. All computations were made on the CDC 6400 at the University of Arizona Computing Center.

First six terms of series for $p_3(200)$

$$\begin{aligned}
 &56669336086583184772691.88617 \\
 &\quad 875179402.76032 \\
 &\quad -33461.76862 \\
 &\quad 64.90130 \\
 &\quad 4.30801 \\
 &\quad .57670 \\
 \text{sum} &= 56669336086584059918702.66388 \\
 \text{exact value} &= 56669336086584059918703
 \end{aligned}$$

First four terms of series for $p_2(100)$

1843645804262.16325
 16512.32889
 - 10.51372
 1.70633
 sum = 1843645820765.68475
 exact value = 1843645820766

APPENDIX

n	$b_4(n)$	$b(n)$	$p_2(n)$	$p_3(n)$
1	1	1	2	3
2	2	2	5	9
3	4	4	10	22
4	7	7	20	51
5	11	12	36	108
6	19	21	65	221
7	29	34	110	429
8	46	56	185	810
9	70	90	300	1479
10	106	143	481	2640
11	156	223	752	4599
12	232	348	1165	7868
13	334	532	1770	13209
14	482	811	2665	21843
15	686	1224	3956	35581
16	971	1834	5822	57222
17	1357	2725	8470	90882
18	1894	4031	12230	1 42769
19	2612	5914	17490	2 21910
20	3592	8638	24842	3 41649
21	4900	12540	35002	5 21196
22	6656	18116	49010	7 88460
23	8980	26035	68150	11 83221
24	12077	37262	94235	17 62462
25	16137	53070	1 29512	26 06604
26	21490	75292	1 77087	38 29437
27	28476	1 06377	2 40840	55 90110
28	37600	1 49738	3 26015	81 11346
29	49422	2 09980	4 39190	117 01998
30	64763	2 93473	5 89128	167 90136
31	84511	4 08734	7 86814	239 64594
32	1 09953	5 67484	10 46705	340 34391
33	1 42539	7 85409	13 86930	481 04069
34	1 84244	10 83817	18 31065	676 79109
35	2 37368	14 91247	24 08658	948 00537
36	3 04996	20 46233	31 57789	1322 30021
37	3 90688	28 00125	41 26070	1836 86994
38	4 99189	38 21959	53 74390	2541 70332
39	6 36059	52 03515	69 78730	3503 71088
40	8 08489	70 67373	90 35539	4812 25800
41	10 25017	95 76142	116 64896	6586 26822
42	12 96595	129 46065	150 18300	8983 65204
43	16 36173	174 62920	192 83830	12213 40197
44	20 60246	235 05298	246 97480	16551 51489
45	25 88440	315 72120	315 51450	22361 38826
46	32 45381	423 21495	402 10481	30120 47298
47	40 60519	566 18358	511 24970	40454 35515
48	50 70574	755 99640	648 54575	54181 00581
49	63 19336	1007 54843	820 88400	72367 23897
50	78 61151	1340 35641	1036 79156	96401 59893

1. S. K. AGARWAL & J. M. GANDHI, "Generalization of a certain function related to the partition function." Private communication.
2. M. S. CHEEMA & C. T. HASKELL, "Multirestricted and rowed partitions," *Duke Math. J.*, v. 34, 1967, pp. 443–451. MR 36 #125.
3. J. M. GANDHI, "Generalization of a certain function related to the partition function," *Mathematica (Cluj)*, v. 11 (34), 1969, pp. 245–251. MR 41 #5316.
4. B. GORDON & L. HOUTEN, "Notes on plane partitions. III," *Duke Math. J.*, v. 36, 1969, pp. 801–824. MR 40 #1358.
5. B. GORDON & L. HOUTEN, "Notes on plane partitions," *J. Combinatorial Theory*, v. 4, 1968, pp. 72–99. MR 36 #1339.
6. G. H. HARDY & S. RAMANUJAN, "Asymptotic formulae in combinatorial analysis," *Proc. London Math. Soc.*, v. 17, 1918, pp. 75–115.
7. P. A. MACMAHON, *Combinatory Analysis*. Vols. I, II, Chelsea, New York, 1960. MR 25 #5003.
8. D. H. LEHMER, "The series for the partition function," *Trans. Amer. Math. Soc.*, v. 43, 1938, pp. 271–295.
9. HANS RADEMACHER, "On the Selberg formula for $A_k(n)$," *J. Indian Math. Soc.*, v. 21, 1957, pp. 41–55. MR 19, 1163.
10. A. L. WHITEMAN, "A sum connected with the series for the partition function," *Pacific J. Math.*, v. 6, 1956, pp. 159–176. MR 18, 195.
11. A. L. WHITEMAN, "A sum connected with the partition function," *Bull. Amer. Math. Soc.*, v. 53, 1947, pp. 598–603. MR 8, 567.
12. E. M. WRIGHT, "Asymptotic partition formula. I. Plane partitions," *Quart. J. Math.*, v. 2, 1931, pp. 177–189.

