50[7].—C. MOSIER & D. D. SHILLADY, *A Fast, Accurate Approximation for $F_0(z)$ Occurring in Gaussian Lobe Basis Electronic Calculations*, Chemistry Department, Virginia Commonwealth University, Richmond, Virginia. Ms. of 3 typewritten pp. and 2 computer sheets deposited in the UMT file.

The mathematical function referred to in the title is defined by the definite integral $F_0(z) = \int_0^1 \exp(-zu^2)\, du$, which is expressible in terms of the error function by the relation $F_0(z) = \frac{1}{2}(\pi/z)^{1/2}\, \mathrm{Erf}(z^{1/2})$, as the authors note.

Specifically, the function $F_0(z)$ is herein approximated for positive z not exceeding 22.5 by a quartic polynomial in $z - s_i$, where the interval i and the corresponding shift s_i are calculated from a given value of z by simple formulas presented in the explanatory text. An accompanying table consists of 16S decimal values (in floating-point form) of the coefficients of this approximating polynomial for $i = 1(1)119$.

The authors claim that the error in their approximation is everywhere less than $4 \cdot 10^{-12}$. Moreover, tests performed by the authors on an IBM 360/50 system have revealed their algorithm to be faster than those based on comparable approximations cited in the literature.

J. W. W.

51[7].—ROBERT PIESSENS & MARIA BRANDERS, *Chebyshev Polynomial Expansions of the Riemann Zeta Function*, 3 pages of tables and 2 pages of explanatory text, reproduced on the microfiche card attached to this issue.

Herein are six 23D tables of the coefficients of the respective expansions of $x\zeta(x+1)$ and $\zeta(x+k)$ for $k = 2(1)5, 8$ in terms of the shifted Chebyshev polynomials $T_n^*(x)$, for $0 \leqq x \leqq 1$.

These tables were calculated on an IBM 1620 at the Computing Centre of the University of Leuven, and each table was checked by calculating $\zeta(x)$ therefrom for several values of x and then comparing the results with corresponding entries in the tables of McLellan [1].

Coefficients of the Chebyshev expansion of $x(\zeta + 1)$ have been published to 20D by Luke [2]; however, several entries are incorrect beyond 16D, as noted by the present authors. As a further check on Table 1 in the set under review, this reviewer has successfully compared it with a similar, unpublished 40D table of Thacher [3].

J. W. W.

1. ALDEN MCLELLAN IV, *Tables of the Riemann Zeta Function and Related Functions*, Desert Research Institute, University of Nevada, Reno, Nevada, 1968. (See *Math. Comp.*, v. 22, 1968, pp. 687–688, RMT **69**.)
2. YUDELL L. LUKE, *The Special Functions and Their Approximations*, v. II, Academic Press, New York and London, 1969.
3. H. C. THACHER, JR., *On Expansions of the Riemann Zeta Function*.

52[7].—GORO SHIMURA, *Introduction to the Arithmetic Theory of Automorphic Functions*, Princeton Univ. Press, Princeton, N.J., 1971, xiv + 267 pp., 24 cm. Price $10.00.

The present book is an advanced work on some of the most fascinating chapters

CHEBYSHEV POLYNOMIAL EXPANSIONS OF THE RIEMANN ZETA FUNCTION

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Introduction

The Riemann zeta function

$$\zeta(x) = \sum_{k=0}^{\infty} k^{-x}$$

occurs in a variety of physical and technical problems. It is thus important to have available approximations, which are convenient for use on a digital computer. We give here Chebyshev series approximations of $\zeta(x)$ for various intervals.

