

## REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

**42[2.10].**—W. ROBERT BOLAND, *Coefficients for Product-Type Quadrature Formulas*, Department of Mathematics, Clemson University, Clemson, South Carolina. Ms. of 16 pp. (undated) deposited in the UMT file.

Product-type quadrature formulas have been introduced by the author [1] in collaboration with C. S. Duris for the numerical approximation of definite integrals of the form  $\int_a^b f(x)g(x) dx$ . Such a formula is said to be "product-interpolatory" if it is derived by integrating  $p_n(x) \cdot q_m(x)$ , where  $p_n(x)$  is the polynomial interpolating  $f(x)$  at the nodes  $x_i$ ,  $i = 0(1)n$ , and  $q_m(x)$  similarly is the polynomial interpolating  $g(x)$  at the nodes  $y_j$ ,  $j = 0(1)m$ . In this case the author proves in [1] that the coefficients in the corresponding quadrature formula

$$\int_a^b f(x)g(x) dx \approx \sum_{i=0}^n \sum_{j=0}^m a_{ij} f(x_i) g(y_j)$$

are given by  $a_{ij} = \int_a^b l_i(x)L_j(x) dx$ , where  $l_i(x)$  and  $L_j(x)$  are, respectively, the  $i$ th and  $j$ th Lagrange interpolation coefficients for the nodes  $x_i$  and  $y_j$ .

In the present tables the range of integration is taken to be  $(-1, 1)$ , and the parameters  $n$  and  $m$  are restricted to the ranges  $n = 1(1)5$  and  $m = 1(1)5$ .

Table 1 consists of the exact (rational) values of the coefficients for the corresponding product Newton-Cotes formulas; Table 2 consists of 16S values (in floating-point form) of the coefficients for the corresponding product Gauss formulas; and Table 3 gives 16S values of the coefficients for the corresponding product Gauss-Newton-Cotes formulas.

The tabular values were calculated on an IBM 360/75 system, using double-precision arithmetic, and the author believes they are correct to at least 15S. As partial confirmation, a spot check by this reviewer revealed no errors exceeding 5 units in the least significant digit. However, in Table 1 a serious printing error was discovered; namely, the least common denominator of the coefficients corresponding to  $n = m = 3$  should read 840 instead of 1.

For an appropriate error analysis of such quadrature formulas, the user of these tables should consult [1], where he will also find some comments on their applicability, in particular to the study of the Fredholm integral equation of the second kind.

J. W. W.

1. W. R. BOLAND & C. S. DURIS, "Product type quadrature formulas," *Nordisk Tidsskr. Informationsbehandling (BIT)*, v. 11, 1971, pp. 139-158.

**43[2.10].**—PAUL F. BYRD & DAVID C. GALANT, *Gauss Quadrature Rules Involving Some Nonclassical Weight Functions*, NASA Technical Note D-5785, Ames