The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

**42[2.10].**—W. Robert Boland, *Coefficients for Product-Type Quadrature Formulas*, Department of Mathematics, Clemson University, Clemson, South Carolina. Ms. of 16 pp. (undated) deposited in the UMT file.

Product-type quadrature formulas have been introduced by the author [1] in collaboration with C. S. Duris for the numerical approximation of definite integrals of the form $\int_a^b f(x)g(x)\,dx$. Such a formula is said to be "product-interpolatory" if it is derived by integrating $p_n(x)\cdot q_m(x)$, where $p_n(x)$ is the polynomial interpolating $f(x)$ at the nodes $x_i$, $i = 0(1)n$, and $q_m(x)$ similarly is the polynomial interpolating $g(x)$ at the nodes $y_j$, $j = 0(1)m$. In this case the author proves in [1] that the coefficients in the corresponding quadrature formula

$$\int_a^b f(x)g(x)\,dx \approx \sum_{i=0}^{n} \sum_{j=0}^{m} a_{ij} f(x_i) g(y_i)$$

are given by $a_{ij} = \int_a^b l_i(x)L_j(x)\,dx$, where $l_i(x)$ and $L_j(x)$ are, respectively, the $i$th and $j$th Lagrange interpolation coefficients for the nodes $x_i$ and $y_j$.

In the present tables the range of integration is taken to be $(-1, 1)$, and the parameters $n$ and $m$ are restricted to the ranges $n = 1(1)5$ and $m = 1(1)5$.

Table 1 consists of the exact (rational) values of the coefficients for the corresponding product Newton-Cotes formulas; Table 2 consists of 16S values (in floating-point form) of the coefficients for the corresponding product Gauss formulas; and Table 3 gives 16S values of the coefficients for the corresponding product Gauss-Newton-Cotes formulas.

The tabular values were calculated on an IBM 360/75 system, using double-precision arithmetic, and the author believes they are correct to at least 15S. As partial confirmation, a spot check by this reviewer revealed no errors exceeding 5 units in the least significant digit. However, in Table 1 a serious printing error was discovered; namely, the least common denominator of the coefficients corresponding to $n = m = 3$ should read 840 instead of 1.

For an appropriate error analysis of such quadrature formulas, the user of these tables should consult [1], where he will also find some comments on their applicability, in particular to the study of the Fredholm integral equation of the second kind.

J. W. W.

1. W. R. Boland & C. S. Duris, "Product type quadrature formulas," *Nordisk Tidskr. Informationsbehandling (BIT)*, v. 11, 1971, pp. 139–158.

**43[2.10].**—Paul F. Byrd & David C. Galant, *Gauss Quadrature Rules Involving Some Nonclassical Weight Functions*, NASA Technical Note D-5785, Ames

Research Center, National Aeronautics and Space Administration, Moffett Field, California, May 1970, iv + 36 pp., 27 cm. Available from National Technical Information Service, Operations Division, Springfield, Virginia 22151. Price $3.00.

Nodes, $t_{iN}$, and weight coefficients, $W_{iN}$, are herein tabulated (Tables 1–12) to 25S (in floating-point form) for twelve Gauss quadrature formulas

$$\int_0^1 w(a, \beta, \gamma; t) f(t)\, dt = \sum_{i=1}^{N} W_{iN} f(t_{iN}) + E_N,$$

where the weight function, $w$, is of the form $t^\gamma(1 - t^\alpha)^\beta$ and $N = 2(2)8(4)16, 24$. The corresponding values of the parameters $\alpha$, $\beta$, $\gamma$ are, respectively, $(3, \pm 1/2, 0)$, $(4, \pm 1/2, 0)$, $(6, \pm 1/2, 0)$, $(8, \pm 1/2, 0)$, $(3, \pm 1/2, \pm 1/2)$, $(2, -3/4, 0)$, and $(2, -2/3, 0)$. The coefficient $k_N$ in the error term

$$E_n = k_N[(2N)!]^{-1} f^{(2N)}(\tau), \qquad 0 < \tau < 1,$$

is tabulated (in Table 13) to 5S for the same values of $N$.

The zeroth moment $M_0 = \int_0^1 w(t)\, dt$ and the coefficients $b_i$, $g_i$ of the three-term recurrence relation for the monic orthogonal polynomials associated with the enumerated weight function are given to 25S in Tables 14–25, for $j = 1(1)26$.

An introduction of 11 pages contains a detailed discussion of the numerical difficulties overcome in the construction of these unique tables and of the checks that were applied to test their accuracy. Appended is a listing of a double-precision version of the algorithm used in calculating these tables. As the authors note, this computer program can be used to find additional quadrature rules of the type considered in this very useful report, which also includes a list of 16 valuable references.

J. W. W.

**44[2.10].**—R. Piessens, *Gaussian Quadrature Formulae for Integrals Involving Bessel Functions*, 30 pages of tables and 3 pages of explanatory text, reproduced on the microfiche card attached to this issue.

These tables consist of 14D values of the abscissas, $x_k$, and weights, $A_k$, in the Gaussian quadrature formula

$$\int_{j_{n,s-1}}^{j_{n,s}} J_n(x) f(x)\, dx = (-1)^{s+1} \sum_{k=1}^{N} A_k f(x_k),$$

for $n = 0, 1, 2$, $s = 1(1)20$, and $N = 2(2)8$. The limits of integration are pairs of successive zeros of $J_n(x)$, for the stated ranges of $n$ and $s$.

The calculation of these tables was performed on an IBM 1620 system at the Computing Centre of the University of Leuven, using algorithms described by Golub & Welsch [1].

The authors refer to similar tables of Longman [2], which consist of 10D entries corresponding to $n = 0, 1$, $s = 1(1)20$, and $N = 16$. The present tables constitute a valuable and unique supplement to these earlier ones.

J. W. W.