Although one could argue with the emphasis of this book, it is a valuable addition to the expanding literature on optimization. It includes some new results not previously published and expounds its subject in a rigorous, yet readable manner.

D. G.

47[2.35,13.35].—Peter L. Falb & Jan L. de Jong, *Some Successive Approximation Methods in Control and Oscillation Theory*, Academic Press, New York, 1969, viii + 240 pp., 24 cm. Price $13.50.

It is with considerable pleasure that I recommend this fine book by Falb and de Jong. It should prove useful to a wide class of readers with interests in applied mathematics, computational techniques, and control and optimization of nonlinear dynamic systems.

The importance of the book lies mainly in its approach to the subject. The significant problems associated with approximating solutions of nonlinear systems and with the efficient application of digital computers to this class of problems are discussed from a central point of view in the rich mathematical setting of functional analysis. This makes possible on the one hand a rigorous treatment of convergence for a number of algorithms that have been found useful in the computer solution of problems in optimal control, and, on the other hand, provides a framework that makes clear the essential features of the various algorithms. It also provides a useful way of comparing the computational efficiency of various methods after first applying them to the solution of a "standard" problem in optimal control. The results obtained by this approach have been much needed since convergence proofs in the past have either been quite restrictive or nonexistent even for algorithms that have proved quite useful in practice.

Such a unification is obtained by observing that the application of the necessary conditions for optimal control generally gives a two-point boundary value problem, and that the latter may be viewed as a nonlinear operator equation on a suitable Banach space. The results obtained by the authors then follow mainly by application of the contraction mapping principle. The work of the distinguished Soviet mathematician Kantorovich was obviously crucial at this point, for he was among the first to realize the power of functional analysis in the unification of the theory of iterative methods and provided the basic theorems that now support much of the results in this area.

The techniques discussed in the book are all indirect methods since they proceed by first finding a solution to the two-point boundary value problem given by the necessary conditions and then establishing that under suitable conditions this solution provides an optimal value for the functional under consideration. The specific algorithms considered in detail are Newton's method and variations of it, and Picard's method. These are applied to several numerical examples and the results are discussed in terms of the convergence theory.

If there is a lack in this book, it is one of scope. The powerful techniques of functional analysis, so aptly applied by the authors, could have been applied in a wider context that would have allowed discussion of a number of the direct methods. These are basically gradient, or hill climbing, procedures, and conjugate direction techniques and have proved of considerable practical value. It would also have provided a point of departure for the development of new methods, topological in nature, that take advantage of the mixed topology inherent in the optimal control problem: the fact that the control space has a weak topology and the state space a strong topology. Apparently, this remains to be exploited: the yield in terms of generality and computational efficiency may be considerable. But this is a small disappointment for one reader who, being stimulated, naturally wishes for more.

This book is an important contribution to the literature on mathematics of computation and has gone a good distance toward filling a serious gap in the field.

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48[3].—J. B. ROSEN, O. L. MANGASARIAN & K. RITTER, Editors, *Nonlinear Programming*, Academic Press, New York, 1970, xii + 490 pp., 24 cm. Price $10.50.

This book presents the proceedings of a Symposium in Nonlinear Programming held at the University of Wisconsin at Madison, on May 4–6, 1970. According to the preface, "... one of the main purposes of this Symposium was to further strengthen the existing relationship between theory and computational aspects of this subject." In this, the editors have succeeded admirably.

Of the 17 papers listed below, the first nine are devoted primarily to computational algorithms. The emphasis in this group of papers is nicely balanced between the practical and algorithmic and the theoretical. This is especially true of the papers by Powell, McCormick, and Ritter. Particularly noteworthy is the paper by Bartels, Golub and Saunders. It applies numerically stable and efficient LU and QR matrix decomposition techniques to linearly constrained problems. Until recently, such techniques have gone practically unnoticed by mathematical programmers. The paper by Zoutendijk is a very clear and concise summary of his feasible directions approach applied to several algorithms. Daniel's paper extends to constrained problems the results on steplength algorithms contained in Chapter 4 of his book "The Approximate Minimization of Functionals", and Polak's paper presents the basic ideas for implementing algorithms that he expounds in particular cases in his book, "Computational Methods in Optimization".

The next four papers deal with theoretical aspects of nonlinear programming. Lemke's paper is a good summary of recent results relating to the complementarity problem.

The last four papers consider the application of nonlinear programming to areas such as mathematical analysis and the physical sciences, statistics and probability, and l_p approximation. These papers do not give nonlinear programming solutions to specific problems, rather they indicate how some of the problems that arise in these basic areas can be viewed in terms of nonlinear programming.