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Some Factorizations of $10^n \pm 1$

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Abstract. Factorizations of $10^n + 1$ and/or $10^n - 1$ are given for a number of values of n.

The factorizations of $10^n \pm 1$ given in Tables 1* and 2 supplement those given by Riesel [4]. Small factors were found by computing $10^n \pm 1$ modulo m, where m runs through the terms of an arithmetic progression determined by Fermat's theorem (e.g. the factors of $10^{34} + 1$, except for 101, are of the form $68k + 1$). One minute's running time on the University of London's CDC 6600 sufficed to try about one million such factors. Larger factors were obtained by the continued fraction method (see, e.g. Knuth [1]). Primality of factors was tested by Lehmer's method [2]. The factorizations of $N - 1$ required for this were all sufficiently simple to make it unnecessary to reproduce them. In these cases small factors were obtained by division by successive odd numbers. The factor 2028119 of $10^{37} - 1$ is due to Ondrejka [3].

The smallest number of the form $10^n \pm 1$ so far unfactorized is $10^{41} + 1$, which is 11 times a 40-digit composite number with no factor less than 92134955.

TABLE 1

n	$(10^n - 1)/9$
31	2791·6943319·57336415063790604359
33	3·37·67·21649·513239·1344628210313298373
37	2028119·247629013·2212394296770203368013
39	3·37·53·79·265371653·900900900900990990990991
41	83·1231·538987·201763709900322803748657942361
43	173·1527791·1963506722254397·2140992015395526641
45	3^2·31·37·41·271·238681·333667·2906161·4185502830133110721

**Editorial note.* Two editors of *Math. Comp.*, and other investigators interested in such problems, were aware that John Brillhart had completely factored

$$(10^p - 1)/9$$

for $p = 31, 37, 41, 43$, and some larger values some time ago, but he had not published them. His factorizations agree exactly with the corresponding four entries in Table 1.

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TABLE 2

n	$10^n + 1$
22	$89 \cdot 101 \cdot 1052788969 \cdot 1056689261$
26	$101 \cdot 521 \cdot 1900381976777332243781$
28	$73 \cdot 137 \cdot 7841 \cdot 127522001020150503761$
29	$11 \cdot 59 \cdot 154083204930662557781201849$
33	$7 \cdot 11^2 \cdot 13 \cdot 23 \cdot 4093 \cdot 8779 \cdot 599144041 \cdot 183411838171$
34	$101 \cdot 28559389 \cdot 1491383821 \cdot 2324557465671829$
35	$11 \cdot 9091 \cdot 909091 \cdot 4147571 \cdot 265212793249617641$
37	$11 \cdot 7253 \cdot 422650073734453 \cdot 296557347313446299$
38	$101 \cdot 722817036322379041 \cdot 1369778187490592461$
39	$7 \cdot 11 \cdot 13^2 \cdot 157 \cdot 859 \cdot 6397 \cdot 216451 \cdot 1058313049 \cdot 388847808493$
40	$17 \cdot 5070721 \cdot 5882353 \cdot 19721061166646717498359681$
42	$29 \cdot 101 \cdot 281 \cdot 9901 \cdot 226549 \cdot 121499449 \cdot 4458192223320340849$
43	$11 \cdot 57009401 \cdot 2182600451 \cdot 7306116556571817748755241$
44	$73 \cdot 137 \cdot 617 \cdot 16205834846012967584927082656402106953$
45	$7 \cdot 11 \cdot 13 \cdot 19 \cdot 211 \cdot 241 \cdot 2161 \cdot 9091 \cdot 29611 \cdot 52579 \cdot 3762091 \cdot 8985695684401$
48	$97 \cdot 353 \cdot 449 \cdot 641 \cdot 1409 \cdot 69857 \cdot 206209 \cdot 66554101249 \cdot 75118313082913$
49	$11 \cdot 197 \cdot 909091 \cdot 5076141624365532994918781726395939035533$

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