

Research Center, National Aeronautics and Space Administration, Moffett Field, California, May 1970, iv + 36 pp., 27 cm. Available from National Technical Information Service, Operations Division, Springfield, Virginia 22151. Price \$3.00.

Nodes, t_{iN} , and weight coefficients, W_{iN} , are herein tabulated (Tables 1–12) to 25S (in floating-point form) for twelve Gauss quadrature formulas

$$\int_0^1 w(a, \beta, \gamma; t) f(t) dt = \sum_{i=1}^N W_{iN} f(t_{iN}) + E_N,$$

where the weight function, w , is of the form $t^\gamma(1 - t^\alpha)^\beta$ and $N = 2(2)8(4)16, 24$. The corresponding values of the parameters α, β, γ are, respectively, $(3, \pm 1/2, 0)$, $(4, \pm 1/2, 0)$, $(6, \pm 1/2, 0)$, $(8, \pm 1/2, 0)$, $(3, \pm 1/2, \pm 1/2)$, $(2, -3/4, 0)$, and $(2, -2/3, 0)$. The coefficient k_N in the error term

$$E_N = k_N [(2N)!]^{-1} f^{(2N)}(\tau), \quad 0 < \tau < 1,$$

is tabulated (in Table 13) to 5S for the same values of N .

The zeroth moment $M_0 = \int_0^1 w(t) dt$ and the coefficients b_j, g_j of the three-term recurrence relation for the monic orthogonal polynomials associated with the enumerated weight function are given to 25S in Tables 14–25, for $j = 1(1)26$.

An introduction of 11 pages contains a detailed discussion of the numerical difficulties overcome in the construction of these unique tables and of the checks that were applied to test their accuracy. Appended is a listing of a double-precision version of the algorithm used in calculating these tables. As the authors note, this computer program can be used to find additional quadrature rules of the type considered in this very useful report, which also includes a list of 16 valuable references.

J. W. W.

44[2.10].—R. PIESSENS, *Gaussian Quadrature Formulae for Integrals Involving Bessel Functions*, 30 pages of tables and 3 pages of explanatory text, reproduced on the microfiche card attached to this issue.

These tables consist of 14D values of the abscissas, x_k , and weights, A_k , in the Gaussian quadrature formula

$$\int_{j_{n,s-1}}^{j_{n,s}} J_n(x) f(x) dx = (-1)^{s+1} \sum_{k=1}^N A_k f(x_k),$$

for $n = 0, 1, 2$, $s = 1(1)20$, and $N = 2(2)8$. The limits of integration are pairs of successive zeros of $J_n(x)$, for the stated ranges of n and s .

The calculation of these tables was performed on an IBM 1620 system at the Computing Centre of the University of Leuven, using algorithms described by Golub & Welsch [1].

The authors refer to similar tables of Longman [2], which consist of 10D entries corresponding to $n = 0, 1$, $s = 1(1)20$, and $N = 16$. The present tables constitute a valuable and unique supplement to these earlier ones.

J. W. W.

1. GENE H. GOLUB & JOHN H. WELSCH, "Calculation of Gauss quadrature rules," *Math. Comp.*, v. 23, 1969, pp. 221-230.

2. I. M. LONGMAN, "Tables for the rapid and accurate numerical evaluation of certain infinite integrals involving Bessel functions," *MTAC*, v. 11, 1957, pp. 166-180.

45[2.10,7].—É. N. GLONTI, *Tablitsy Kornei i Kvadrurnykh Koeffitsientov Polinomov Iakobi* (Tables of the Roots and Quadrature Coefficients of Jacobi Polynomials), Computing Center, Acad. Sci. USSR, Moscow, 1971, xiv + 236 pp., 27 cm. Price 2.14 rubles.

This is an extensive tabulation of the zeros $x_k^{(n)}$ and Christoffel numbers $A_k^{(n)}$ of the Jacobi polynomials $P_n(p, q, z)$ orthogonal on the interval $(0, 1)$ with weight function $x^{q-1}(1-x)^{p-q}$. The parameters of p, q range through the values $q = 0.1(0.1)1.0, p = (2q-1)(0.1)(q+1)$, while $n = 2(1)15$. The precision is 15S in the zeros and 15D in the coefficients. The only published table that is comparable in scope is that of Krylov et al. [1], which covers a somewhat larger region of the parameters, but is restricted to $n \leq 8$ and a precision of only 8S.

An additional table of quadrature errors

$$e_j^{(n)} = \left| \int_0^1 x^{q-1}(1-x)^{p-q} x^{2n+j} dx - \sum_{k=1}^n A_k^{(n)} [x_k^{(n)}]^{2n+j} \right|,$$

for $0 \leq j \leq 16$ and $2 \leq n \leq 15$, appears in the introduction. These errors grow slightly as q increases for fixed p and sharply increase with p for fixed q . Consequently, only the errors for $q = 1, p = 2$ and $q = 0.1, p = -0.8$ are tabulated, representing the largest and smallest errors, respectively, in the tabular range.

The introduction also includes a collection of formal relationships satisfied by Jacobi polynomials, comments on the computation and use of the tables, and information facilitating interpolation.

W. G.

1. V. I. KRYLOV, V. V. LUGIN & L. A. IANOVICH, *Tablitsy dlia Chislennogo Integrirvaniia Funktsii so Stepennymi Osobennostiami*, Izdat. Akad. Nauk BSSR, Minsk, 1963.

46[2.35, 6, 13.35].—JAMES W. DANIEL, *The Approximate Minimization of Functionals*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1971, xi + 228 pp., 24 cm. Price \$9.50 (cloth).

This book presents the basic theory relevant to problems of approximately minimizing functionals and an analysis of some of the numerical methods available for their solution. It is an informative, useful, and very readable book. The author's exposition is clear, his prose is smooth, and the text is attractively printed. Exercises, almost all of a theoretical nature, are interspersed throughout the text, and references to nearly 200 books and papers, most of them quite recent, are well documented. Although practical methods are discussed, theoretical considerations dominate. There is little discussion of the comparative merits of these methods, although some very rough guidelines are given in an epilogue.

In Chapter 1, "Variational Problems in an Abstract Setting", basic functional analysis relevant to minimization problems is reviewed, various notions of convexity

GAUSSIAN QUADRATURE FORMULAE FOR INTEGRALS
INVOLVING BESSEL FUNCTIONS

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Introduction

Many physical and technical calculations lead to integrals of the form

$$\int_0^b J_n(x) f(x) dx \quad (1)$$

where b is finite or infinite and where $J_n(x)$ is the Bessel function of the first kind and of order n . Important examples of integrals of this type are the Fourier-Bessel coefficients. For the numerical evaluation of (1), Longman [1] has split up the integral (1) into a sum of integrals

$$\int_{j_{n,s-1}}^{j_{n,s}} J_n(x) f(x) dx \quad (2)$$

where $j_{n,0} = 0$ and $j_{n,s}$ is the s -th positive zero of $J_n(x)$.

In the case $b = \infty$, Euler's transformation can be applied to speed up the convergence of the resulting series.

Longman has given tables which facilitate the evaluation of (2) using a 16-point Gauss-Legendre quadrature formula.

We here present abscissae and weights for the Gaussian quadrature formulae

$$(-1)^{s+1} \int_{j_{n,s-1}}^{j_{n,s}} J_n(x) f(x) dx = \sum_{k=1}^N A_k f(x_k) \quad (3)$$

for $n=0,1,2, \dots, s=1(1)20$ and $N=2(2)8$.

Abscissae and weights are given to 14D.

Computation of abscissae and weights

Using the method of Golub and Welsch [2], we have calculated abscissae u_k and weights B_k of the Gaussian quadrature formula

$$\int_c^1 J_n(j_{n,s} u) g(u) du = \sum_{k=1}^N B_k g(u_k) \quad (4)$$

where $c = j_{n,s-1} / j_{n,s}$

The required coefficients x_k and A_k of formula (3) are given by

$$\begin{aligned} x_k &= j_{n,s} u_k \\ A_k &= j_{n,s} B_k \end{aligned} \quad (5)$$

The zeros $j_{n,s}$ are given to 18D in [3].

