1. GENE H. GOLUB & JOHN H. WELSCH, "Calculation of Gauss quadrature rules," *Math. Comp.*, v. 23, 1969, pp. 221–230.
2. I. M. LONGMAN, "Tables for the rapid and accurate numerical evaluation of certain infinite integrals involving Bessel functions," *MTAC*, v. 11, 1957, pp. 166–180.

45[2.10,7].—É. N. GLONTI, *Tablit͡sy Korneĭ i Kvadraturnykh Koeffit͡sientov Polinomov Iakobi* (*Tables of the Roots and Quadrature Coefficients of Jacobi Polynomials*), Computing Center, Acad. Sci. USSR, Moscow, 1971, xiv + 236 pp., 27 cm. Price 2.14 rubles.

This is an extensive tabulation of the zeros $x_k^{(n)}$ and Christoffel numbers $A_k^{(n)}$ of the Jacobi polynomials $P_n(p, q, z)$ orthogonal on the interval (0, 1) with weight function $x^{q-1}(1 - x)^{p-q}$. The parameters of p, q range through the values $q = 0.1(0.1)1.0$, $p = (2q - 1)(0.1)(q + 1)$, while $n = 2(1)15$. The precision is 15S in the zeros and 15D in the coefficients. The only published table that is comparable in scope is that of Krylov et al. [1], which covers a somewhat larger region of the parameters, but is restricted to $n \leqq 8$ and a precision of only 8S.

An additional table of quadrature errors

$$e_j^{(n)} = \left| \int_0^1 x^{q-1}(1 - x)^{p-q} x^{2n+j}\, dx - \sum_{k=1}^{n} A_k^{(n)} [x_k^{(n)}]^{2n+j} \right|,$$

for $0 \leqq j \leqq 16$ and $2 \leqq n \leqq 15$, appears in the introduction. These errors grow slightly as q increases for fixed p and sharply increase with p for fixed q. Consequently, only the errors for $q = 1$, $p = 2$ and $q = 0.1$, $p = -0.8$ are tabulated, representing the largest and smallest errors, respectively, in the tabular range.

The introduction also includes a collection of formal relationships satisfied by Jacobi polynomials, comments on the computation and use of the tables, and information facilitating interpolation.

W. G.

1. V. I. KRYLOV, V. V. LUGIN & L. A. IANOVICH, *Tablit͡sy dli͡a Chislennogo Integrirovanii͡a Funkt͡siĭ so Stepennymi Osobennosti͡ami*, Izdat. Akad. Nauk BSSR, Minsk, 1963.

46[2.35, 6, 13.35].—JAMES W. DANIEL, *The Approximate Minimization of Functionals*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1971, xi + 228 pp., 24 cm. Price $9.50 (cloth).

This book presents the basic theory relevant to problems of approximately minimizing functionals and an analysis of some of the numerical methods available for their solution. It is an informative, useful, and very readable book. The author's exposition is clear, his prose is smooth, and the text is attractively printed. Exercises, almost all of a theoretical nature, are interspersed throughout the text, and references to nearly 200 books and papers, most of them quite recent, are well documented. Although practical methods are discussed, theoretical considerations dominate. There is little discussion of the comparative merits of these methods, although some very rough guidelines are given in an epilogue.

In Chapter 1, "Variational Problems in an Abstract Setting", basic functional analysis relevant to minimization problems is reviewed, various notions of convexity are introduced, and some results on minimizing sequences are proved. Most of this material is presented in the context of Banach spaces.

Chapter 2, "Theory of Discretization", presents theory applicable to the convergence of approximate solutions of discretized variational problems.

Chapter 3, "Examples of Discretization", presents specific examples of discretization schemes that can be analyzed by this theory. Particular examples are taken from regularization, optimal control, Chebyshev solution of differential equations, calculus of variations, two-point boundary value problems and the Ritz method.

After introducing criticizing sequences and presenting some general convergence results, Chapter 4, "General Banach-Space Methods of Gradient Type", analyzes various steplength algorithms in great detail. There is also a very brief discussion of direction algorithms and penalty function methods for constrained problems. As the author states in his preface, "$\cdots$ this is not a text on mathematical programming."

Chapter 5, "Conjugate-Gradient and Similar Methods in Hilbert Space", is devoted to a concise but thorough survey of the theory of the conjugate gradient method in Hilbert space. Much of the original work in this area was done by the author.

