50[7].—C. Mosier & D. D. Shillady, *A Fast, Accurate Approximation for $F_0(z)$ Occurring in Gaussian Lobe Basis Electronic Calculations*, Chemistry Department, Virginia Commonwealth University, Richmond, Virginia. Ms. of 3 typewritten pp. and 2 computer sheets deposited in the UMT file.

The mathematical function referred to in the title is defined by the definite integral $F_0(z) = \int_0^1 \exp(-zu^2)\, du$, which is expressible in terms of the error function by the relation $F_0(z) = \frac{1}{2}(\pi/z)^{1/2} \operatorname{Erf}(z^{1/2})$, as the authors note.

Specifically, the function $F_0(z)$ is herein approximated for positive z not exceeding 22.5 by a quartic polynomial in $z - s_i$, where the interval i and the corresponding shift s_i are calculated from a given value of z by simple formulas presented in the explanatory text. An accompanying table consists of 16S decimal values (in floating-point form) of the coefficients of this approximating polynomial for $i = 1(1)119$.

The authors claim that the error in their approximation is everywhere less than $4 \cdot 10^{-12}$. Moreover, tests performed by the authors on an IBM 360/50 system have revealed their algorithm to be faster than those based on comparable approximations cited in the literature.

J. W. W.

51[7].—Robert Piessens & Maria Branders, *Chebyshev Polynomial Expansions of the Riemann Zeta Function*, 3 pages of tables and 2 pages of explanatory text, reproduced on the microfiche card attached to this issue.

Herein are six 23D tables of the coefficients of the respective expansions of $x\zeta(x+1)$ and $\zeta(x + k)$ for $k = 2(1)5, 8$ in terms of the shifted Chebyshev polynomials $T_n^*(x)$, for $0 \leqq x \leqq 1$.

These tables were calculated on an IBM 1620 at the Computing Centre of the University of Leuven, and each table was checked by calculating $\zeta(x)$ therefrom for several values of x and then comparing the results with corresponding entries in the tables of McLellan [1].

Coefficients of the Chebyshev expansion of $x(\zeta + 1)$ have been published to 20D by Luke [2]; however, several entries are incorrect beyond 16D, as noted by the present authors. As a further check on Table 1 in the set under review, this reviewer has successfully compared it with a similar, unpublished 40D table of Thacher [3].

J. W. W.

1. Alden McLellan IV, *Tables of the Riemann Zeta Function and Related Functions*, Desert Research Institute, University of Nevada, Reno, Nevada, 1968. (See *Math. Comp.*, v. 22, 1968, pp. 687–688, RMT **69**.)

2. Yudell L. Luke, *The Special Functions and Their Approximations*, v. II, Academic Press, New York and London, 1969.

3. H. C. Thacher, Jr., *On Expansions of the Riemann Zeta Function.*

52[7].—Goro Shimura, *Introduction to the Arithmetic Theory of Automorphic Functions*, Princeton Univ. Press, Princeton, N.J., 1971, xiv + 267 pp., 24 cm. Price $10.00.

The present book is an advanced work on some of the most fascinating chapters in number theory. The author's two main topics are the theory of complex multiplication of elliptic and elliptic modular functions and applications of the theory of Hecke operators to zeta functions of algebraic curves and abelian varieties. Since the prerequisites for studying these topics are quite formidable, probably only the most sophisticated readers will be able to comprehend the book in its entirety.

Professor Shimura has divided his book into three parts which make successively increasing demands upon the reader's mathematical background. The first of these consists of chapters on Fuchsian groups of the first kind, automorphic forms and functions, and the zeta functions associated with modular forms. This material is accessible to those who have mastered the usual introductory graduate courses. However, the Riemann-Roch Theorem and a theorem of Wedderburn about an algebra with radical are stated and used without proof. The next section encompasses elliptic curves, Abelian extensions of imaginary quadratic fields, complex multiplication of elliptic curves, and modular functions of higher level. The classical results of Kronecker, Webber, Takagi and Hasse concerning the construction of maximal Abelian extensions of imaginary quadratic fields by adjoining special values of elliptic and elliptic modular functions are derived by modern methods. Specifically, the adele-theoretic formulation of class field theory as presented in Weil's *Basic Number Theory* and some concepts from algebraic geometry are used freely. The final section treats zeta functions of algebraic curves and Abelian varieties and arithmetic Fuchsian groups. The Hasse-Weil Conjecture, the construction of class fields over real quadratic fields and Fuchsian groups obtained from quaternion algebras are among the topics discussed.

Professor Shimura has made his book even more useful by providing an appendix summarizing the required algebraic geometry and an extensive bibliography.

MARVIN TRETKOFF

Stevens Institute of Technology
Castle Point Station
Hoboken, New Jersey 07030

53[8].—G. W. HILL, *Reference Table*: *"Student's" t-Distribution Quantiles to* 20D, Technical Paper No. 35, Division of Mathematical Statistics, Commonwealth Scientific and Industrial Research Organization, Melbourne, Australia, 1972, 24 pp., 25 cm. Copy deposited in the UMT file.

Quantiles of Student's t-distribution corresponding to the two-tail probability levels $P(t \mid n) = 0.9(-0.1)0.1, \{5, 2, 1\} \times 10^{-r}$ for $r = 2(1)5$, 10^{-s} for $s = 6(1)10(5)30$, and for $n = 1(1)30(2)50(5)100(10)150, 200, \{24, 30, 40, 60, 120\} \times 10^{r}$ where $r = 1(1)3$, and also for $n = \infty$, are herein tabulated to 20D for $t < 10^3$, otherwise to 20S. These numbers were originally calculated to about 25S on a CDC 6400 system prior to rounding to the tabular precision, and elaborate checks applied at successive stages of the calculations and to the final reproduction inspire acceptance of the author's claim of accuracy of the tabular entries to within half a unit in the least significant digit.

We are informed in the introduction that this table is not intended for daily use, but rather has been designed to provide reference values for resolving discrepancies in previous tables and for determining errors of various approximations.