The calculation of the moments which are necessary to the application of the method of Golub and Welsch, can be carried out according to the formula

$$\int_c^1 J_n(j_{n,s} u) u^m du = M_m(j_{n,s}) - \left(\frac{j_{n,s-1}}{j_{n,s}}\right)^{m+1} M_m(j_{n,s-1}) \quad (6)$$

where

$$M_m(j_{n,s}) = \int_0^1 J_n(j_{n,s} u) u^m du$$

It is known (see Luke [4]) that $M_m(j_{n,s})$ satisfies a recurrence relation

$$M_m(j_{n,s}) = -\frac{J_{n-1}(j_{n,s})}{j_{n,s}} + \frac{(m+n-1)(n-m+1)}{j_{n,s}^2} M_{m-2}(j_{n,s}) \quad (7)$$

To have numerical stability, (7) must be used in forward direction if $m \leq (j_{n,s}^2 + (n-1)^2)^{1/2}$. For larger values of m , backward recursion must be applied.

All calculations were carried out on the IBM-1620 computer of the Computing Centre of the University of Leuven.

References

1. I.M. Longman, Tables for the rapid and accurate numerical evaluation of certain infinite integrals involving Bessel functions, *MTAC*, 11, 1957, pp. 166-180
2. G.H. Golub and J.H. Welsch, Calculation of Gauss quadrature rules, *Math. Comp.*, 23, 1969, pp. 221-230.
3. P. Detournay and R. Piessens, Zeros of Bessel Functions and Zeros of Cross Products of Bessel Functions, Report TW7, Applied Mathematics Division, University of Leuven, 1971.
4. Y.L. Luke, Integrals of Besselfunctions, McGraw-Hill, New York, 1962.

$$(-1)^{B+1} \int_{j_{0,B-1}}^{j_{0,B}} J_0(x) f(x) dx = \sum_{k=1}^N A_k f(x_k)$$

S	N	x_k	A_k	
1	2	0.39882922822995	0.90108882774262	
		1.56194850615902	0.56921121564155	
	4	0.14473822825570	0.35943229514774	
		0.68411144537350	0.60043637583620	
		1.41258236533148	0.41175504216916	
		2.06769209300286	0.09867633023108	
	6	0.07339090660313	0.18567018912283	
		0.36659942705566	0.37603006078970	
		0.82318891184541	0.42592256812299	
		1.35083692479061	0.31454440420181	
		1.85067678032127	0.14082517927539	
		2.22814702187129	0.02730764187146	
	8	0.04415168227093	0.11241452816076	
		0.22550464414805	0.24287219834492	
		0.52500763246448	0.32298950298153	
		0.90401792091887	0.32551603010775	
		1.31711944274235	0.25319543981293	
		1.71683415081715	0.14651481617332	
		2.05706854694283	0.05660021995336	
		2.29687903908346	0.01019730784960	
	2	2	3.24530646611014	0.41513278125292
			4.60630512173795	0.38632142981301
		4	2.76734061911444	0.09956250246132
			3.49898577507422	0.32073897309296
4.37279061418102			0.29915508250833	
5.13210398252647			0.08199765300331	
6		2.60363513856059	0.03208896506293	
		3.03465618312165	0.14210182717209	
		3.62027269832971	0.24865636501633	
		4.26406292205469	0.23427106072747	
		4.86183849178307	0.11962902200930	
		5.31032140586202	0.02470697107780	
8		2.52978116224146	0.01303915472841	
		2.80933716941964	0.06552756129283	
		3.21089743529117	0.14498606002447	
		3.69130423851183	0.20082234285906	
		4.20098241566359	0.19083206957815	
		4.68768061496920	0.12484959444854	
		5.09962392258045	0.05174129185413	
		5.38954725618336	0.00965613628033	

S	N	X_k	A_k
3	2	6.38426843763965	0.30554705223059
		7.74968824357975	0.29377546321483
	4	5.89187736904971	0.07091540555768
		6.63385715595154	0.23673297701159
		7.51099440956639	0.22785865191462
		8.26821266447014	0.06381548096153
	6	5.72325237534609	0.02240004995697
		6.16129391316855	0.10206871123038
		6.75257273027006	0.18421608589130
		7.39909800991468	0.17827947792438
		7.99712927189181	0.09291195015627
	8	8.44468955643414	0.01944624028612
5.64742837156860		0.00899215275340	
5.93147822333563		0.04618321997435	
6.33781752060490		0.10493193224608	
6.82200146655804		0.14920037954094	
7.33399608424485		0.14506836448934	
7.82169051647855		0.09667996469727	
8.23374350921060		0.04061909281417	
8.52340121029460	0.00764740892987		
4	2	9.52569539389943	0.25290464577015
		10.89211687700662	0.24614497467337
	4	9.02843698417596	0.05803185815485
		9.77370828658947	0.19607807512870
		10.65159799477811	0.19097492985179
		11.40739662056320	0.05396475730819
	6	8.85822933593985	0.01820952599728
		9.29844044062610	0.08373652403923
		9.89142987717250	0.15275858487002
		10.53857227610608	0.14934187851481
		11.13623539960997	0.07848182474293
	8	11.58300694550210	0.01652128227926
8.78179034834599		0.00728151975845	
9.06719191811286		0.03765030398164	
9.47496966260703		0.08629345189551	
9.96023300918492		0.12383682639832	
10.47275024572454		0.12145751363792	
10.96044603179589		0.08155185089794	
11.37217010678902		0.03446410661859	
11.66142998535677	0.00651404725517		

S	N	X_k	A_k
5	2	12.66734968101222	0.22052500469567
		14.03415140098986	0.21601010762881
	4	12.16765264939948	0.05030875592729
		12.91452128672628	0.17102093190601
		13.79269637533593	0.16761063756418
		14.54760869995916	0.04759478692700
	6	11.99666885176896	0.01573409732540
		12.43791810882215	0.07268001136442
		13.03171245495085	0.13331267015135
		13.67908886058116	0.13102864501679
		14.27645683492945	0.06917119460953
	8	14.72271428267796	0.01460849385700
11.91993248266430		0.00627956828219	
12.20597231220889		0.03257609120865	
12.61442638053717		0.07498787782526	
13.10018069313495		0.10812195195439	
13.61289609740387		0.10653110440897	
14.10051109766581		0.07182012928850	
14.51198019130551		0.03045023937364	
14.80096349330559	0.00576814998289		
6	2	15.80902564059889	0.19805455434352
		17.17601141345626	0.19476800493725
	4	15.30786487953550	0.04502350662653
		16.05567302138838	0.15361668147030
		16.93398619784171	0.15113357031579
		17.68830328285757	0.04304880086815
	6	15.13642098884404	0.01405328663747
		15.57827573204508	0.06509025468484
		16.17253388942674	0.11978566069494
		16.82002351545568	0.11812235292132
		17.41717445715894	0.06253637918502
	8	17.86308006273045	0.01323462515716
15.05950981368121		0.00560231358595	
15.34591884104648		0.02911907909493	
15.75476259001626		0.06720417373053	
16.24079326390014		0.09717738431837	
16.75360454739527		0.09601876464529	
17.24114171833230		0.06489799629382	
17.65242511277094		0.02757239530829	
17.94121420567474	0.00523045230359		