Tables of coefficients of the Chebyshev expansions

Table 1 presents the coefficients $c_k^{(1)}$ of the expansion

$$x\zeta(x+1) = \sum_{k=0}^{\infty} c_k^{(1)} T_k^*(x) \qquad 0\leqslant x\leqslant 1$$

(this table is a correction of a table given by Luke [1]. His table is accurate to 16 decimal places.)
In the other tables, we give the coefficients $c_k^{(i)}$ of the Chebyshev expansions

$$\zeta(x+2) = \sum_{k=0}^{\infty} c_k^{(2)} T_k^*(x) \quad , 0\leqslant x\leqslant 1 \quad \text{(table 2)}$$

$$\zeta(x+3) = \sum_{k=0}^{\infty} c_k^{(3)} T_k^*(x) \quad , 0\leqslant x\leqslant 1 \quad \text{(table 3)}$$

$$\zeta(x+4) = \sum_{k=0}^{\infty} c_k^{(4)} T_k^*(x) \quad , 0\leqslant x\leqslant 1 \quad \text{(table 4)}$$

$$\zeta(x+5) = \sum_{k=0}^{\infty} c_k^{(5)} T_k^{*}(x/3) \quad , 0\leq x\leq 3 \quad \text{(table 5)}$$

$$\zeta(x+8) = \sum_{k=0}^{\infty} c_k^{(6)} T_k^{*}(x/4) \quad , 0\leq x\leq 4 \quad \text{(table 6)}$$

The coefficients are computed utilizing the orthogonality property with respect to summation, as described in [2]. Each table is checked by calculating $\zeta(x)$ for a number of values of x, and comparing the results with the tables of Mc Lellan IV [3].
The computations are carried out on the IBM 1620 of the Computing Centre of the University of Leuven.

<u>References</u>

1. Y.L. Luke, The Special Functions and their Approximations, vol. 2, Academic Press, New York, 1969.

2. C.W. Clenshaw, Chebyshev Series for Mathematical Functions, National Physical Lab. Math. Tables, vol. 5, HMSO, London, 1962.

3. A. Mc Lellan IV, The Riemann Zeta Function to High Precision, Desert Research Institute, University of Nevada, 1968.

4. H.C. Thacher, Jr., On Expansions of the Riemann Zeta Function, Math. Comp. (To appear).

Table 1

k	$c_k^{(1)}$
0	1.31432 83582 88986 45852 146
1	.32263 19132 50488 54378 760
2	.00813 96830 05218 55040 106
3	-.00016 50282 34494 21896 727
4	-.00000 10044 07619 23114 425
5	.00000 01484 07396 83416 742
6	-.00000 00034 64534 60597 441
7	.00000 00000 00780 30567 222
8	.00000 00000 02061 46811 674
9	-.00000 00000 00058 25743 603
10	.00000 00000 00000 57885 437
11	.00000 00000 00000 01100 244
12	-.00000 00000 00000 00053 871
13	.00000 00000 00000 00000 983
14	-.00000 00000 00000 00000 006

Table 2

k	$c_k^{(2)}$
0	1.38126 44621 28791 07082 670
1	-.21408 17876 89329 08307 654
2	.04096 80074 15081 95795 845
3	-.00714 01441 90389 45911 241
4	.00122 58461 27821 69160 184
5	-.00021 02727 41554 83076 445
6	.00003 60751 73415 37067 652
7	-.00000 61894 95049 75335 998
8	.00000 10619 49768 30358 566
9	-.00000 01822 01796 72266 315
10	.00000 00312 60886 59057 715
11	-.00000 00053 63520 19829 427
12	.00000 00009 20234 58180 543
13	-.00000 00001 57887 29310 475
14	.00000 00000 27089 17684 397
15	-.00000 00000 04647 76795 938
16	.00000 00000 00797 43091 230
17	-.00000 00000 00136 81751 444
18	.00000 00000 00023 47417 434
19	-.00000 00000 00004 02753 159
20	.00000 00000 00000 69101 517
21	-.00000 00000 00000 11855 946
22	.00000 00000 00000 02034 159
23	-.00000 00000 00000 00349 006
24	.00000 00000 00000 00059 880
25	-.00000 00000 00000 00010 274
26	.00000 00000 00000 00001 763
27	-.00000 00000 00000 00000 302
28	.00000 00000 00000 00000 052
29	-.00000 00000 00000 00000 009
30	.00000 00000 00000 00000 002