Tables of Partition Numbers

$b_4(n)$, $b(n)$, $p_2(n)$, $p_3(n)$

which satisfy

$$(1-x)(1-x^2) \prod_{i=1}^{\infty} (1-x^i)^{-2} = \sum_{n=0}^{\infty} b_4(n) x^n$$

$$\prod_{i=1}^{\infty} (1-x^i)^{-\left[\frac{i+1}{2}\right]} = \sum_{n=0}^{\infty} b(n) x^n$$

$$\prod_{i=1}^{\infty} (1-x^i)^{-k} = \sum_{n=0}^{\infty} p_k(n) x^n$$

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Appendix

n	$b_4(n)$	$b(n)$	$P_2(n)$	$P_3(n)$
1	1	1	2	3
2	2	2	5	9
3	4	4	10	22
4	7	7	20	51
5	11	12	36	108
6	19	21	65	221
7	29	34	110	429
8	46	56	185	810
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48	50 70574	755 99640	648 54575	54181 00581
49	63 19336	1007 54843	820 88400	72367 23897
50	78 61151	1340 35641	1036 79156	96401 59893

n	$b_4(n)$	$b(n)$	$P_2(n)$	$P_3(n)$
51	97 60947	1779 91834	1306 73928	1 28087 13258
52	120 98596	2359 53419	1643 63280	1 69760 54058
53	149 69658	3122 59351	2063 27710	2 24443 45566
54	184 91168	4125 59627	2585 08230	2 96035 82728
55	228 02852	5441 95940	3232 75512	3 89560 80441
56	280 75176	7167 01932	4035 31208	5 11476 44598
57	345 11640	9424 34111	5028 10130	6 70069 39237
58	423 59179	12373 94514	6254 25005	8 75953 98657
59	519 12503	16222 72321	7766 16430	11 42700 97032
60	635 27989	21237 92565	9627 59294	14 87631 65694
61	776 30153	27764 24253	11915 80872	19 32815 63784
62	947 30804	36245 84487	14724 54540	25 06326 77790
63	1154 39052	47254 28331	18167 15170	32 43816 99990
64	1404 86477	61524 32614	22380 75315	41 90491 10067
65	1707 42895	79999 34941	27530 78840	54 03574 92807
66	2072 50172	1 03889 40580	33816 89157	69 55403 06883
67	2512 44858	1 34744 71111	41479 37540	89 37268 27236
68	3042 04393	1 74549 40928	50807 52250	114 64223 20191
69	3678 80067	2 25840 09590	62148 80700	146 81052 16993
70	4443 58487	2 91856 60250	75920 53897	187 69699 80867
71	5361 09869	3 76732 45924	92622 92216	239 58487 83159
72	6460 70712	4 85736 53625	1 12855 36125	305 33550 42465
73	7777 11656	6 25577 89418	1 37334 86100	388 52989 27059
74	9351 49786	8 04791 72553	1 66918 79795	493 64392 01418
75	11232 44364	10 34225 58912	2 02630 74134	626 26468 18988
76	13477 46824	13 27653 62504	2 45692 14653	793 35761 94687
77	16154 36128	17 02549 47501	2 97558 45120	1003 59570 93903
78	19343 20386	21 81060 60992	3 59963 06025	1267 76495 92650
79	23138 23888	27 91232 72951	4 34967 60380	1599 26304 79809
80	27650 59357	35 68550 43423	5 25022 80642	2014 71219 20736
81	33010 86605	45 57870 60819	6 33038 21602	2534 71122 98617
82	39372 76461	58 15850 83518	7 62466 18325	3184 75801 60095
83	46916 68345	74 13992 80551	9 17398 27630	3996 37906 89714
84	55854 57927	94 42458 39597	11 02680 82280	5008 51217 44867
85	66434 85079	120 14845 59610	13 24047 76664	6269 19633 53418
86	78948 88695	152 74167 49728	15 88279 20009	7837 63602 65406
87	93737 71817	194 00324 90230	19 03383 86210	9786 71952 92003
88	1 11200 74980	246 19446 75328	22 78816 04535	12206 08912 52613
89	1 31804 81724	312 15548 54008	27 25725 52460	15205 87972 91889
90	1 56095 84303	395 45082 94185	32 57253 55088	18921 26829 57873
91	1 84711 06297	500 55077 39575	38 88874 09310	23518 00376 09481
92	2 18394 58922	633 05739 09024	46 38796 70860	29199 12407 09658
93	2 58013 89188	799 98595 57802	55 28431 14270	36213 10665 85700
94	3 04580 48150	1010 11517 88869	65 82934 23970	44863 75119 18588
95	3 59272 08796	1274 42264 98799	78 31840 76176	55522 15095 10041
96	4 23460 13451	1606 62605 05248	93 09803 99327	68641 18385 37549
97	4 98739 41887	2023 85520 31009	110 57449 93420	84773 03695 60425
98	5 86964 58854	2547 48627 29494	131 22377 75425	1 04590 38430 64817
99	6 90289 78602	3204 17635 82294	155 60313 48120	1 28911 95675 92367
100	8 11216 90641	4027 14608 00459	184 36458 20766	1 58733 39267 61125

