The paper [3] which describes the algorithm used to generate this tree appears elsewhere in this issue. Each node of the tree corresponds to a restriction on the canonical decomposition of an odd perfect number (hereafter denoted by n); and since these restrictions exhaust the logical possibilities, all odd perfect numbers are accounted for. The tree is finite, since no branching is permitted from a node at which it can be determined that the "associated" odd perfect numbers all exceed 10^{36}. Thus, there are only two "least prime divisor" nodes (of level 1) from which branching is permitted. For if the smallest prime divisor of n is neither 3 nor 5, then it follows easily from the tables to be found in [2] that $n > 10^{41}$. Also, as soon as it is known that $p^{2\alpha} \mid n$ and $p^{2\alpha} > 10^{18}$ the tree is truncated, since then $n \geqq p^{2\alpha} \cdot \sigma(p^{2\alpha}) > 10^{36}$. Truncation nodes of the latter type have not been printed out here, and the reviewer would like to suggest that if and when similar trees are generated the program be modified so that such nodes are presented explicitly. Truncation also occurs when the numbers associated with a node can be shown to be either abundant or to possess a prime divisor less than the "least prime divisor." Such nodes *are* printed out here.

Since the algorithm would detect any odd perfect number which did not satisfy the restrictions at a truncation node this tree shows that (i) $n > 10^{36}$, (ii) if neither 3 nor 5 divides n, then $p^{2\alpha} \mid n$ where $p^{2\alpha} > 10^{18}$. ((i) has been improved by the reviewer [1].)

In general, the branching process is dependent on the determination of the prime factors of $F_q(p)$ where p is a known prime divisor of n, q is a prime and $F_q(x)$ is the qth cyclotomic polynomial. (For if $p^\alpha \parallel n$ it follows that $F_q(p) \mid n$ if $q \mid (\alpha + 1)$.) The complete factorizations of all of the relevant $F_q(p)$ are given here (except when a "least prime" contradiction occurs); and it was the large expenditure of time and effort required for this phase in the execution of the algorithm that necessitated the truncation at 10^{36}. With the steady development of faster computers and more efficient tests of primality it is obviously only a matter of time until Tuckerman's algorithm is utilized to establish a *much better* lower bound for n. Unless, of course, the smallest odd perfect number is discovered in the process.

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1. P. HAGIS, JR., "A lower bound for the set of odd perfect numbers," *Math. Comp.*, v. 27, 1973, pp. 951-953.
2. K. K. NORTON, "Remarks on the number of factors of an odd perfect number," *Acta Arith.*, v. 6, 1961, pp. 365–374.
3. B. TUCKERMAN, "A search procedure and lower bound for odd perfect numbers," *Math. Comp.*, v. 27, 1973, pp. 943-949.

53 [9].—PETER HAGIS, JR., *If n is Odd and Perfect, then $n > 10^{45}$. A Case Study Proof with a Supplement in which the Lower Bound is Improved to* 10^{50}, Temple University, Philadelphia, Pennsylvania, 1972, ms. of 83 pp. deposited in the UMT file.

This manuscript comprises mainly the detailed case study that supports the author's paper [1] appearing elsewhere in this issue. Here the author first assembles twelve criteria (1)–(12), of which the first eight are classical (such as results of Euler and others, and properties of $\sigma(\cdot)$); the ninth and tenth are proved in the present manuscript; the eleventh is due to Muskat. The twelfth is due to Hagis and McDaniel [2], also appearing in this issue.

The author then subdivides the set of odd perfect numbers n (if any) into cases (or subcases), repeatedly branching and drawing conclusions, until a lower bound $\geqq 10^{45}$ is derived in each case. Each such lower bound is a product (or a minimum of such products) of known factors and/or known underbounds for unknown necessary factors of every n (if any) in this case. The tools used are judiciously chosen in each case from the aforementioned criteria (1)–(12), results of Kanold and Norton, properties of $\sigma(\cdot)$, deductions from incomplete factorizations and about sources for 3's, etc.

The branching is done first on divisibility by various combinations of 3, 5, 7; then, primarily, on powers or groups of powers, first of 7 or 3, and then of other primes successively generated by $\sigma(\cdot)$ and branching. Some of the above tools are used to pre-exclude certain branches. At times, presumably for greater efficiency, this pattern is varied by the use of other branchings, such as on $11 \mid n$ versus $11 \nmid n$. A computer was used to find factors, generally the ones $< 10^5$, of the relevant $\sigma(p^\beta)$.

This case study is followed on page 47 by a useful outline which gives, for each case and subcase, its name, its defining restrictions, remarks (in some cases), and the deduced lower bound.

The supplement (pages 64–81), which raises the lower bound to 10^{50}, uses two additional tools, (14) due to Tuckerman and (15) due to Robbins and to Pomerance, together with further application of the previous methods.

The author's paper [1] includes 11 typical cases and subcases selected and edited from this manuscript. These illustrate most of the methods used; however, the specialist will want to consult the complete manuscript also.

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1. PETER HAGIS, JR., "A lower bound for the set of odd perfect numbers," *Math. Comp.*, v. 27, 1973, pp. 951-953.
2. PETER HAGIS, JR., & WAYNE L. MCDANIEL, "On the largest prime divisor of an odd perfect number," *Math. Comp.*, v. 27, 1973, pp. 955-957.

54 [10].—P. A. MORRIS, *A Catalogue of Trees*, University of the West Indies, St. Augustine, Trinidad, West Indies, October 1972. Ms. of 10 pp. + 46 computer sheets deposited in the UMT file.

This catalogue lists all unlabeled mathematical trees, without duplication, up to 13 nodes, inclusive. The trees are described by their node pairs, preceded by a code giving the number of edges; thus, for example, the tabular entry 04003, 0102, 0203, 0204 refers to the tree on 4 nodes with the 3 edges (1, 2), (2, 3), (2, 4).

AUTHOR'S SUMMARY