S	M	X_k	A_k		
7	2	18.95069293692949	0.18129311957081		
		20.31778158488716	0.17876371696218		
	4	18.44855639773336	0.04111567955638		
		19.19698209584964	0.14062810409420		
		20.07537244138803	0.13871677042572		
		20.82926442014468	0.03959628245668		
	6	18.27680830492153	0.01281660649542		
		18.71905879639375	0.05946808502181		
		19.31361738531078	0.10968150556042		
		19.96117028084584	0.10840109081179		
		20.55815873474435	0.05750269993294		
	21.00381035772162	0.01218684871061			
8	2	18.19978220452049	0.00510541816158		
		18.48643117448062	0.02657033628169		
		18.89552796331855	0.06142786519298		
		19.38173398626847	0.08899629112677		
		19.89459884841946	0.08810433878438		
		20.38207103410535	0.05965286582986		
		20.79321601151989	0.02538031641742		
		21.08186294677337	0.00481940473830		
		8	2	22.09234815442154	0.16817256811429
				23.49950007652645	0.16614841155441
4	21.58951474491165		0.03807526678906		
	22.33837694363783		0.13045791960261		
	23.21681474754707		0.12892823777574		
	23.97038860267225		0.03685955550129		
6	21.41755066869660		0.01185766637526		
	21.86007973944828		0.05508842203252		
	22.45484834836810		0.10176507555244		
	23.10244015322938		0.10074027711257		
	23.69930343353495		0.05351568758588		
24.14476366884041	0.01135385101004				
8	2	21.34044330350925	0.00472086271001		
		21.62726047366750	0.02459129399462		
		22.03653316165274	0.05692259675168		
		22.52286116086941	0.08258341142634		
		23.03575897258349	0.08186950152772		
		23.52317802965495	0.05550209644031		
		23.93421664169045	0.02363914836556		
		24.22275551164581	0.00449206845247		

S	N	X_k	A_k
9	2	25.23399273268616	0.15754118453908
		26.60118632705126	0.15587388765267
	4	24.73063677087437	0.03562263034497
		25.47982369753300	0.12221558119087
26.35829275534542		0.12095550945848	
6	27.11161984920106	0.03462135119743	
	24.55851144692037	0.01108600943502	
	25.00124712039390	0.05155229612466	
	25.59617063122067	0.09534650116028	
	26.24378805958970	0.09450229621592	
8	26.84055231896436	0.05025687161388	
	27.28586337476252	0.01067109764199	
	24.48134359830344	0.00441184796767	
	24.76828511248740	0.02299719635962	
	25.17768800092031	0.05328182745973	
	25.66410458242701	0.07738206960230	
	26.17702409284353	0.07679395490115	
	26.66439951480252	0.05211173185917	
27.07535411767476	0.02221300597223		
27.36380824480785	0.00422343806988		
2	28.37562870727189	0.14870012351381	
	29.74285119456903	0.14729588281354	
4	27.87186639284432	0.03358996228250	
	28.62130425919489	0.11536031682743	
	29.49979498551987	0.11429900882027	
	30.25292519862849	0.03274671839715	
6	27.69961610461560	0.01044768228852	
	28.14251104595197	0.04861970607989	
	28.73755343642115	0.09000635416997	
	29.38518862953618	0.08929529678124	
	29.98187275421764	0.04752868708186	
	30.42706435531633	0.01009827992586	
8	27.62240149548652	0.00415650052551	
	27.90943862266694	0.02167755635993	
	28.31894144990923	0.05026042620253	
	28.80542543237927	0.07305348762797	
	29.31835998645035	0.07255812278884	
	29.80569917069343	0.04927492375477	
	30.21658586149600	0.02101714432702	
	30.50497183097916	0.00399784474078	

S	N	X_k	A_k
11	2	31.51725785106629	0.14119753919939
		32.88450118996761	0.13999377979067
	4	31.01317051178295	0.03186966686218
		31.76280807633665	0.10954230849969
		32.64131444000777	0.10863249527239
		33.39428396934832	0.03114684835579
	6	30.84082054215471	0.00990824837333
		31.28384198048343	0.04613649249901
		31.87897844126911	0.08547306947701
		32.52662645443758	0.08486350015441
		33.12324447075985	0.04520124969358
	33.56833830718896	0.00960875879272	
8	30.76356870560441	0.00394089551308	
	31.05068161636360	0.02056171820028	
	31.46026353782896	0.04770066479342	
	31.94680047598937	0.06937816187598	
	32.45974588641141	0.06895349515499	
	32.94705460487474	0.04685585015758	
	33.35788531498511	0.01999562525492	
33.64621531416330	0.00380490803981		
12	2	34.65888156345639	0.13472662155254
		36.02614044203718	0.13367979351934
	4	34.15452833731468	0.03038916007488
		34.90432857918431	0.10452393877827
		35.78284659633066	0.10373271863411
		36.53568252438770	0.02976059758461
	6	33.98209700522263	0.00944455933228
		34.42522132345713	0.04399851779499
		35.02043405380653	0.08156205810838
		35.66809162085895	0.08103193847647
		36.26465420645689	0.04318520894838
	36.70966657563559	0.00918413241139	
8	33.90481483129648	0.00375569151752	
	34.19198927174666	0.01960211670275	
	34.60163532939656	0.04549585222681	
	35.08821494482190	0.06620680906884	
	35.60116844570870	0.06583748939234	
	36.08845124744824	0.04476116711366	
	36.49923504692246	0.01910984543957	
36.78751828181799	0.00363744361039		

S	M	X_k	A_k
13	2	37.80050092119639	0.12907076167884
		39.16777168991609	0.12814949864493
	4	37.29592615464303	0.02909746540654
		38.04586146246642	0.10013740375409
		38.92438839607673	0.09944107735721
		39.67711150589790	0.02854431380592
	6	37.12342715647295	0.00904040420722
		37.56663677697008	0.04213256633145
		38.16191257816290	0.07814289465164
		38.80957745518277	0.07767634940444
		39.40609291480581	0.04141681802199
	39.85103636547099	0.00881122770702	
8	37.04611978586374	0.00359435730137	
	37.33334517000380	0.01876540266059	
	37.74304427142092	0.04357089784134	
	38.22965899738486	0.06343390411980	
	38.74261868853304	0.06310887299390	
	39.22987920396943	0.04292433205256	
	39.64062313984798	0.01833219281854	
	39.92886672849125	0.00349030053567	
14	2	40.94211675081384	0.12407209476309
		42.30939681960116	0.12325315648410
	4	40.43735455131309	0.02795759092071
		41.18740378643344	0.09626038234526
		42.06593769410744	0.09564138892298
		42.81856427821866	0.02746588905824
	6	40.26479839490895	0.00868404681562
		40.70807988439713	0.04048546168635
		41.30340870260170	0.07512046402552
		41.95107929713725	0.07470572934448
		42.54755421134598	0.03984921556875
	42.99243860779064	0.00848033380647	
8	40.18746976460965	0.00345217077849	
	40.47473801724022	0.01802740945078	
	40.88448171197083	0.04187121615560	
	41.37112582046678	0.06098246391232	
	41.88409035334701	0.06069352662634	
	42.37133151690973	0.04129646118611	
	42.78204116862028	0.01764232629460	
	43.07025072508895	0.00335967684295	

S	N	X_k	A_k
15	2	44.08372969074906	0.11961248816127
		45.45101717066663	0.11887824062196
	4	43.57880685319733	0.02694195468657
		44.32895347483008	0.09280132480351
		45.20749294000540	0.09224633901341
		45.96003600485970	0.02650111027973
	6	43.40620177599013	0.00836675357384
		43.84954463902730	0.03901752589104
		44.44491864161397	0.07242355308138
		45.09259379232016	0.07205170212761
		45.68903346067265	0.03844708018926
		46.13386669549959	0.00818411392009
8	43.32885496767924	0.00332562188731	
	43.61615978965466	0.01737013911394	
	44.02594149343741	0.04035604852803	
	44.51261054625404	0.05879483402036	
	45.02557893740522	0.05853577198489	
	45.51280314995367	0.03984072965618	
	45.92348300790277	0.01702488661249	
	46.21166303650272	0.00324269698003	
16	2	47.22534023890692	0.11560154584750
		48.59263372000153	0.11493833476216
	4	46.72027819637551	0.02602951539568
		47.47050901929309	0.08969016726580
		48.34905298462697	0.08918887063552
		49.10152307730222	0.02563132731267
	6	46.54763077533050	0.00808187301810
		46.99102665666697	0.03769847165992
		47.58643962392413	0.06999764542834
		48.23411846395930	0.06966176552361
		48.83052721540486	0.03718321710624
		49.27531570104383	0.00791690787345
8	46.47026824625400	0.00321203991001	
	46.75760462984200	0.01677987538405	
	47.16741911366697	0.03899426387977	
	47.65410960349666	0.05682686696341	
	48.16708111886209	0.05659286461372	
	48.65429036698918	0.03852879899031	
	49.06494409726677	0.01646803036950	
	49.35309826603017	0.00313714049888	