n	$b_4(n)$	$b(n)$	$P_2(n)$	$P_3(n)$
51	97 60947	1779 91834	1306 73928	1 28087 13258
52	120 98596	2359 53419	1643 63280	1 69760 54058
53	149 69658	3122 59351	2063 27710	2 24443 45566
54	184 91168	4125 59627	2585 08230	2 96035 82728
55	228 02852	5441 95940	3232 75512	3 89560 80441
56	280 75176	7167 01932	4035 31208	5 11476 44598
57	345 11640	9424 34111	5028 10130	6 70069 39237
58	423 59179	12373 94514	6254 25005	8 75953 98657
59	519 12503	16222 72321	7766 16430	11 42700 97032
60	635 27989	21237 92565	9627 59294	14 87631 65694
61	776 30153	27764 24253	11915 80872	19 32815 63784
62	947 30804	36245 84487	14724 54540	25 06326 77790
63	1154 39052	47254 28331	18167 15170	32 43816 99990
64	1404 86477	61524 32614	22380 75315	41 90491 10067
65	1707 42895	79999 34941	27530 78840	54 03574 92807
66	2072 50172	1 03889 40580	33816 89157	69 55403 06883
67	2512 44858	1 34744 71111	41479 37540	89 37268 27236
68	3042 04393	1 74549 40928	50807 52250	114 64223 20191
69	3678 80067	2 25840 09590	62148 80700	146 81052 16993
70	4443 58487	2 91856 60250	75920 53897	187 69699 80867
71	5361 09869	3 76732 45924	92622 92216	239 58487 83159
72	6460 70712	4 85736 53625	1 12855 36125	305 33550 42465
73	7777 11656	6 25577 89418	1 37334 86100	388 52989 27059
74	9351 49786	8 04791 72553	1 66918 79795	493 64392 01418
75	11232 44364	10 34225 58912	2 02630 74134	626 26468 18988
76	13477 46824	13 27653 62504	2 45692 14653	793 35761 94687
77	16154 36128	17 02549 47501	2 97558 45120	1003 59570 93903
78	19343 20386	21 81060 60992	3 59963 06025	1267 76495 92650
79	23138 23888	27 91232 72951	4 34967 60380	1599 26304 79809
80	27650 59357	35 68550 43423	5 25022 80642	2014 71219 20736
81	33010 86605	45 57870 60819	6 33038 21602	2534 71122 98617
82	39372 76461	58 15850 83518	7 62466 18325	3184 75801 60095
83	46916 68345	74 13992 80551	9 17398 27630	3996 37906 89714
84	55854 57927	94 42458 39597	11 02680 82280	5008 51217 44867
85	66434 85079	120 14845 59610	13 24047 76664	6269 19633 53418
86	78948 88695	152 74167 49728	15 88279 20009	7837 63602 65406
87	93737 71817	194 00324 90230	19 03383 86210	9786 71952 92003
88	1 11200 74980	246 19446 75328	22 78816 04535	12206 08912 52613
89	1 31804 81724	312 15548 54008	27 25725 52460	15205 87972 91889
90	1 56095 84303	395 45082 94185	32 57253 55088	18921 26829 57873
91	1 84711 06297	500 55077 39575	38 88874 09310	23518 00376 09481
92	2 18394 58922	633 05739 09024	46 38796 70860	29199 12407 09658
93	2 58013 89188	799 98595 57802	55 28431 14270	36213 10665 85700
94	3 04580 48150	1010 11517 88869	65 82934 23970	44863 75119 18588
95	3 59272 08796	1274 42264 98799	78 31840 76176	55522 15095 10041
96	4 23460 13451	1606 62605 05248	93 09803 99327	68641 18385 37549
97	4 98739 41887	2023 85520 31009	110 57449 93420	84773 03695 60425
98	5 86964 58854	2547 48627 29494	131 22377 75425	1 04590 38430 64817
99	6 90289 78602	3204 17635 82294	155 60313 48120	1 28911 95675 92367
100	8 11216 90641	4027 14608 00459	184 36458 20766	1 58733 39267 61125

