20[7].—HARRY HOCHSTADT, *The Functions of Mathematical Physics*, John Wiley & Sons, Inc., New York, 1971, xi + 322 pp., 24 cm. Price $17.50.

The topics which make up the subject "The Functions of Mathematical Physics," which is also known simply as "The Special Functions," were first studied in the eighteenth and nineteenth centuries by many eminent mathematicians. Their mathematical research on the subject was most often stimulated by the physical applications in which the topics arose. To a large extent, this tendency has persisted to the present day. However, in current times the subject has acquired a mathematical character of its own and has applications in a number of fields far removed from mathematical physics.

The intent of the author of the volume under review is to reflect this historic interplay noted above by developing topics of interest to both applied workers and to mathematicians. He hopes that the selection will enable the reader to consult more specialized treatises and to get new results as needed. Obviously, the author had to exercise considerable judgment in selecting material, as there is a large mass of material available. I find his selection refreshing and informative. However, it should be recognized that no attempt is made to unify various topics as, for example, in the manner of my own work on the subject. (See Y. L. Luke, "*The Special Functions and Their Approximations*," Vols. 1 and 2, Academic Press, 1969; and also *Math. Comp.*, v. 26, 1972, pp. 297–299.)

Chapters 1 and 2 deal with orthogonal polynomials in general and with the classical orthogonal polynomials in particular. The gamma function is the subject of Chapter 3. Chapter 4 is titled "Hypergeometric Functions," but only the ${}_2F_1$ is treated. Legendre functions, a special case of the ${}_2F_1$, are studied in Chapter 5. Chapter 6 treats spherical harmonics in an arbitrary number of dimensions. Confluent hypergeometric functions and Bessel functions are treated in Chapters 7 and 8, respectively. Chapter 9 takes up Hill's equation.

Each chapter contains a set of exercises. There is a subject index but no notation index. The bibliography consists of texts only. Here, some important volumes have been omitted.

Y. L. L.

21[7].—M. M. AGREST & M. S. MAKSIMOV, *Theory of Incomplete Cylindrical Functions and Their Applications*, translated from the Russian by H. E. Fettis, J. W. Goresh and D. A. Lee, Springer-Verlag, New York, 1971, 330 pp., 24 cm. Price $24.50.

A cylinder function is any linear combination of the functions which satisfy Bessel's differential equation. An example is the cylinder function $C_\nu(z) = AJ_\nu(z) + BY_\nu(z)$, where $J_\nu(z)$ and $Y_\nu(z)$ are the familiar Bessel functions of the first and second kind, respectively, and A and B are independent of z. Now, both $J_\nu(z)$ and $Y_\nu(z)$ have a number of integral representations, say of the form $\int_a^b K(x, t)g(t)\,dt$, where a and b are constants independent of x, for example $(a, b) = (0, 1), (0, \pi/2), (1, \infty)$ or $(0, \infty)$. Such integrals are called complete. If either a or b depend on a variable y, then the integral is said to be incomplete. The incomplete function then satisfies a nonhomogeneous differential equation where the homogeneous part is that satisfied by the

Bessel function itself. Incomplete functions are also known as associated Bessel functions. Clearly, there are as many incomplete functions associated with $J_\nu(z)$ as there are integral representations for $J_\nu(z)$ of the kind specified. Remarks similar to the above also hold for $I_\nu(z)$, $K_\nu(z)$ and the Hankel functions. The wordings 'complete' and 'incomplete' are used in a similar fashion for other types of special functions.

There are a number of texts on the special functions that satisfy linear homogeneous differential equations. Except for original works, no textual information on solutions of nonhomogeneous equations exists, except for the rather recent volume by A. W. Babister, *Transcendental Functions Satisfying Nonhomogeneous Linear Differential Equations*, The Macmillan Co., New York, 1967 (see *Math. Comp.*, v. 22, 1968, pp. 223–226). Though the volume under review treats a special differential equation, it is a welcome and valuable addition to the literature, in view of its applicability to numerous problems in mathematical physics and other applied disciplines. Furthermore, there is little overlap with the Babister volume.

The first six chapters deal with various incomplete functions and their mathematical properties, including integral representations, differential and difference equations, series expansions, asymptotic expansions, integrals, etc., i.e., all the properties one normally associates with complete functions. Chapters VII–IX describe numerous applied problems from the fields of wave propagation and diffraction, solid state theory, electromagnetism, atomic and nuclear physics, acoustics, plasma and gas-dynamics and exchange processes between liquid and solid phases which lead to incomplete cylindrical functions.

Finally, Chapter X is a compendium of tables and formulas for evaluation of incomplete cylindrical functions. There are a list of symbols, and author and subject indices. The bibliography of 84 items in the original Russian edition is fairly complete. The translators have added five items to the list, but other references should have been added in both editions.

Y. L. L.

22 [7].—Henry E. Fettis & James C. Caslin, *A Table of the Inverse Sine-Amplitude Function in the Complex Domain*, Report ARL 72-0050, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, April 1972, iv + 174 pp., 28 cm. Copies available from the National Technical Information Service, Springfield, Virginia 22151. Price \$3.00.

The Jacobian elliptic functions with complex argument arise in numerous applications, e.g., conformal mapping, and tabular values are available in [1] and [2]. Often, one desires the inverse function. This could be obtained by inverse interpolation in the above tables. However, such a procedure is inconvenient and of doubtful accuracy, especially in some regions where a small change in the variable produces a large change in the function. Charts are available in [1] from which qualitatively correct values of the inverse could be deduced, but no prior explicit tabulation is known.

Consider