Table 3

k	$c_k^{(3)}$
0	1.13437 67720 07717 65260 451
1	-.05902 29103 31023 87602 954
2	.00772 72325 23762 10924 983
3	-.00083 52373 68964 45161 728
4	.00008 51870 11765 42873 041
5	-.00000 85984 55858 88758 478
6	.00000 08679 45007 92108 359
7	-.00000 00876 64534 73764 293
8	.00000 00088 55805 97125 752
9	-.00000 00008 94622 19566 105
10	.00000 00000 90375 35256 014
11	-.00000 00000 09129 76719 134
12	.00000 00000 00922 29378 814
13	-.00000 00000 00093 17059 232
14	.00000 00000 00009 41214 115
15	-.00000 00000 00000 95081 934
16	.00000 00000 00000 09605 226
17	-.00000 00000 00000 00970 325
18	.00000 00000 00000 00098 023
19	-.00000 00000 00000 00009 902
20	.00000 00000 00000 00001 000
21	-.00000 00000 00000 00000 101
22	.00000 00000 00000 00000 010
23	-.00000 00000 00000 00000 001

Table 4

k	$c_k^{(4)}$
0	1.05715 11369 96683 76630 955
1	-.02249 07900 54801 39476 977
2	.00245 89126 21956 75635 579
3	-.00020 58373 21003 59625 478
4	.00001 53651 90028 69536 470
5	-.00000 11061 99408 25365 491
6	.00000 00792 08841 80305 420
7	-.00000 00056 79252 76929 688
8	.00000 00004 07630 86777 025
9	-.00000 00000 29266 47043 196
10	.00000 00000 02101 28220 369
11	-.00000 00000 00150 86647 042
12	.00000 00000 00010 83174 100
13	-.00000 00000 00000 77768 403
14	.00000 00000 00000 05583 520
15	-.00000 00000 00000 00400 879
16	.00000 00000 00000 00028 782
17	-.00000 00000 00000 00002 066
18	.00000 00000 00000 00000 148
19	-.00000 00000 00000 00000 011
20	.00000 00000 00000 00000 001

Table 5

k	$c_k^{(5)}$
0	1.01611 86295 99375 58571 714
1	-.01556 90364 43911 88441 481
2	.00424 55657 44683 92159 293
3	-.00083 59929 96940 29162 473
4	.00013 55454 07399 22072 856
5	-.00001 97812 47728 75780 244
6	.00000 27611 28739 63934 693
7	-.00000 03813 19742 92945 745
8	.00000 00527 53150 51297 979
9	-.00000 00073 20187 68406 105
10	.00000 00010 17262 64132 396
11	-.00000 00001 41414 84558 906
12	.00000 00000 19658 30392 485
13	-.00000 00000 02732 56553 637
14	.00000 00000 00379 82427 903
15	-.00000 00000 00052 79487 223
16	.00000 00000 00007 33838 725
17	-.00000 00000 00001 02002 256
18	.00000 00000 00000 14178 134
19	-.00000 00000 00000 01970 736
20	.00000 00000 00000 00273 929
21	-.00000 00000 00000 00038 076
22	.00000 00000 00000 00005 292
23	-.00000 00000 00000 00000 736
24	.00000 00000 00000 00000 102
25	-.00000 00000 00000 00000 014
26	.00000 00000 00000 00000 002

Table 6

k	$c_k^{(6)}$
0	1.00155 43116 22136 33947 832
1	-.00177 82465 72386 68853 761
2	.00058 31170 70190 89630 655
3	-.00013 38095 24910 46381 596
4	.00002 38305 34712 08035 489
5	-.00000 35240 26818 68760 206
6	.00000 04559 78869 48590 926
7	-.00000 00540 14540 24097 318
8	.00000 00060 93936 77232 804
9	-.00000 00006 75206 66489 783
10	.00000 00000 74831 79915 587
11	-.00000 00000 08348 62832 631
12	.00000 00000 00936 79697 965
13	-.00000 00000 00105 41773 314
14	.00000 00000 00011 87097 427
15	-.00000 00000 00001 33651 045
16	.00000 00000 00000 15042 290
17	-.00000 00000 00000 01692 646
18	.00000 00000 00000 00190 451
19	-.00000 00000 00000 00021 429
20	.00000 00000 00000 00002 411
21	-.00000 00000 00000 00000 271
22	.00000 00000 00000 00000 031
23	-.00000 00000 00000 00000 003