n	$b_4(n)$	$b(n)$
151	14033 69942 91008	2070 17665 38311 83838
152	16041 40095 43983	2527 20183 16157 57502
153	18329 03479 67537	3083 80488 06275 77667
154	20934 64799 99540	3761 40308 93128 19053
155	23901 32914 24946	4585 96187 19253 19501
156	27277 86046 04423	5588 94855 84287 58236
157	31119 45091 10205	6808 48374 88238 01408
158	35488 56273 49522	8290 73066 07407 25473
159	40455 84020 81564	10091 57107 24590 90062
160	46101 15603 52754	12278 62627 28119 43757
161	52514 78660 97216	14933 69319 21608 96995
162	59798 73514 44801	18155 67999 61171 11033
163	68068 21708 46379	22064 14233 05677 17745
164	77453 33122 32680	26803 54167 01054 04099
165	88100 93492 54122	32548 37149 21345 45335
166	1 00176 75231 44028	39509 32606 87073 94900
167	1 13867 73888 68368	47940 72146 53691 07853
168	1 29384 73808 66327	58149 41998 83269 92123
169	1 46965 45963 59625	70505 55915 07188 51204
170	1 66877 82343 09319	85455 44583 70073 66026
171	1 89423 70669 97531	1 03537 04761 42278 96833
172	2 14943 14841 45401	1 25398 59834 26456 57251
173	2 43819 05857 94999	1 51820 93705 26030 89373
174	2 76482 49890 46445	1 83744 32070 12869 31304
175	3 13418 59484 67501	2 22300 59670 24461 84461
176	3 55173 16100 48910	2 68851 79459 82115 44210
177	4 02360 11521 20648	3 25036 40330 28348 86035
178	4 55669 78234 31540	3 92824 84745 29275 83183
179	5 15878 18234 84276	4 74585 97117 80413 17696
180	5 83857 42692 00034	5 73166 68919 16971 22000
181	6 60587 34307 50210	6 91987 38429 03647 40049
182	7 47168 47690 70842	8 35156 13003 54238 68739
183	8 44836 62522 99700	10 07605 41283 28893 05102
184	9 54979 08379 93904	12 15255 73701 90728 90575
185	10 79152 79638 19946	14 65211 34143 45992 26898
186	12 19104 63685 51608	17 65994 26198 27530 30469
187	13 76794 05377 46630	21 27824 17217 22925 10579
188	15 54418 36311 35827	25 62952 85877 52745 22297
189	17 54440 97435 85969	30 86063 88523 37184 72227
190	19 79622 90132 59709	37 14750 01216 73638 15695
191	22 33057 91178 32961	44 70083 34240 96588 93630
192	25 18211 74773 31311	53 77296 00889 59475 46456
193	28 38965 85532 94609	64 66592 61197 44990 60307
194	31 99666 15509 52930	77 74119 63866 60451 73639
195	36 05177 39586 52138	93 43121 87874 38527 66692
196	40 60943 74415 44152	112 25321 53204 39024 42895
197	45 73056 28099 15362	134 82562 44461 74145 83257
198	51 48328 20606 18253	161 88769 91555 78790 30148
199	57 94378 57918 31245	194 32286 01485 36564 17634
200	65 19725 58041 76049	233 18651 62179 63536 57014