S	N	X_k	A_k
17	2	50.36694878775682	0.11196876123906
		51.73424719632177	0.11136582731989
	4	49.86176495247570	0.02520390459575
		50.61206929746734	0.08687225577037
		51.49061695778330	0.08641651727118
		52.24302274846944	0.02484191092165
	6	49.68908052215402	0.00782423824724
		50.13252266237901	0.03650472575350
		50.72796957381771	0.06780020422236
		51.37565144307567	0.06749484800992
		51.97203285824481	0.03603630261331
	52.41678187461692	0.00767426971261	
8	49.61170426311484	0.00310935190783	
	49.89906816202427	0.01624595735188	
	50.30891120467726	0.03776161965959	
	50.79562031261410	0.05504410912266	
	51.30859439319311	0.05483137162445	
	51.79579033773806	0.03733845812126	
	52.20642097135860	0.01596245886106	
	52.49455230640147	0.00304126191021	
18	2	53.50855565000498	0.10865822313641
		54.87585815400436	0.10810695805752
	4	53.00326435982878	0.02445217196416
		53.75363345685734	0.08430424619761
		54.63218418821542	0.08388756013872
		55.38453289010897	0.02412120289343
	6	52.83054730669892	0.00758976641987
		53.27403016048350	0.03541762929510
		53.86950690561117	0.06579748760072
		54.51719129256975	0.06551829653659
		55.11354836617720	0.03498934917895
	55.55826231212986	0.00745265216269	
8	52.75315895300612	0.00301592102068	
	53.04054705127492	0.01575995673218	
	53.45041519631185	0.03663891858606	
	53.93714062327266	0.05341922562597	
	54.45011683589909	0.05322471662407	
	54.93730087855071	0.03625201919417	
	55.34791094964519	0.01550075581611	
	55.63602197637697	0.00295366759468	

S	N	X_k	A_k
19	2	56.65016107740799	0.10562494785578
		58.01746702211676	0.10511837129066
	4	56.14477427929485	0.02376391920036
		56.89520083826046	0.08195126310922
		57.77375414968966	0.08156835402221
		58.52605182781238	0.02345978281465
	6	55.97202825378612	0.00737518284771
		56.41554721576361	0.03442219456076
		57.01105038698757	0.06396234108687
		57.65873688913680	0.06370578077327
		58.25507215077245	0.03402863404519
		58.69975472951377	0.00724918583263
	8	55.89462916512950	0.00293043520411
		56.18203871154116	0.01531511064551
		56.59192909333079	0.03561073441574
		57.07866893999941	0.05193021469526
57.59164694381790		0.05175147194781	
58.07882027773450		0.03525519841724	
58.48941192616804		0.01507692424686	
58.77750477373406		0.00287322957391	
20	2	59.79176527465634	0.10283227840756
		61.15907413782477	0.10236466249374
	4	59.28629302822623	0.02313068786328
		60.03677092388373	0.07978488481588
		60.91532642375776	0.07943142370913
		61.66757822615050	0.02284994451301
	6	59.11352110064712	0.00717782632566
		59.55707230447693	0.03350622571023
		60.15259904568369	0.06227263146886
		60.80028734195935	0.06203580158193
		61.39660294738491	0.03314293452789
		61.84125730563873	0.00706152128672
	8	59.03611241970265	0.00285182879892
		59.32354110651668	0.01490592110933
		59.73345132298524	0.03466451012963
		60.22020400243412	0.05055914052069
60.73318352556539		0.05039414350098	
61.22034717440782		0.03433631771129	
61.63092222333577		0.01468605535529	
61.91899870327104		0.00279902377518	

$$(-1)^{s+1} \int_{j_{1,s-1}}^{j_{1,s}} J_1(x) f(x) dx = \sum_{k=1}^N A_k f(x_k)$$

S	N	x_k	A_k	
1	2	1.06490359717708	0.71352730570934	
		2.72131533209114	0.68923208999321	
	4	0.45894552032648	0.16288779170304	
		1.36622668450166	0.55523923260296	
		2.43260541628559	0.53670441517958	
	6	3.35654135706968	0.14792795621698	
		0.25050630522870	0.05081720068574	
		0.78800957908357	0.23625182964311	
	8	1.50976003202129	0.43333579777075	
		2.29683537470147	0.42085764350012	
		3.02585792001862	0.21694128204246	
	2	3.57417420884993	0.04455564206039	
		0.15682331770170	0.02024226700548	
		0.50569474413235	0.10565639958157	
	4	1.00301791830541	0.24426855346537	
		1.59383568463673	0.35160778001875	
		2.21766918296365	0.34288843027054	
	6	2.81224795846689	0.22683437609859	
		3.31591277526150	0.09387426764509	
		3.67129809520792	0.01738732161716	
	2	2	4.70332406638757	0.36020958440536
			6.09178045076188	0.34266556382333
		4	4.20697213576599	0.08452497234615
			4.95884539269611	0.27885106483800
5.85061361925253			0.26565579559800	
6		6.62216816583074	0.07384331544654	
		4.03701532863946	0.02688456826219	
		4.48055760313514	0.12131514008113	
8		5.08058729933126	0.21673774909876	
		5.73780700902665	0.20792281351893	
		6.34649495226934	0.10760573369767	
2		6.80247437188872	0.02240914357000	
		3.96051332776246	0.01083815916324	
		4.24813692263017	0.05524510276883	
4		4.66018449066159	0.12440793674340	
		5.15181848941217	0.17538408254399	
		5.67223448864598	0.16925351380240	
6		6.16835314072349	0.11209339680686	
		6.58779860890087	0.04686096060493	
		6.88280734910309	0.00879199579503	

S	N	X_k	A_k
3	2	7.89046436064263 9.26593924270673	0.27929007805562 0.27053055152835
	4	7.39170757475265 8.14083657782799 9.02449927572835 9.78609188765246	0.06437798706407 0.21647998141399 0.20987160088239 0.05909106022352
	6	7.22096254355460 7.66335365547810 8.25977427800549 8.91114809828879 9.51308869985421 9.96327758753308	0.02025527744530 0.09279911613778 0.16857713151624 0.16415447941316 0.08597894317765 0.01805568189385
	8	7.14424491000703 7.43107797892434 7.84110840671306 8.32930560139514 8.84516100718880 9.33622755835058 9.75093437676105 10.04236560288399	0.00811251439736 0.04183104639714 0.09554120544721 0.13661165961795 0.13353256077320 0.08939338678600 0.03768702234580 0.00711123381932
4	2	11.05084529773110 12.42249393588882	0.23676807430746 0.23129620999826
	4	10.55035622095976 11.29928685896219 12.18055857148761 12.93861358759833	0.05414829320833 0.18359692781534 0.17946503614699 0.05085402713506
	6	10.37908011859982 10.82150370244048 11.41713545871110 12.06678478862178 12.66646552337644 13.11458128752893	0.01695913597148 0.07818625893186 0.14308201053225 0.14031521325320 0.07393029885625 0.01559136676067
	8	10.30219121871710 10.58900558742281 10.99867021695616 11.48599827763724 12.00050849696499 12.48994348508737 12.90302980069295 13.19319401204112	0.00677416882182 0.03509158580382 0.08062834049676 0.11602315344840 0.11409625089018 0.07678795219881 0.03251050711382 0.00615232553210

