Consider

$$z = \mathrm{sn}(\omega, k), \qquad z = a + ib, \qquad \omega = u + iv,$$

$$u + iv = \sin^{-1}(a + ib) = F(\psi, k),$$

$$a + ib = \sin \psi = \sin(\theta + i\varphi),$$

where $F(\psi, k)$ is the incomplete elliptic integral of the first kind, and k is the usual notation for the modulus. Let C, D, E, and F stand for certain ranges on the parameters. Thus:

$$C\colon 0(0.1)1; \qquad D\colon 0.9(0.01)1;$$

$$E\colon 0.01(0.01)0.1; \qquad F\colon 0.1(0.1)1.$$

Let K and K' be the complete elliptic integrals of the first kind of modulus k and $k' = (1 - k^2)^{1/2}$, respectively. Then the tables give 5D values of $u/k + iv/k'$ for

$$k = \sin \theta, \qquad \theta = 5°(5°)85°(1°)89°,$$

and the ranges

$$a = C, b = C; \quad a = D, b = C; \quad a = C, b^{-1} = E; \quad a = C, b^{-1} = F;$$

$$a^{-1} = E, b = C; \quad a^{-1} = F, b = C; \quad a^{-1} = E, b^{-1} = E;$$

$$a^{-1} = F, b^{-1} = F.$$

The headings for each page were machine printed and here no confusion should arise provided one understands that $K = \sin 5$, for example, should read $k = \sin 5°$.

The method of computation and other pertinent formulas are given in the introduction.

Y. L. L.

1. H. E. Fettis & J. C. Caslin, *Elliptic Functions for Complex Arguments,* Report ARL 67-0001, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, January, 1967. (See *Math. Comp.,* v. 22, 1968, pp. 230–231.)

2. F. M. Henderson, *Elliptic Functions with Complex Arguments,* Univ. of Michigan Press, Ann Arbor, Mich., 1960. (See *Math. Comp.,* v. 15, 1961, pp. 95, 96.)

51 [9].—Bryant Tuckerman, *Odd Perfect Numbers: A Search Procedure, and a New Lower Bound of* 10^{36}, IBM Research Paper RC-1925, October 20, 1967, original report (marked "scarce") and one Xerox copy deposited in the UMT file, 59 pages.

This is the original (1967) much more detailed version of Tuckerman's paper printed elsewhere in this issue. It established the lower bound of 10^{36}. See the following review for a description of the UMT supplement to his present paper.

D. S.

52 [9].—Bryant Tuckerman, *Odd-Perfect-Number Tree to* 10^{36}, IBM, Thomas J. Watson Research Center, Yorktown Heights, New York, 1972, ms. of 9 computer sheets, deposited in the UMT file.

The paper [3] which describes the algorithm used to generate this tree appears elsewhere in this issue. Each node of the tree corresponds to a restriction on the canonical decomposition of an odd perfect number (hereafter denoted by n); and since these restrictions exhaust the logical possibilities, all odd perfect numbers are accounted for. The tree is finite, since no branching is permitted from a node at which it can be determined that the "associated" odd perfect numbers all exceed 10^{36}. Thus, there are only two "least prime divisor" nodes (of level 1) from which branching is permitted. For if the smallest prime divisor of n is neither 3 nor 5, then it follows easily from the tables to be found in [2] that $n > 10^{41}$. Also, as soon as it is known that $p^{2\alpha} \mid n$ and $p^{2\alpha} > 10^{18}$ the tree is truncated, since then $n \geqq p^{2\alpha} \cdot \sigma(p^{2\alpha}) > 10^{36}$. Truncation nodes of the latter type have not been printed out here, and the reviewer would like to suggest that if and when similar trees are generated the program be modified so that such nodes are presented explicitly. Truncation also occurs when the numbers associated with a node can be shown to be either abundant or to possess a prime divisor less than the "least prime divisor." Such nodes *are* printed out here.

Since the algorithm would detect any odd perfect number which did not satisfy the restrictions at a truncation node this tree shows that (i) $n > 10^{36}$, (ii) if neither 3 nor 5 divides n, then $p^{2\alpha} \mid n$ where $p^{2\alpha} > 10^{18}$. ((i) has been improved by the reviewer [1].)

In general, the branching process is dependent on the determination of the prime factors of $F_q(p)$ where p is a known prime divisor of n, q is a prime and $F_q(x)$ is the qth cyclotomic polynomial. (For if $p^\alpha \| n$ it follows that $F_q(p) \mid n$ if $q \mid (\alpha + 1)$.) The complete factorizations of all of the relevant $F_q(p)$ are given here (except when a "least prime" contradiction occurs); and it was the large expenditure of time and effort required for this phase in the execution of the algorithm that necessitated the truncation at 10^{36}. With the steady development of faster computers and more efficient tests of primality it is obviously only a matter of time until Tuckerman's algorithm is utilized to establish a *much better* lower bound for n. Unless, of course, the smallest odd perfect number is discovered in the process.

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1. P. HAGIS, JR., "A lower bound for the set of odd perfect numbers," *Math. Comp.*, v. 27, 1973, pp. 951-953.
2. K. K. NORTON, "Remarks on the number of factors of an odd perfect number," *Acta Arith.*, v. 6, 1961, pp. 365–374.
3. B. TUCKERMAN, "A search procedure and lower bound for odd perfect numbers," *Math. Comp.*, v. 27, 1973, pp. 943-949.

53 [9].—PETER HAGIS, JR., *If n is Odd and Perfect, then* $n > 10^{45}$. *A Case Study Proof with a Supplement in which the Lower Bound is Improved to* 10^{50}, Temple University, Philadelphia, Pennsylvania, 1972, ms. of 83 pp. deposited in the UMT file.

This manuscript comprises mainly the detailed case study that supports the author's