n	$P_2(n)$			$P_3(n)$		
151	4	49015	73109 97300		22640	87911 16475 69456
152	5	16123	58849 35745		26858	62511 36666 00973
153	5	93005	09298 90730		31845	20062 17373 62771
154	6	81047	59838 28715		37737	75760 07407 85634
155	7	81830	43202 08646		44697	39540 33269 46231
156	8	97150	79787 51054		52913	25396 11213 79729
157	10	29053	08242 41190		62607	29195 53411 07034
158	11	79862	01101 33120		74039	86221 75325 30338
159	13	52220	13577 04820		87516	21461 79374 57835
160	15	49130	22039 49504	1	03394	07755 35187 16102
161	17	74003	13176 18420	1	22092	49325 31528 57974
162	20	30711	95153 07905	1	44102	11001 15398 78926
163	23	23653	07998 23200	1	69997	16668 49109 45779
164	26	57815	23097 45365	2	00449	44203 66179 07760
165	30	38857	29435 14782	2	36244	48452 76288 80593
166	34	73196	19765 80975	2	78300	48783 24483 41068
167	39	68105	99992 18760	3	27690	23468 06008 56744
168	45	31829	64131 51280	3	85666	59779 30598 63887
169	51	73704	90321 48690	4	53692	16297 00070 97052
170	59	04306	36803 90529	5	33473	62738 44923 80594
171	67	35605	33663 85470	6	27001	72752 53971 99245
172	76	81149	94987 72710	7	36597	56809 99910 55590
173	87	56267	97705 62650	8	64966	35777 11993 28925
174	99	78295	08919 96335	10	15259	71259 99715 85310
175	113	66831	71122 53776	11	91147	86641 76664 97548
176	129	44031	98437 36371	13	96903	33264 13124 33735
177	147	34928	72161 14460	16	37497	79812 33280 04538
178	167	67798	77710 28595	19	18714	30116 80557 85850
179	190	74573	69668 90960	22	47277	05792 21057 62841
180	216	91301	17910 50129	26	31001	66005 39757 18517
181	246	58663	44176 17704	30	78968	77863 87391 56306
182	280	22559	40108 02715	36	01724	98241 44464 86178
183	318	34758	28897 14990	42	11514	82180 51732 35276
184	361	53633	33208 93905	49	22548	95368 59164 19479
185	410	44985	01636 26126	57	51313	79721 85877 18040
186	465	82964	69633 56649	67	16929	03172 76580 41226
187	528	51110	43438 35500	78	41560	18843 62421 60461
188	599	43508	47747 01850	91	50894	66652 93097 12916
189	679	66095	18987 66670	106	74690	73993 31136 25416
190	770	38116	13428 92729	124	47410	53725 31074 96486
191	872	93760	75847 90510	145	08949	59889 07910 94350
192	988	83993	45062 47880	169	05477	47214 48565 21306
193	1119	78603	93014 40270	196	90405	93015 01708 96799
194	1267	68502	77738 50570	229	25503	83233 21206 11273
195	1434	68290	65276 95138	266	82180	42563 03119 82111
196	1623	19133	24416 49590	310	42962	06699 80197 88196
197	1835	91977	40054 09520	361	03190	98441 77251 21475
198	2075	91148	19799 82225	419	72978	85054 24074 76755
199	2346	58370	93355 73400	487	79452	69245 48347 91174
200	2651	77267	31143 22154	566	69336	08658 40599 18703

n	$b_4(n)$	$b(n)$
201	73 33890 31270 21135	279 73919 75990 00880 67322
202	82 47512 35017 60022	335 48600 65460 22153 88527
203	92 72478 29401 26100	402 22357 73099 60774 92787
204	104 22064 81004 21789	482 09596 04093 33820 79246
205	117 11097 70143 51245	577 66111 00029 13711 22562
206	131 56128 82208 54497	691 96996 58276 09684 78171
207	147 75632 74067 11053	828 66049 14721 72386 14931
208	165 90225 36545 29973	992 06946 92462 92504 80085
209	186 22906 87630 02577	1187 36537 10034 76498 92141
210	208 99331 66726 61830	1420 70623 86643 90711 08199
211	234 48108 17957 56120	1699 42723 51836 95590 05953
212	263 01131 92995 15275	2032 26338 67533 98329 23555
213	294 93955 26530 17765	2429 61405 42516 10676 91128
214	330 66197 88158 54025	2903 85687 47790 03462 08222
215	370 62002 43260 10215	3469 72033 61175 56383 99734
216	415 30540 15874 31163	4144 72582 76559 59916 13027
217	465 26571 83021 06077	4949 71199 73706 46749 54785
218	521 11070 02067 47532	5909 45659 07851 91059 71402
219	583 51909 18579 70118	7053 41371 91893 11889 50297
220	653 24630 88305 58360	8416 58777 70592 66290 89274
221	731 13292 04378 17268	10040 56909 24033 70122 04069
222	818 11405 23836 75727	11974 76095 44450 99342 17398
223	915 22980 59263 18317	14277 83304 51698 29339 29895
224	1023 63680 24529 91173	17019 44265 13868 57554 36541
225	1144 62097 12775 09967	20282 27252 45744 55329 00489
226	1279 61171 32138 33020	24164 44308 98418 81704 73537
227	1430 19758 35270 87666	28782 36712 15744 00852 26910
228	1598 14365 55037 81375	34274 12728 25081 66406 73423
229	1785 41073 95402 99637	40803 47139 52337 54868 69427
230	1994 17665 37711 97804	48564 53736 85845 27445 35715
231	2226 85975 90908 98582	57787 43979 21917 44492 59951
232	2486 14499 67214 79235	68744 87387 67850 57178 83947
233	2775 01268 71831 87173	81759 92028 54094 51673 94920
234	3096 77037 98244 78666	97215 26721 15149 58884 51545
235	3455 08806 74805 15328	1 15564 10468 27698 94063 03450
236	3854 03711 71331 89113	1 37342 99152 48171 78149 49829
237	4298 13329 85662 57901	1 63187 04890 48745 74582 45261
238	4792 38433 65124 60312	1 93847 89729 71345 95415 14649
239	5342 34244 98408 39566	2 30214 82772 03470 07518 91630
240	5954 16239 34563 17231	2 73339 78512 88331 33955 51819
241	6634 66556 50319 65911	3 24466 84416 25938 69607 47455
242	7391 41080 11539 97657	3 85066 97774 39670 69827 35024
243	8232 77254 40733 59759	4 56879 06037 11240 66684 36439
244	9168 02713 50800 80697	5 41958 21406 49890 98141 96721
245	10207 44805 94332 03701	6 42732 80006 89579 95459 02079
246	11362 41105 74342 20917	7 62071 58861 16823 33806 43448
247	12645 51009 99500 16601	9 03362 90821 79923 42027 49449
248	14070 68533 41503 54463	10 70607 89209 93062 65158 71174
249	15653 36420 68004 49153	12 68530 31016 36678 77253 06651
250	17410 61710 11962 25420	15 02705 91063 28716 49447 41750