S	N	x_k	A_k
5	2	14.20277575787670	0.20932709575021
		15.57278234669750	0.20549768296632
	4	13.70112431857185	0.04766032511259
		14.45021713135490	0.16234994271852
15.33046246515310		0.15945714382099	
16.08675319295981		0.04535736706443	
6	13.52949639069419	0.01488951030566	
	13.97208609997972	0.06888063176443	
	14.56749001389021	0.12657727157124	
	15.21639905736028	0.12463972955381	
	15.81499974221485	0.06590347471404	
	16.26207430983887	0.01393416080735	
8	13.45248887959130	0.00593874108096	
	13.73938383955387	0.03084074327811	
	14.14898409753593	0.07109560171501	
	14.63599699814604	0.10267500393337	
	15.14992977616329	0.10132543478276	
	15.63860690099289	0.06840794646009	
	16.05090580018212	0.02903667495063	
	16.34043785377379	0.00550463251559	
6	2	17.35092698812979	0.18970022603913
		18.72007909266465	0.18682852077385
	4	16.84845641421354	0.04306844803306
		17.59775963504003	0.14714383408084
18.47747079871647		0.14497405293578	
19.23271769201442		0.04134241176329	
6	16.67658172662086	0.01343342354605	
	17.11933575546043	0.06227939121373	
	17.71467737446271	0.11475217786061	
	18.36320113828653	0.11329871660107	
	18.96118363320837	0.06004731238130	
	19.40763867169557	0.01271772521021	
8	16.59948923636146	0.00535299867118	
	16.88647163799561	0.02784242428785	
	17.29608559649272	0.06431831673959	
	17.78296386349010	0.09310317720664	
	18.29659611815122	0.09209071288486	
	18.78484953629112	0.06230281824724	
	19.19669163304717	0.02649042164153	
	19.48584926786576	0.00502787713408	

S	N	X_k	A_k	
7	2	20.49705609254248	0.17475207549155	
		21.86570708211134	0.17249590032658	
	4	19.99398053248468	0.03959583052434	
		20.74348029993656	0.13555842622704	
		21.62287817660110	0.13385351508517	
		22.37743984467512	0.03824020398158	
	6	19.82192401691780	0.01233660018250	
		20.26481970960968	0.05727979784184	
		20.86015741744453	0.10573634858524	
		21.50845521617976	0.10459420313855	
		22.10604258494835	0.05552638570997	
	8	2	22.55208904972942	0.01177464036003
4		19.74476811507798	0.00491278291276	
		20.03182831136635	0.02558017938988	
		20.44147770693811	0.05917797109355	
		20.92829403483559	0.08580114417808	
		21.44175003491934	0.08500550408386	
		21.92973796290051	0.05759447758928	
6	22.34128481088701	0.02451837472683		
	22.63019636639549	0.00465754184389		
8	2	23.64197518735555	0.16287132172503	
		25.01030720168835	0.16103826500164	
	4	23.13843560033686	0.03684978107070	
		23.88810677164586	0.12634823622064	
		24.76730522318109	0.12496295316161	
		25.52138446156331	0.03574861627373	
	6	22.96623980339104	0.01147170536422	
		23.40925437141132	0.05332217813855	
		24.00461021948335	0.09856537148592	
		24.65276417798326	0.09763730588612	
		25.25007942029145	0.05189771924356	
	8	2	25.69583713211917	0.01101530660830
		4	22.88903492024681	0.00456621576717
			23.17616145090228	0.02379425431259
			23.58585074728766	0.05510478219379
			24.07263840699840	0.07999093944177
			24.58598221103153	0.07934441339498
			25.07379039253687	0.05381825203434
	6	25.48513197940088	0.02293178787179	
		25.77387008740123	0.00435894171025	

S	N	X_k	A_k
9	2	26.78611221271032	0.15313179756883
		28.15422866549673	0.15160429865680
	4	26.28220585457282	0.03460723676223
		27.03202403391981	0.11879676395095
		27.91108771152571	0.11764233740114
		28.66480976707720	0.03368975811131
	6	26.10990005723056	0.01076688598109
		26.55301430195048	0.05008775471199
		27.14839629396918	0.09268370490406
		27.79645304325782	0.09191027614112
		28.39357050514045	0.04890081182288
	8	28.83911374621965	0.01038666266449
26.03265625881287		0.00428413438669	
26.31983895600518		0.02233764069481	
26.72956689705462		0.05177338320979	
27.21634242204669		0.07522395684573	
27.72961040264892		0.07468514657999	
28.21728987533060		0.05070130763060	
28.62848107588203		0.02161905935282	
28.91709063323064	0.00411146752520		
10	2	29.92971433767791	0.14495762654666
		31.29767830146744	0.14365928161010
	4	29.42551097627073	0.03273072785073
		30.17545514982937	0.11245824543205
		31.05442348611501	0.11147696846477
		31.80787087615724	0.03195096640921
	6	29.25311615074587	0.01017807839474
		29.69631456349030	0.04737968511760
		30.29172484542490	0.08774539098734
		30.93971282718667	0.08708795203744
		31.53668058576039	0.04637084753524
	8	31.98205845709385	0.00985495408441
29.17584071536247		0.00404870526944	
29.46307115348347		0.02111999343935	
29.87283467507896		0.04898248478792	
30.35960672221288		0.07122061846096	
30.87282106407263		0.07076260655079	
31.36040438228614		0.04807124034347	
31.77148095967056		0.02050928389283	
32.05999157242079	0.00390197541200		

S	M	X_k	A_k
11	2	33.07293439010022	0.13796915895496
		34.44078651871712	0.13684787953902
	4	32.56848570767936	0.03113025287759
		33.31853839313401	0.10703868911058
		34.19743680532443	0.10619121353210
		34.95066665192012	0.03045688297371
	6	32.39601737351183	0.00967654441320
		32.83928751727454	0.04506890504827
		33.43472571740958	0.08352209602283
		34.08266326445098	0.08295429232390
		34.67951406194809	0.04419767669363
	8	35.12476065115589	0.00939752399216
32.31871572514054		0.00384832289107	
32.60598704360893		0.02008229413986	
33.01578269562095		0.04659991671221	
33.50255592952149		0.06779629397443	
34.01573093229943		0.06740072354815	
34.50323999403220		0.04581294542073	
34.91422647328121		0.01955491674542	
35.20265865358104	0.00372162506212		
12	2	36.21587198611030	0.13190448482213
		37.58363966216580	0.13092337755995
	4	35.71121730394508	0.02974406667167
		36.46136408154340	0.10233521745797
		37.34020968988606	0.10159367031623
		38.09326311102544	0.02915490793620
	6	35.53868726025435	0.00924262853210
		35.98201911786795	0.04306676726455
		36.57748380775512	0.07985616140234
		37.22538326850223	0.07935932291605
		37.82214033617590	0.04230446919747
	8	38.26728024256537	0.00899851306956
35.46136354505399		0.00367506350356	
35.74867016164001		0.01918412571919	
36.15849462948662		0.04453479500185	
36.64527157775997		0.06482344215162	
37.15841687325561		0.06447730832406	
37.64586704212707		0.04384620341488	
38.05678093222201		0.01872270589840	
38.34514945718403	0.00356421836852		

S	N	X_k	A_k
13	2	39.35859485261642	0.12657623852276
		40.72629719412573	0.12570834431915
	4	38.85376476741806	0.02852819888671
		39.60399372728105	0.09820266237163
		40.48279848497979	0.09754667425175
		41.23570603004174	0.02800704733182
	6	38.68118219050590	0.00886236852439
		39.12456762502793	0.04131008456351
		39.72005698686985	0.07663472324118
		40.36792698289473	0.07619520525900
		40.96460736056206	0.04063576227012
	8	41.40965890165334	0.00864643898370
38.60383964694296		0.00352330652749	
38.89117699403287		0.01839674855532	
39.30102727328779		0.04272227825750	
39.78780927290001		0.06221072535963	
40.30093158595251		0.06190452340985	
40.78833401622618		0.04211314522500	
41.19918823147534		0.01798859370928	
41.48750408829036	0.00342526179783		
14	2	42.50115065095921	0.12184633620617
		43.86880141051006	0.12107143657435
	4	41.99616943060349	0.02745036765720
		42.74647068226595	0.09453401933269
		43.62524318798194	0.09394831275902
		44.37802815541159	0.02698507303161
	6	41.82354159580227	0.00852553503330
		42.26697393263323	0.03975243080424
		42.86248606970250	0.07377455720266
		43.51033280302867	0.07338212548942
		44.10694933727586	0.03915037023234
	8	44.55192651085262	0.00833275401856
41.74618280052668		0.00338893896746	
42.03354711080345		0.01769908890650	
42.44342054450037		0.04111468852096	
42.93020825010044		0.05989077170782	
43.44331241828887		0.05961737242872	
43.93067544085729		0.04057082374659	
44.34147976671489		0.01733468201386	
44.62975134058122	0.00330140648862		

