

tabulation is carried out until both R and I are equal to 0.50000. For $y < 0$, the terminal value of x is either 20 or that value for which subsequent values of R and I are of the order of 10^8 or greater.

The arrangement of the tables is somewhat inconvenient, inasmuch as the second of the four columns on each page is not a continuation of the first column on that page but instead is that of the first column of some subsequent page.

The tables are prefaced by a description of their contents and use, their method of calculation, and means of finding values corresponding to arguments outside the tabular range.

The entries (given in floating-point decimal format) were subjected to a spot check against corresponding values computed independently by the reviewer, and no discrepancies were found.

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- 9 [9].—MORRIS NEWMAN, *A Table of $\tau(p)$ modulo p , p prime, $3 \leq p \leq 16067$* , National Bureau of Standards, August 1972, 7 pp. of computer output deposited in the UMT file.

Let $\tau(n)$ denote the Ramanujan function, defined by

$$\sum_{n=1}^{\infty} \tau(n)x^n = x \prod_{n=1}^{\infty} (1 - x^n)^{24}.$$

Then $\tau(n)$ satisfies the recurrence formula

$$\tau(np) = \tau(n)\tau(p) - p^{11}\tau(n/p),$$

where p is a prime and $\tau(n/p)$ is defined to be zero if p does not divide n . Thus, if p happens to divide $\tau(p)$, then p divides $\tau(np)$ for all n .

As stated in the title, this table lists the values of $\tau(p)$ modulo p for all primes p such that $3 \leq p \leq 16067$. In addition to the known cases $p = 2, 3, 5$, and 7 , the table shows that there is just one more prime p in the indicated range that divides $\tau(p)$; namely, $p = 2411$.

The table was computed by means of the congruence

$$\tau(n) \equiv 540 \sum_{k=1}^n \sigma_3(k)\sigma_3(n-k) \pmod{n},$$

where $\sigma_3(n)$ denotes the sum of the cubes of the divisors of n .

This table was motivated by the unresolved question as to the existence of an n for which $\tau(n) = 0$.

AUTHOR'S SUMMARY

EDITORIAL NOTE: From D. H. Lehmer's table of $\tau(n)$ for $n = 1(1)10000$ (*Math. Comp.*, v. 24, 1970, pp. 495–496, UMT 41) several $\tau(p)$ were selected and reduced (mod p) and no discrepancies were found when the results were compared with those in the present table. For $p = 2411$, one finds

$\tau(p)/p = 1883882662835292$, which ratio is not itself divisible by p . The ratio $\tau(p) \pmod{p}/p$ appears to be distributed uniformly between 0 and 1. That implies that the number of such "Newman primes" (i.e., 2, 3, 5, 7, 2411, ...) that do not exceed N should be asymptotic to $\sum_{p \leq N} (1/p) \sim \ln \ln N$. Since the normal order of magnitude of $\tau(p)$ is $\pm p^{1/2}$, and since there are no other Newman primes ≤ 16067 , it is therefore very improbable that $\tau(n)$, which is multiplicative, will have a zero.

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10 [9].—SAMUEL YATES, *Prime Period Lengths*, RCA Defense Electronic Products, Moorestown, New Jersey. Ms. (undated) of 525 pp. deposited in the UMT file.

This voluminous unpublished table gives the length of the decimal period of the reciprocal of each of the 105000 odd primes (excluding 5) from 3 to 1370471, inclusive. This compilation evolved over the past four years from calculations performed on a succession of electronic computers such as IBM 7090, XDS Sigma 7, RCA Spectra 70/45, and (mainly) RCA Spectra 70/55 at the Moorestown computer facility.

The author has supplied supplementary detailed information relating to the density of those tabulated primes having 10 as a primitive root, from which we find, for example, that there are precisely 39447 such primes in the tabular range. On the other hand, Artin's conjecture [1] predicts a count of 39266 in the same range; however, there exists heuristic reasoning [2] to support the observation that the density of such primes generally exceeds the predicted density. (This reviewer has found the first exception to occur for the interval ending with the prime 138289.) It may be noted here that Cunningham [3] erroneously gave 3618, instead of 3617, as the count of such primes less than 10^5 . Also, D. H. Lehmer & Emma Lehmer [4] reported a count of 8245 such primes below $2.5 \cdot 10^5$, attributed to Miller, but the latter in an unpublished table [5] has given this count as 8255, in agreement with one based on the present table.

The range of this new table is more than tenfold that of any of the previous tables of this type, as listed by Lehmer [6]. The table has materially assisted its author in his continuing search for new prime factors of integers of the form $10^n - 1$ [7].

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1. A. E. WESTERN & J. C. P. MILLER, *Tables of Indices and Primitive Roots*, Royal Society Mathematical Tables, v. 9, Cambridge Univ. Press, London, 1968, p. xli.

2. DANIEL SHANKS, *Solved and Unsolved Problems in Number Theory*, Spartan Books, Washington, D.C., 1962, pp. 80–83.

3. A. CUNNINGHAM, "On the number of primes of the same residuacity," *Proc. London Math. Soc.*, (2), v. 13, 1914, pp. 258–272.

4. D. H. LEHMER & EMMA LEHMER, "Heuristics, anyone?," *Studies in Mathematical Analysis and Related Topics*, Stanford Univ. Press, Stanford, Calif., 1962, pp. 202–210.

5. J. C. P. MILLER, *Primitive Root Counts*, University Mathematical Laboratory, Cambridge, England. Ms. deposited in UMT file; *Math. Comp.*, v. 26, 1972, p. 1024, RMT 54.

6. D. H. LEHMER, *Guide to Tables in the Theory of Numbers*, National Research Council Bulletin No. 105, Washington, D.C., 1941, p. 15.

7. SAMUEL YATES, *Partial List of Primes with Decimal Periods Less than 3000*, Moorestown, New Jersey, Ms. deposited in UMT file; *Math. Comp.*, v. 26, 1972, p. 1024, RMT 55.

11 [10].—P. A. MORRIS, *Self-Complementary Graphs and Digraphs*, 24 pp. deposited in the UMT file.

Two graphs, G and \bar{G} , on the same set of nodes, are *complementary* if two nodes are joined in G if, and only if, they are not joined in \bar{G} . Two digraphs D and \bar{D} , on