n	$P_2(n)$	$P_3(n)$
201	2995 78380 35969 34464	658 11914 78329 19883 32801
202	3383 44789 08774 43240	764 02442 97917 11839 00108
203	3820 18380 43001 81650	886 66054 52800 48287 38334
204	4312 06853 96811 12215	1028 62252 59326 57004 85104
205	4865 91543 01182 01870	1192 90061 75279 33473 40331
206	5489 36145 37199 86932	1382 93938 55003 96794 04118
207	6190 96467 15637 87640	1602 70550 14848 89613 30821
208	6980 31294 88201 02675	1856 76546 31874 76948 89937
209	7868 14523 54269 05960	2150 37467 73250 77209 15430
210	8866 48682 93558 82825	2489 57953 75314 07601 46315
211	9988 80019 77584 42230	2881 33435 93554 43049 47065
212	11250 15311 09869 34370	3333 63529 66274 64554 60826
213	12667 40603 20425 11540	3855 67366 18674 08417 73063
214	14259 42092 40868 57705	4458 01141 30057 15802 06034
215	16047 29386 94684 45090	5152 78195 50462 74637 44373
216	18054 61416 31002 22418	5953 91984 47748 39611 85308
217	20307 75282 67839 15880	6877 42348 63566 60124 79604
218	22836 18382 06224 40740	7941 65547 38218 51961 18061
219	25672 84157 61641 04320	9167 68588 21842 46031 23411
220	28854 51887 88331 87540	10579 68454 27236 75515 97306
221	32422 30955 48126 68388	12205 36917 15490 74390 79095
222	36422 10090 98654 27335	14076 51716 73271 55021 62475
223	40905 12139 17712 26500	16229 54996 90787 74590 33764
224	45928 54954 92769 76620	18706 20008 53293 23490 37188
225	51556 19100 24602 85752	21554 27229 09339 89100 32817
226	57859 23087 31798 68892	24828 51205 94671 29197 46045
227	64917 06990 98902 65690	28591 59608 18194 24476 48618
228	72818 25343 61136 30205	32915 26174 35401 86317 03665
229	81661 50321 23643 26640	37881 59472 59822 74618 91475
230	91556 86339 23588 88959	43584 49649 51290 80918 86193
231	1 02626 97292 77004 83976	50131 35638 72879 43655 02757
232	1 15008 47810 44165 25530	57644 95633 81145 10946 60662
233	1 28853 60032 69413 07720	66265 64008 19179 86976 66467
234	1 44331 87588 34818 27940	76153 78292 98491 28746 62525
235	1 61632 08617 34871 25458	87492 60308 36517 07347 42228
236	1 80964 39884 71608 34791	1 00491 36092 99496 97141 55571
237	2 02562 74243 57323 90210	1 15388 99897 16084 09715 00517
238	2 26687 43944 59185 59855	1 32458 28208 14008 48971 75110
239	2 53628 12548 43309 54840	1 52010 50571 46595 70269 89965
240	2 83706 98488 79734 41716	1 74400 84871 10781 03004 05836
241	3 17282 33649 14172 02612	2 00034 45748 55590 28045 64177
242	3 54752 60669 62136 87145	2 29373 35990 54578 24270 48585
243	3 96560 73084 37308 07800	2 62944 32014 71372 27494 22847
244	4 43199 02818 36073 73030	3 01347 76051 06191 79528 13973
245	4 95214 60039 05576 97386	3 45267 89276 34509 81801 96211
246	5 53215 30878 78684 83533	3 95484 22032 73592 68194 04378
247	6 17876 39109 47688 24490	4 52884 59378 78209 30073 24975
248	6 89947 78482 62299 65100	5 18480 02610 60248 18139 52636
249	7 70262 23133 99307 55210	5 93421 50089 18125 58587 26648
250	8 59744 24217 25881 21240	6 79019 03754 58256 81825 91330