S	N	X_k	A_k	
15	2	45.64357390004278	0.11761045576331	
		47.01118322358923	0.11691305036754	
	4	45.13846111222209	0.02648626271035	
		45.88882638416447	0.09124843497762	
		46.76757298160521	0.09072129696925	
		47.52025353441799	0.02606751147363	
	6	44.96579388399654	0.00822443828315	
		45.40926759375085	0.03835882163030	
		46.00480067270101	0.07121275848086	
		46.65262871890762	0.07085956636527	
		47.24919132118055	0.03781697644586	
	6	47.69410505922681	0.00805094492541	
		8	44.88842092006103	0.00326887204448
			45.17580905916471	0.01707529365688
			45.58570332185360	0.03967607503203
			46.07249699019568	0.05781264524468
	46.58558658247688		0.05756658251072	
	47.07291656964095		0.03918659963432	
	47.48367859046948		0.01674733980200	
	47.77191242253808		0.00319009820576	
	16	2	48.78589021662078	0.11378810573026
			50.15346575301652	0.11315608524912
		4	48.28066186653750	0.02561717897082
			49.03108418883850	0.08828351821023
49.90980966044582			0.08780579784484	
50.66240022544993			0.02523769595350	
6		48.10796004051998	0.00795317046085	
		48.55147049871203	0.03710232425177	
		49.14702281067104	0.06890077307378	
		49.79483561923223	0.06858069007839	
		50.39135208833354	0.03661128394850	
6		50.83621108813131	0.00779594916609	
		8	48.03057461557192	0.00316073479345
			48.31798395313777	0.01651317602561
			48.72789701662815	0.03837874584599
			49.21469667094557	0.05593702165557
49.72777437789669			0.05571402480122	
50.21507630391176			0.03793516090553	
50.62580201242936			0.01621597696647	
50.91400330138187			0.00308934998556	

S	N	X_k	A_k
17	2	51.92811901262204	0.11031604869548
		53.29566663730247	0.10973979710742
	4	51.42278835679352	0.02482845627094
		52.17326181204774	0.08559024723172
		53.05196983145582	0.08515467749973
		53.80448205394907	0.02448246480050
	6	51.25005590426661	0.00770710736133
		51.69359920863740	0.03596182006161
		52.28916919013400	0.06680044242879
		52.93696941076580	0.06650860006179
		53.53344598807254	0.03551411203770
	8	53.97825728035036	0.00756376385168
51.17265943539115		0.00306267277997	
51.46008774554217		0.01600319213795	
51.87001784072271		0.03720097680732	
52.35682336815282		0.05423299013317	
52.86989125655439		0.05402966758820	
53.35716907309136		0.03679653332719	
53.76786328369924		0.01573222339700	
54.05603622822478	0.00299758963209		
18	2	55.07027526735835	0.10714381656885
		56.43779956814002	0.10661558777007
	4	54.56485340799734	0.02410841935482
		55.31537293591134	0.08312949584257
		56.19406637168034	0.08273022269723
		56.94650978450983	0.02379126644430
	6	54.39209365435467	0.00748257052126
		54.83566648468894	0.03492048346724
		55.43125271920927	0.06488130462111
		56.07904242094546	0.06461378092798
		56.67548417567861	0.03451008842391
	8	57.12025352347755	0.00735117637741
54.31468733035039		0.00297321180421	
54.60213271476071		0.01553774661327	
55.01207829547076		0.03612545274054	
55.49888950590482		0.05267587054703	
56.01194918959737		0.05248949010361	
56.49920608427126		0.03575471411722	
56.90987272100713	0.01528936386314		
57.19802075989611	0.00291355454990		

S	N	X_k	A_k
19	2	58.21237072491146	0.10423057380280
		59.57987533639628	0.10374404869583
	4	57.70686705212284	0.02344763892360
		58.45742829563067	0.08086960341586
		59.33610942032654	0.08050185114200
		60.08849192380959	0.02315552901718
	6	57.53408280957620	0.00727659210551
		57.97768231920183	0.03396472072363
		58.57328352961525	0.06311870576120
		59.22106435161173	0.06287230117520
		59.81747545466850	0.03358672838255
		60.26220763991821	0.00715557435054
8	57.45666763737980	0.00289116314638	
	57.74412846518473	0.01511070896374	
	58.15408817602479	0.03513817927872	
	58.64090483810609	0.05124568551743	
	59.15395759563375	0.05107401819916	
	59.64119617451871	0.03479671045777	
	60.05183847658487	0.01488193947279	
	60.33996446108795	0.00283621746263	
20	2	61.35441472313654	0.10154287426056
		62.72190256204669	0.10109283965436
	4	60.84883724934118	0.02283840474280
		61.59943643269268	0.07878463692706
		62.47810707002704	0.07844446552809
		63.23043528330407	0.02256820671697
	6	60.67603091724635	0.00708674753112
		61.11965464822981	0.03308341284968
		61.71526968539366	0.06149244886718
		62.36304294305720	0.06126452369186
		62.95942686578161	0.03273377237188
		63.40412589942178	0.00697480860319
8	60.59860775720813	0.00281555617783	
	60.88608261787253	0.01471706911980	
	61.29605526653612	0.03422770608289	
	61.78287712985785	0.04992606705931	
	62.29592398722686	0.04976727390485	
	62.78314640639017	0.03391184817840	
	63.19376707780948	0.01450546056398	
	63.48187339781834	0.00276473282787	

$$(-1)^{s+1} \int_{j_{2,s-1}}^{j_{2,s}} J_2(x) f(x) dx = \sum_{k=1}^N A_k f(x_k)$$

S	N	x_k	A_k
1	2	1.83557559386286	0.63354307439217
		3.84562939480103	0.74007473113547
	4	0.89030008436721	0.09998840342473
		2.11598420590046	0.50062791562582
		3.44125395917942	0.59386256293280
		4.56068947413491	0.17913892354430
	6	0.51569196206427	0.02108053942818
		1.29127404479050	0.15978363670852
		2.24009408549737	0.39999918721029
		3.23358862993870	0.46840746060974
		4.13838445746304	0.26708110900833
	4.81642281505437	0.05726587256258	
8	0.33412894838105	0.00590759717632	
	0.85835681836614	0.05300229809476	
	1.53847546771988	0.17815399544911	
	2.31049322863400	0.33072783510314	
	3.10618871812769	0.38126080135927	
	3.85549921456657	0.27936941019601	
	4.48755967295445	0.12207101680667	
	4.93378624790561	0.02312485134237	
2	2	6.04108929268297	0.33399899927855
		7.47112430655471	0.32184046664168
	4	5.52494751073982	0.07745525080710
		6.30242690369448	0.25877443696103
		7.22103313105769	0.24962089061447
		8.01361532147904	0.06998888753763
	6	5.34833637257170	0.02447251879291
		5.80720777619558	0.11146104143152
		6.42666996004173	0.20139690539053
		7.10374903779758	0.19527872883486
		7.72981132703964	0.10190026166770
	8.19835074170119	0.02133000980271	
8	5.26894779377374	0.00982780921031	
	5.56643899066389	0.05043131991642	
	5.99211807297228	0.11459507190869	
	6.49931661539005	0.16313744662924	
	7.03550222501068	0.15888071985094	
	7.54610911097531	0.10601789488111	
	7.97748775268141	0.04456142048342	
	8.28076577076926	0.00838778304008	

