2 [2.05].—ARNOLD SCHÖNHAGE, *Approximationstheorie*, Walter de Gruyter & Co,. Berlin, New York, 1971, 212 pp., 24 cm. Price DM44.— or $14.—.

This concisely written text offers an excellent opportunity to become acquainted with classical problems and achievements of approximation theory. There are eight chapters covering: (i) Basic tools and generalities from functional analysis. (ii) Density theorems of Weierstrass, Stone, Bernstein and Müntz. (iii) Orthogonal polynomials. (iv) Trigonometric approximation. (v) Interpolation. (vi) Chebyshev approximation. (vii) L^1-approximation. (viii) Degree of approximation. Though moderately sized, the book contains a substantial amount of nontrivial matter in almost every chapter. One of the welcome contributions of the book is an introduction to approximation by functions of class G_ν, i.e., entire functions of exponential type and degree at most ν, bounded along and restricted to the real line. This theory, largely due to S. N. Bernstein, significantly extends and deepens trigonometric approximation; portions are worked into Chapters 2, 5 and 8.

Other highlights are: In Chapter 1, duality, strict and uniform convexity, Haar uniqueness theorem, precompactness and "uniform approximability," Bernstein's lethargy theorem, and orthogonal projections. In Chapter 3, separation and monotonicity properties of zeros of orthogonal polynomials, and a detailed discussion of Jacobi polynomials. In Chapter 4, pointwise and uniform convergence of Fourier series under the various classical hypotheses, the Kharshiladze-Lozinski theorem, and Fejér and de la Vallée-Poussin sums. In Chapter 5, approximation by interpolation, and the Markov's inequality in a refined version of Duffin and Schaeffer. Chapter 6 includes Solotarev polynomials with a weight function. Chapter 7 has a discrete analog of Nagy's criterion [1, p. 184] to be applied in the last chapter. There, one finds the generalization to G_ν-approximation of Jackson's second theorem; the latter is obtained as a corollary. Converse theorems of Bernstein and Zygmund and theorems (by Bernstein, Krein, Akhiezer and Favard) on the error in approximating classes of Lipschitz and holomorphic functions conclude the book. There is a moderate number of problems, a very brief bibliography and a usable index.

The material in this result-oriented book is tightly packed and its substance and detail should impress every reader. Proofs are usually complete and are kept as direct and elementary, hence often computational, as possible. In thus coping with limited space, the author nevertheless brings out the interplay of ideas on many occasions. The author's claim to a "modern presentation" needs some qualification since there is no reflection of the attempts by recent authors ([2], [3], etc.) to establish a unifying and more penetrating view of approximation theory. This is essentially a classical analysis book, accessible to second year graduate students in the U.S.A. Computer applications are barely mentioned in the introduction.

There are few misprints or other errors (the reviewer found an erroneous formula after (1.1) on p. 10; Theorem 1.10 is misstated; a factor $|f|$ is missing in problem 5.10; etc.). Credit is given erratically and sometimes erroneously: Bernstein's name occurs in ϵ-proportion to the great role of his results in this book. His generalization of Jackson's theorem is credited to the latter, Lozinski's theorem to Berman, and the student will never know that he learned, e.g., about Bernstein's lethargy result, Lebesgue constants, the Dini-Lipschitz condition or de la Vallée-Poussin sums.

But he will be excellently prepared to reach for more extensive treatises that include such helpful inessentials.

HERMANN G. BURCHARD

Department of Mathematics and Statistics
Oklahoma State University
Stillwater, Oklahoma 74074

1. N. I. AKHIEZER, *Theory of Approximation,* Ungar, New York, 1956.
2. P. L. BUTZER & H. BERENS, *Semigroups of Operators and Approximation,* Die Grundlehren der math. Wissenschaften, Band 145, Springer-Verlag, New York, 1967.
3. H. S. SHAPIRO, *Topics in Approximation,* Lecture Notes in Math., vol. 187, Springer-Verlag, New York and Berlin, 1971.

3 [2.05, 2.25, 2.40, 6, 7].—A. O. GEL'FOND, *Calculus of Finite Differences* (authorized English translation of the third Russian edition), Hindustan Publishing Corp., Delhi-7, India, 1971, vi + 451 pp., 23 cm. Price $10.00.

The first edition of this important treatise was published in 1952. A second, revised and enlarged, edition appeared in 1959, and the third edition (which is essentially identical with the second) in 1967, a year before the author's death. The book has been translated into several languages, including German, French, Chinese, Czechoslovakian, and Romanian. This appears to be the first translation into English. (For a review of the French edition, see Review 3, this Journal, v. 18, 1964, p. 514.)

The calculus of finite differences relates to three broad areas of analysis: interpolation and approximation, summation of functions, and difference equations. The present author places emphasis on the first of these areas, devoting to it three chapters, or about two-thirds of the book. Approximation processes in the complex plane receive particular attention.

Chapter I starts out with Lagrange's and Newton's interpolation formulas and some elementary facts on divided differences. The discussion then moves on to a general interpolation problem associated with an infinite triangular array of nodes, and to resulting interpolation series. There is a discussion of best approximation by polynomials, in preparation to a convergence result for the Lagrange interpolation process. Other polynomial approximation processes are studied, both for real and complex domains. In a final section, a general interpolation problem is conceived as a moment problem in the complex plane. Chapter II is concerned with convergence and representation properties of Newton's series. These are special interpolation series; the cases of equidistant, as well as arbitrary, interpolation points are studied in detail. Some number-theoretic applications are also included, e.g., the author's own proof of the transcendence of e and π. Chapter III, the most advanced and most technical chapter, deals with the problem of constructing an entire function from a denumerable set of data, e.g., from the function values at a sequence of points accumulating at infinity. Problems of this sort do not have unique solutions, but can be treated in a meaningful way by imposing suitable restrictions on the growth of the entire function. They have also bearing on the problem of solving linear differential equations of infinite order with constant coefficients, as is shown at the end of the chapter. The remaining two chapters return to a more elementary level, and to more standard topics, Chapter IV dealing with the problem of summation, Ber-