n	$b_4(n)$				$b(n)$						
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252	21526	38841	61167	85044	21	07320	23945	08349	73210	22000	
253	23928	89646	79762	65204	24	94680	56451	15882	27017	74441	
254	26594	39618	99955	73171	29	52596	50863	53287	13841	74513	
255	29551	12606	21459	31369	34	93803	74205	34923	10529	18433	
256	32830	29899	34140	37999	41	33316	32021	75200	86872	29413	
257	36466	40969	69344	12213	48	88830	69056	28456	89205	95905	
258	40497	57323	49636	35336	57	81200	71373	18053	94736	27735	
259	44965	89782	78580	24788	68	34996	07415	46538	44337	07612	
260	49917	89533	80398	56074	80	79158	59291	36052	75544	52690	
261	55404	93315	70074	41706	95	47773	46010	90354	55518	80426	
262	61483	73160	25270	76106	112	80975	33673	14298	52437	61730	
263	68216	91131	51494	49764	133	26012	71002	13393	24931	33099	
264	75673	59559	66183	71315	157	38497	90641	13527	35294	87892	
265	83930	07309	25786	74229	185	83874	87160	34510	06594	70877	
266	93070	52676	41972	16689	219	39142	33401	34436	90247	47976	
267	1	03187	83564	63568	94933	258	94876	41101	19771	97815	65746
268	1	14384	45653	82533	51251	305	57604	25507	14462	94106	19894
269	1	26773	39343	68603	22813	360	52589	15347	81327	18598	98876
270	1	40479	26329	82890	09989	425	27097	80588	95539	66767	38932
271	1	55639	46751	01911	55665	501	54232	55982	67621	54252	33720
272	1	72405	47938	36632	51962	591	37425	47817	54592	73373	76180
273	1	90944	25893	07832	08020	697	15707	58670	22471	61754	05492
274	2	11439	80729	89205	86974	821	69885	90209	78222	03767	22656
275	2	34094	87438	12761	35640	968	29783	33699	13771	62772	78913
276	2	59132	83444	22102	78017	1140	82722	85974	21113	31365	23551
277	2	86799	74597	18577	80643	1343	83467	98545	40656	09810	09678
278	3	17366	61355	86583	95868	1582	65867	52705	57726	97903	53698
279	3	51131	87121	59499	75992	1863	56494	39972	39716	63792	54276
280	3	88424	10847	65345	91788	2193	90617	14850	22894	29507	76413
281	4	29605	06254	05526	03416	2582	30899	92280	66417	70384	52542
282	4	75072	90200	10665	40023	3038	89293	20982	65566	31821	25214
283	5	25265	83002	92444	29509	3575	52655	34231	55047	49602	21954
284	5	80666	03756	99264	77754	4206	12735	45560	36478	77115	05051
285	6	41804	03991	56886	11430	4947	01254	32698	07779	27600	13952
286	7	09263	43320	75042	58897	5817	30942	88274	05017	94034	97872
287	7	83686	11077	49417	42183	6839	43542	00801	55080	26916	02614
288	8	65777	98301	62387	40298	8039	65935	02148	96043	37998	30409
289	9	56315	24853	49199	84812	9448	75779	71339	28467	94590	86688
290	10	56151	26875	87110	38716	11102	78234	76569	26779	39644	08130
291	11	66224	10305	75766	82698	13043	95640	99966	94698	87916	51322
292	12	87564	76674	38337	88570	15321	72327	41742	62863	37749	08457
293	14	21306	28004	93235	56832	17993	97072	61778	12103	97657	66560
294	15	68693	58255	80275	52239	21128	46172	24197	87878	00712	68106
295	17	31094	39437	86922	62293	24804	50552	36947	71036	80440	03139
296	19	10011	11293	25146	53513	29114	90938	54706	50364	24502	46536
297	21	07093	84233	62934	08081	34168	25753	53820	54135	00936	32511
298	23	24154	66138	35900	88497	40091	57189	19561	40372	12231	50501
299	25	63183	24577	37720	69521	47033	41796	83225	11705	31594	76103