S	N	X_k	A_k
3	2	9.30732384056833	0.26327643030934
		10.70199064367538	0.25638480165527
	4	8.79967849611624 9.56045163664055 10.45648017355947 11.22778466478286	0.06040004252111 0.20411838944614 0.19891835157081 0.05622444842656
6	8	8.62595296645366	0.01895420388451
		9.07528699065097	0.08714903137893
		9.68058617347766	0.15902810923306
		10.34109702721271	0.15554772114568
		10.95104663144022	0.08176905714237
8	11	11.4070677800949	0.01721310918007
		8.54794050668571	0.00758007452677
		8.83924028621130	0.03918485450639
		9.25547460293179	0.08981112984530
		9.75079590304641	0.12892363008989
		10.27389939066624	0.12650035981811
		10.77163472773463	0.08496502625995
11.19182698174883	0.03590957850489		
4	2	11.48704801871993	0.00678657841332
		12.50594523866158	0.22566006845203
	4	13.88874743202553	0.22109119643196
		12.00035464567996	0.05147618052582
		12.75600601273691	0.17500294335728
		13.64445883206654	0.17155284993932
	6	14.40816177038954	0.04871929106158
		11.82736930211663	0.01609975841496
		12.27379381056635	0.07436495988163
		12.87456110779089	0.13641862206639
		13.52951318558302	0.13410837242060
8	14	14.13384718249212	0.07080599750631
		14.58531105820107	0.01495355459410
		11.74973718539678	0.00642577201863
		12.03912530262745	0.03333139164060
		12.45237239641464	0.07672665017382
		12.94382475541785	0.11064249535649
		13.46253962102757	0.10903352366123
		13.95584643278424	0.07351670401993
14.37211573295908	0.03117044660608		
14.66447572677423	0.00590428140723		

S	M	X_k	A_k
5	2	15.68071476003226 17.05803150517749	0.20103568278310 0.19772291474177
	4	15.17574410719367 15.92922404117540 16.81417394826207 17.57416839412833	0.04569936910067 0.15592880252866 0.15342632428578 0.04370410160975
	6	15.00300445092219 15.44820365136678 16.04696412274400 16.69934594150262 17.30099632230282 17.75026331670232	0.01426448657254 0.06606663797629 0.12158945897911 0.11991337596175 0.06348861912494 0.01343601891023
	8	14.92551325039795 15.21408685970514 15.62602641071322 16.11573632675523 16.63242205569585 17.12363562260706 17.53801907937861 17.82898786268005	0.00568662550560 0.02955591211807 0.06821241621722 0.09864140964480 0.09747395616845 0.06588584736729 0.02799244618802 0.00530998431541
6	2	18.84411176157675 20.21843069408089	0.18320427655247 0.18066050563954
	4	18.33930648523096 19.09179227851008 19.97473772712297 20.73260353441189	0.04154814113199 0.14211052069478 0.14018856354379 0.04001755682144
	6	18.16664993496016 18.61124248481992 19.20895216673294 19.85992926127910 20.46006735561648 20.90807119176827	0.01295153748702 0.06009321665769 0.11083816548702 0.10955073910208 0.05811466114161 0.01231646231659
	8	18.08921713171157 18.37738224985046 18.78864560425927 19.27742728726571 19.79300404772537 20.28304975175115 20.69636456010225 20.98653660794059	0.00515922768928 0.02684965890746 0.06207368048059 0.08993516891145 0.08903836683021 0.06028753601693 0.02565051675093 0.00487062660515

S	N	X_k	A_k
7	2	22.00115999024791	0.16948417437162
		23.37365854732550	0.16745142752071
	4	21.49635183580416	0.03837165142133
		22.24815017526072	0.13147535353941
		23.13002306050035	0.12993931677095
		23.88653561300444	0.03714928016065
	6	21.32371513854012	0.01195003902469
		21.76797683727839	0.05551703251120
		22.36507565527230	0.10255900744921
		23.01519972694387	0.10153001324188
		23.61439238180328	0.05393644519919
	24.06159179560733	0.01144306446617	
8	21.24630662949607	0.00475767351124	
	21.53424559460690	0.02478263020718	
	21.94512112555745	0.05736552795568	
	22.43335336346859	0.08322790380653	
	22.94825670472314	0.08251109134007	
	23.43757906834189	0.05593837480369	
	23.85022228424511	0.02382505690560	
	24.13988869524707	0.00452734336234	
8	2	25.15429200762589	0.15848731169756
		26.52560122358732	0.15681453206717
	4	24.64941521994206	0.03583620450451
		25.40086247158694	0.12294954111102
		26.28197578102811	0.12168541188317
		27.03756500708549	0.03483068626602
	6	24.47677166051336	0.01115251851040
		24.92084145545620	0.05186124805806
		25.51755869142497	0.09591927150009
		26.16712543028191	0.09507238706031
		26.76568289056555	0.05056082014563
	27.21233253279678	0.01073559849023	
8	24.39937177832746	0.00443833775035	
	24.68717716506677	0.02313502057643	
	25.09781401035961	0.05360117705378	
	25.58569641514276	0.07784705428734	
	26.10015977182225	0.07725708574899	
	26.58900040880581	0.05242683951943	
	27.00119018684910	0.02234737669032	
27.29051208725821	0.00424895213809		

S	M	X_k	A_k
9	2	28.30483420067677	0.14940910010901
		29.67532319806841	0.14800117783916
	4	27.79986436275680	0.03374987275747
		28.55109634344965	0.11591032534996
		29.43168581083763	0.11484629213210
		30.18661145543442	0.03290378770864
	6	27.62720246489142	0.01049744840788
		28.07115653245428	0.04885108449022
		28.66762259116540	0.09043559568123
		29.31680496527883	0.08972273834368
		29.91491240672749	0.04775670975778
	30.36116631117056	0.01014670126739	
8	27.54980355664337	0.00417631352846	
	27.83752654586598	0.02174071952677	
	28.24800883968818	0.05049963501858	
	28.73565623365190	0.07340196795773	
	29.24981615573849	0.07290535771448	
	29.73831833178820	0.04951128612156	
	30.15018536482800	0.02111799342484	
30.43925996430644	0.00401700465575		
10	2	31.45357304442834	0.14174502593494
		32.82347231852917	0.14053869220416
	4	30.94850358721114	0.03199307230008
		31.69959999323216	0.10996704761255
		32.57981282331519	0.10905532845738
		33.33424218051411	0.03126826976908
	6	30.77581875627325	0.00994663426790
		31.21970151200881	0.04631508462135
		31.81599553771590	0.08580456201120
		32.46490155746648	0.08519373372530
		33.06267689114788	0.04537749730701
	33.50863438486530	0.00964620620635	
8	30.69841698129388	0.00395617273171	
	30.98608769318204	0.02064131181423	
	31.39646605857231	0.04784532762083	
	31.88394894776961	0.06964729370959	
	32.39789070389831	0.06922175585172	
	32.88614507117747	0.04703852357869	
	33.29777287468754	0.02007360320336	
33.58666270950660	0.00381972962897		