The remaining four chapters treat minimization in real finite-dimensional spaces. In Chapter 6, "Gradient Methods in R^l", the results of Chapters 4 and 5 are strengthened. In particular, the nature of the limit sets of criticizing sequences are explored, and improved convergence results are given for variants of the conjugate gradient method. There is an oversight in the proof of one of these results, i.e., Theorem 6.4.1, which makes it false as stated. We must have $\bar{p}_0 = r_0$ as well as $z_0 = x_n$ for $z_l = h_n$. Therefore, the theorem is true only for the restart algorithm.

Variable metric methods are described in Chapter 7, "Variable-Metric Gradient Methods in R^l". The emphasis is on proofs of finite termination for quadratics and convergence. In the discussion preceding Theorem 7.4.2, the following statement appears regarding the one-parameter family of variable metric methods first considered by Broyden: $\cdots$ "If $\langle \nabla f(x_{n+1}), p_n \rangle = 0$ for all n, then the directions $p_n/||p_n||$ are independent of the parameters $\{\beta_i\}$." This remarkable result was just recently proved by Dixon. However, although the above statement is true, it does not follow directly from Theorem 7.4.2 or from its proof.

Newton's methods and variants of it, generalized versions of iterative linear methods such as the Jacobi, Gauss-Seidel, SOR and ADI methods, and methods for solving nonlinear least squares problems are the topics covered in Chapter 8, "Operator-Equation Methods". "Only the barest outline", to quote the author, of this important area is presented in this very brief chapter.

"Avoiding Calculation of Derivatives" is the subject and title of Chapter 9. Modifications of the Davidon-Fletcher-Powell, Newton, Gauss-Newton, and Levenberg-Marquardt methods that employ difference approximations to derivatives are described. There is also a section on methods which use only function values, but which are not based upon difference approximations to derivatives. The focus is primarily on Zangwill's version of Powell's method. Here, as elsewhere in the book, the outcome of an isolated computational experiment with a particular algorithm is presented. However, precise details of the algorithm are not given. Numerical results presented in this manner serve no useful purpose in this reviewer's opinion.

Although one could argue with the emphasis of this book, it is a valuable addition to the expanding literature on optimization. It includes some new results not previously published and expounds its subject in a rigorous, yet readable manner.

D. G.

47[2.35,13.35].—PETER L. FALB & JAN L. DE JONG, *Some Successive Approximation Methods in Control and Oscillation Theory*, Academic Press, New York, 1969, viii + 240 pp., 24 cm. Price $13.50.

It is with considerable pleasure that I recommend this fine book by Falb and de Jong. It should prove useful to a wide class of readers with interests in applied mathematics, computational techniques, and control and optimization of nonlinear dynamic systems.

The importance of the book lies mainly in its approach to the subject. The significant problems associated with approximating solutions of nonlinear systems and with the efficient application of digital computers to this class of problems are discussed from a central point of view in the rich mathematical setting of functional analysis. This makes possible on the one hand a rigorous treatment of convergence for a number of algorithms that have been found useful in the computer solution of problems in optimal control, and, on the other hand, provides a framework that makes clear the essential features of the various algorithms. It also provides a useful way of comparing the computational efficiency of various methods after first applying them to the solution of a "standard" problem in optimal control. The results obtained by this approach have been much needed since convergence proofs in the past have either been quite restrictive or nonexistent even for algorithms that have proved quite useful in practice.

Such a unification is obtained by observing that the application of the necessary conditions for optimal control generally gives a two-point boundary value problem, and that the latter may be viewed as a nonlinear operator equation on a suitable Banach space. The results obtained by the authors then follow mainly by application of the contraction mapping principle. The work of the distinguished Soviet mathematician Kantorovich was obviously crucial at this point, for he was among the first to realize the power of functional analysis in the unification of the theory of iterative methods and provided the basic theorems that now support much of the results in this area.

The techniques discussed in the book are all indirect methods since they proceed by first finding a solution to the two-point boundary value problem given by the necessary conditions and then establishing that under suitable conditions this solution provides an optimal value for the functional under consideration. The specific algorithms considered in detail are Newton's method and variations of it, and Picard's method. These are applied to several numerical examples and the results are discussed in terms of the convergence theory.

If there is a lack in this book, it is one of scope. The powerful techniques of functional analysis, so aptly applied by the authors, could have been applied in a wider context that would have allowed discussion of a number of the direct methods. These are basically gradient, or hill climbing, procedures, and conjugate direction techniques and have proved of considerable practical value. It would also have