n	$P_2(n)$					$P_3(n)$					
251	9	59420	01763	32482	79576	7	76763	31143	83479	33607	97795
252	10	70428	41688	20224	30650	8	88350	16605	54670	15523	37510
253	11	94033	08881	06588	54190	10	15708	39776	54262	54503	02679
254	13	31635	88424	94285	78435	11	61031	24316	40731	43081	91308
255	14	84791	68224	02109	33344	13	26812	05454	96887	00886	84033
256	16	55224	77667	23946	95588	15	15884	71174	31251	74836	68077
257	18	44846	98436	01114	62710	17	31469	38909	48303	95506	72680
258	20	55777	65202	72588	60290	19	77224	37610	16025	22798	59718
259	22	90365	75754	28336	52200	22	57304	73971	02417	55983	17186
260	25	51214	32054	80209	05854	25	76428	71736	81433	37051	69927
261	28	41207	35922	06031	39470	29	39952	84360	10088	79125	02793
262	31	63539	65382	83174	69230	33	53956	94093	72995	43655	34611
263	35	21749	60381	60491	52610	38	25340	25013	23707	90674	26532
264	39	19755	49402	03818	53685	43	61930	13686	45163	31268	65089
265	43	61895	51710	06922	11294	49	72604	99461	41975	90101	61876
266	48	52971	93406	92221	29058	56	67433	16880	31484	74367	57595
267	53	98299	79279	58893	81600	64	57829	95827	16483	17140	92210
268	60	03760	66630	26726	50615	73	56735	00996	17723	35730	26365
269	66	75861	91846	62002	25970	83	78812	71478	37242	38043	69074
270	74	21802	05527	12725	40974	95	40678	54103	14901	72516	67240
271	82	49542	77494	49912	35994	108	61154	61081	93183	43286	78376
272	91	67888	39113	37267	66960	123	61558	23980	41366	55528	59680
273	101	86573	36973	82287	06000	140	66027	62650	93671	03824	93132
274	113	16358	79322	58848	23940	160	01889	40111	13876	94830	89068
275	125	69138	64621	16628	98620	182	00073	33157	56610	68447	99660
276	139	58056	90414	15292	94577	206	95580	14535	76848	94618	81257
277	154	97636	50309	91651	49900	235	28009	16626	91879	91882	93053
278	172	03921	37458	76899	41725	267	42153	29835	37781	97033	77880
279	190	94632	84476	12757	73040	303	88669	82268	47107	18638	92981
280	211	89341	82472	63351	56653	345	24836	52116	33003	57869	54161
281	235	09658	35744	04735	91384	392	15403	81738	66066	47052	04350
282	260	79440	23940	65995	15020	445	33554	94382	42268	78058	33769
283	289	25022	60214	99823	79260	505	61987	62406	73644	14666	65534
284	320	75470	52168	60347	80650	573	94132	41799	20492	38564	77959
285	355	62856	92434	51062	56320	651	35524	73777	75726	11077	38047
286	394	22568	27708	86629	16607	739	05349	62783	80858	82737	66799
287	436	93640	79052	26761	34460	838	38180	73867	52746	48678	39068
288	484	19130	12628	24715	35045	950	85937	54341	14651	31701	94926
289	536	46517	88825	14047	37710	1078	20087	77979	28455	36815	77857
290	594	28158	49276	99111	77011	1222	34125	38716	28696	47042	25015
291	658	21770	35779	64210	62374	1385	46357	88906	39166	73041	86806
292	728	90975	72905	87612	90245	1570	03041	29437	84043	90205	09888
293	807	05893	87413	45947	32440	1778	81905	20534	61345	41262	18455
294	893	43792	82795	49625	12550	2014	96115	98753	54088	37957	66055
295	988	89805	36740	94882	17038	2281	98731	63979	22067	48310	49284
296	1094	37715	43416	23706	50661	2583	87708	47376	95141	56041	82149
297	1210	90821	81595	31897	63230	2925	11526	95365	39776	05282	56405
298	1339	62886	54408	96622	85350	3310	75512	14731	88711	25007	84817
299	1481	79176	17165	42534	67440	3746	48933	30090	34606	94842	00507