S	N	X_k	A_k
11	2	34.60100507368449	0.13515982859075
		35.97046605655288	0.13411113995729
	4	34.09583734140955	0.03048675405513
		34.84684720188337	0.10486002665815
		35.72678010742274	0.10406742881829
		36.48082641040109	0.02985675901647
	6	33.92312823936818	0.00947490392744
		34.36696680720992	0.04413977727186
		34.96313930935220	0.08182437120620
		35.61183993695184	0.08129334146329
		36.20936211312307	0.04332477497635
	36.65509050436412	0.00921379970290	
8	33.84572168815615	0.00376776511768	
	34.13335870973947	0.01966505016800	
	34.54366515863719	0.04564193061527	
	35.03102908822675	0.06641978149417	
	35.54480869924232	0.06604983204563	
	36.03287559006558	0.04490580816859	
	36.44432027533990	0.01917161248051	
	36.73306763028119	0.00364918845819	
12	2	37.74745948352683	0.12942051856858
		39.11658579836799	0.12849787329926
	4	37.24219824913862	0.02917622422242
		37.99315284946863	0.10040875422424
		38.87287201058593	0.09971140423896
		39.62661500477498	0.02862200918223
	6	37.06946510607235	0.00906488405234
		37.51327664638840	0.04224657786702
		38.10936117739121	0.07835468719416
		38.75790497194910	0.07788746571973
		39.35522902456875	0.04152957209276
	39.80077576950711	0.00883520494183	
8	36.99205289073644	0.00360409523649	
	37.27966812206251	0.01881618193578	
	37.68992371196134	0.04368881029829	
	38.17719932062744	0.06360586281040	
	38.69085512124347	0.06328036373786	
	39.17877634709139	0.04304117700346	
	39.59007715643768	0.01838210629161	
39.87871178427615	0.00349979455395		

S	N	X_k	A_k
13	2	40.89316326812318	0.12435930106357
		42.26202823181170	0.12353931181312
	4	40.38781462442769	0.02802223994641
		41.13873450511384	0.09648320735607
		42.01828674682971	0.09586343559644
		42.77178444794824	0.02752972997777
	6	40.21505841148294	0.00870413523731
		40.65885407871567	0.04057905781545
		41.25487366668673	0.07529438619241
		41.90329498565752	0.07487913728350
		42.50046056370287	0.03994185594614
	8	42.94586022502730	0.00850004040189
40.13764019943093		0.00346016032301	
40.42524148250290		0.01806908571801	
40.83546051500599		0.04196802268149	
41.32266910953168		0.06112367763741	
41.83622822300665		0.06083438449976	
42.32403373771758		0.04139245621393	
42.73521903263685		0.01768334469062	
43.02376256156351	0.00336748111246		
14	2	44.03827808556366	0.11985176208883
		45.40693504119769	0.11911669942743
	4	43.53284843568026	0.02699579678204
		44.28374738496401	0.09298696416223
		45.16316677593409	0.09243137466501
		45.91646254981189	0.02655432590697
	6	43.36007044633097	0.00838347981244
		43.80385763992086	0.03909548328498
		44.39982852387473	0.07256845374586
		45.04815236483427	0.07219620392310
		45.64518883079296	0.03852429652879
	8	46.09046727074896	0.00820054422110
43.28264621032234		0.00333227315437	
43.57023883141070		0.01740484449521	
43.98043127886442		0.04043668559058	
44.46758809252898		0.05891248610658	
44.98107025300279		0.05865314804473	
45.46878211878118		0.03992073743632	
45.87987296671687		0.01705908202438	
46.16834155175299	0.00324920466409		

S	N	X_k	A_k
15	2	47.18292221865925	0.11580338751882
		48.55141090800140	0.11513953328081
	4	46.67741779755233	0.02607492155250
		47.42830522878095	0.08984676644369
		48.30761715101780	0.08934499344904
		49.06074416473259	0.02567623935441
	6	46.50461943199939	0.00809597558182
		46.94840313226307	0.03776421990014
		47.54433702771520	0.07011988106454
		48.19258201448812	0.06978368641660
		48.78951162670843	0.03724838077280
	8	49.23468864525033	0.00793077706374
		46.42718931151745	0.00321764707849
		46.71477689712976	0.01680914011442
		47.12494986734109	0.03906227626659
		47.61206604957769	0.05692611762210
48.12548595946533		0.05669189745868	
48.61312081272105		0.03859631492153	
49.02413323084482		0.01649689288912	
49.31253921952885	0.00314263444871		
16	2	50.32718422562051	0.11214088722856
		51.69553486682650	0.11153743817320
	4	49.82161088567079	0.02524261636130
		50.57249330452091	0.08700580102848
		51.45171705763920	0.08654968096745
		52.20470118171964	0.02488022704453
	6	49.64879354006721	0.00783625937034
		50.09257712970569	0.03656078432374
		50.68848265426308	0.06790444648806
		51.33666294776704	0.06759883816404
		51.933350292095452	0.03609189305679
	8	52.37859397712885	0.00768610399879
		49.57135776252823	0.00311413091457
		49.85894283330753	0.01627090520744
		50.26910148661497	0.03781961178338
		50.75618536359965	0.05512875092664
51.26955420210874		0.05491583895716	
51.75712484393467		0.03739605265304	
52.16807125507631		0.01598708467750	
52.45642427271638	0.00304595028203		

S	M	X_k	A_k
17	2	53.47113173539563	0.10880641829058
		54.83936771597287	0.10825473512004
	4	52.96549486511870	0.02448549432396
		53.71637680305035	0.08441922530557
		54.59552732414218	0.08400222955537
		55.34838908464042	0.02415420422572
	6	52.79265987631542	0.00760011227064
		53.23644566102894	0.03546588635409
		53.83232925999617	0.06588723884340
		54.48045582018613	0.06560784315092
		55.07721970200299	0.03503722636054
	55.52223707262737	0.00746284643101	
8	52.71521871158540	0.00302003358035	
	53.00280303532086	0.01578142965063	
	53.41295117693347	0.036688884245148	
	53.90000907578525	0.05349210193914	
	54.41333549514408	0.05329745132529	
	54.90085194994577	0.03630162039889	
	55.31174215735526	0.01552196743888	
	55.60004983652127	0.00295770662597	
18	2	56.61481729156391	0.10575362524811
		57.98295699605298	0.10524670534836
	4	56.10912180124935	0.02379284728798
		56.86000642503202	0.08205109969361
		57.73909545593572	0.08166793628254
		58.49185138323730	0.02348844733234
	6	55.93627042141525	0.00738416306018
		56.38005996841343	0.03446409011252
		56.97592653227588	0.06404027324680
		57.62400797461775	0.06378354488584
		58.22070659066539	0.03407021768550
	58.66566017150976	0.00725804160563	
8	55.85882415536058	0.00293400457168	
	56.14640897790580	0.01533375056301	
	56.55654945094731	0.03565407884915	
	57.04358625352869	0.05199349476452	
	57.55687705512747	0.05181463571995	
	58.04434729468691	0.03529827790404	
	58.45518915254638	0.01509534964166	
	58.74345763680934	0.00287673858246	

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19	2	59.75828234003197	0.10294485705213
		61.12634042041591	0.10247695622132
	4	59.25253268004432	0.02315599238136
		60.00342219881029	0.07987223135440
		60.88245909747595	0.07951855919121
		61.63512268045053	0.02287503034647
	6	59.07966606743365	0.00718568063748
		59.52346043045460	0.03354287510923
		60.11931371666783	0.06234081469893
		60.76735690747100	0.06210384535215
		61.36399902555091	0.03317932511590
	8	61.80889689849515	0.00706927235975
		59.00221498660230	0.00285495038932
		59.28980118635761	0.01492222512378
		59.69993613882359	0.03470242839194
		60.18695566279925	0.05061450505605
60.70021626717507		0.05044941152092	
61.18764671782415	0.03437401611818		
	61.59844660354638	0.01470218138178	
	61.88668090364384	0.00280209529149	
	20	2	62.90156001432275
64.26954829188634			0.09991552360750
4		62.39576020812505	0.02256780374421
		63.14665615056705	0.07785862498511
		64.02564846704497	0.07753084487214
		64.77823084214945	0.02230741778119
6		62.22287942648317	0.00700242548319
		62.66667929825404	0.03269196305431
		63.26252222542081	0.06077017335952
		63.91053270446454	0.06055055170695
		64.50712551137928	0.03235503375529
8		64.95197435430727	0.00689454402340
		62.14542380942026	0.00278197477896
		62.43301200275525	0.01454222469880
		62.84314307262545	0.03382330424132
		63.33014833923223	0.04933998489475
	63.84338311970310	0.04918697702631	
64.33077902509277	0.03351893749879		
	64.74154217899614	0.01433829650579	
	65.02974642992312	0.00273299173794	