

20 [7].—HARRY HOCHSTADT, *The Functions of Mathematical Physics*, John Wiley & Sons, Inc., New York, 1971, xi + 322 pp., 24 cm. Price \$17.50.

The topics which make up the subject “The Functions of Mathematical Physics,” which is also known simply as “The Special Functions,” were first studied in the eighteenth and nineteenth centuries by many eminent mathematicians. Their mathematical research on the subject was most often stimulated by the physical applications in which the topics arose. To a large extent, this tendency has persisted to the present day. However, in current times the subject has acquired a mathematical character of its own and has applications in a number of fields far removed from mathematical physics.

The intent of the author of the volume under review is to reflect this historic interplay noted above by developing topics of interest to both applied workers and to mathematicians. He hopes that the selection will enable the reader to consult more specialized treatises and to get new results as needed. Obviously, the author had to exercise considerable judgment in selecting material, as there is a large mass of material available. I find his selection refreshing and informative. However, it should be recognized that no attempt is made to unify various topics as, for example, in the manner of my own work on the subject. (See Y. L. Luke, “*The Special Functions and Their Approximations*,” Vols. 1 and 2, Academic Press, 1969; and also *Math. Comp.*, v. 26, 1972, pp. 297–299.)

Chapters 1 and 2 deal with orthogonal polynomials in general and with the classical orthogonal polynomials in particular. The gamma function is the subject of Chapter 3. Chapter 4 is titled “Hypergeometric Functions,” but only the  ${}_2F_1$  is treated. Legendre functions, a special case of the  ${}_2F_1$ , are studied in Chapter 5. Chapter 6 treats spherical harmonics in an arbitrary number of dimensions. Confluent hypergeometric functions and Bessel functions are treated in Chapters 7 and 8, respectively. Chapter 9 takes up Hill’s equation.

Each chapter contains a set of exercises. There is a subject index but no notation index. The bibliography consists of texts only. Here, some important volumes have been omitted.

Y. L. L.

21 [7].—M. M. AGREST & M. S. MAKSIMOV, *Theory of Incomplete Cylindrical Functions and Their Applications*, translated from the Russian by H. E. Fettis, J. W. Goresh and D. A. Lee, Springer-Verlag, New York, 1971, 330 pp., 24 cm. Price \$24.50.

A cylinder function is any linear combination of the functions which satisfy Bessel’s differential equation. An example is the cylinder function  $C_\nu(z) = AJ_\nu(z) + BY_\nu(z)$ , where  $J_\nu(z)$  and  $Y_\nu(z)$  are the familiar Bessel functions of the first and second kind, respectively, and  $A$  and  $B$  are independent of  $z$ . Now, both  $J_\nu(z)$  and  $Y_\nu(z)$  have a number of integral representations, say of the form  $\int_a^b K(x, t)g(t) dt$ , where  $a$  and  $b$  are constants independent of  $x$ , for example  $(a, b) = (0, 1)$ ,  $(0, \pi/2)$ ,  $(1, \infty)$  or  $(0, \infty)$ . Such integrals are called complete. If either  $a$  or  $b$  depend on a variable  $y$ , then the integral is said to be incomplete. The incomplete function then satisfies a nonhomogeneous differential equation where the homogeneous part is that satisfied by the