## REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

1 [1].—PETR BECKMANN, *A History of $\pi$ (Pi)*, Second Edition, The Golem Press, Boulder, Colorado, 1971, 196 pp., 24 cm. Price \$6.30.

Bernard Baruch once remarked that "Every man has a right to his own opinion, but no man has a right to be wrong in his facts." It is with this in mind that I shall review the present book. The author admits he is neither an historian nor a mathematician and uses this to excuse his avoidance of mathematical rigor and lack of hesitation in giving vent to (controversial) opinions. Pi is looked upon as "a quaint little mirror of the history of man." We shall not discuss the opinions expressed by the author concerning the Roman Empire or his scathing remarks all through the book concerning the Soviet Union (pp. 52, 54, 55, 76, 80, 125, 147, etc.) except to forewarn the reader that this book is more than a technical history of the calculation of $\pi$. The author, however, hopes "that this little book might stimulate nonmathematical readers to become interested in mathematics." It is inevitable, then, that we must look at errors in history and mathematics, errors in which this book abounds.

This is the second edition of the work, appearing not quite a year after the first. The most astonishing thing about the first edition was the absence of any mention of the calculation of $\pi$ after 1946; not one word was said about electronic computers and the calculation of $\pi$ to 100,000 decimals by Daniel Shanks and John W. Wrench, Jr. (*Math. Comp.*, v. 16, 1962, pp. 76–99). It appears that the present edition was rushed into print to remedy that omission while sales were brisk. The book has a new chapter (18) on the computer age, and the end sheets of the book have been imprinted with the first 10,000 decimals of $\pi$ as found by Shanks and Wrench. Unfortunately, even here on the end sheets, it is erroneously stated that the computation was programmed for, and performed on, an IBM 704, instead of an IBM 7090 (as correctly stated on page 180).

What is more, Chapter 18 is marred by several errors of statement regarding the computing time that was required in the evaluation of $\pi$ to 100,265 decimal places on the IBM 7090 system. Page 181 is quite garbled and incorrect. What it should report is that the computation of 24 arctan(1/8) required only 2 hours and 34 minutes (because of using shifting on the binary machine instead of regular division, which would have required 6 hours and 7 minutes), the computation of 8 arctan(1/57) required 3 hours and 7 minutes, and the computation of 4 arctan(1/239) required 2 hours and 20 minutes. This information is contained in the paper by Shanks and Wrench, a copy of which was sent to the author by Dr. Wrench soon after the first edition of the book appeared, so that this reviewer wonders how the author dreamed up the incorrect story that only 34 minutes was needed to compute 4 arctan(1/239) and that this was possible because the third term "converges fastest because of the small argument."

Also on page 181, there is an incorrect account of the later calculations of $\pi$.

Again, this information was sent to the author by Dr. Wrench (letter of 10 December 1970), so that we still must wonder at the careless manner in which facts are reported. What we should read in paragraph 3 is that the calculation to 250,000 decimals was done by Jean Gilloud and Jean Filliatre; whereas the calculation a year later to 500,000 places was done by Jean Gilloud and Michele Dichampt.

On page 99 it is erroneously stated that Abraham Sharp used an arcsine series to compute $\pi$ to 72 decimal digits. The series he did use was the arctangent series displayed at the top of page 140.

On page 100 the author fails to mention that Levi B. Smith and John W. Wrench, Jr. jointly calculated $\pi$ to 808 decimal places and their final, corrected result was published in a joint paper with D. F. Ferguson in *Mathematical Tables and Other Aids to Computation* (now *Math. Comp.*), v. 3, 1948–1949, pp. 18–19. This fact, as well as the account of the unpublished extension of this approximation to 1157 decimal places by Smith and Wrench, appears in the paper of Wrench cited in the bibliography on page 191.

On page 37 there is an incorrect argument to show the existence of infinitely many primes. Assuming there is a largest prime $p$, the author would have us believe that $p! + 1$, which is not divisible by $2, 3, \cdots, p$, "is therefore a prime" when the simple counterexample $5! + 1 = 121 \neq$ prime reveals the fallacy here. This is a common mistake in textbooks.

On page 109 we are told that "The Goldbach conjecture has been proved for numbers greater than $10^{10}$." No reference is cited and this reviewer would like to see such a proof, as he believes no such proof has been given.

The account on page 47 does not explain why certain constructions using compass and straightedge are inadmissible (sliding measurements used to make two segments of equal length, as in certain trisections).

The index is virtually unchanged from the first edition, so that the new names are not possible to retrieve save by a page-by-page perusal.

Among the interesting misprints and other errors in this book, we offer finally the following sampling: on page 92, line 19, for "enginnering" read "engineering"; on page 95, next to the last line, for "and" read "an"; on page 101, line 14 from bottom, for "exhibite" read "exhibited"; on page 102, line 13 from bottom, Dase was born in 1824, not 1840, but the correct date is given on page 100; on page 108, line 11 from bottom, we should read "1920's"; on page 152, equation (18) is wrong, for "tan" read "tanh"; on page 165, line 10 from bottom, for "ansered" read "answered"; on pages 180–181 and the end sheets, the author is uncertain whether to use one or two "*m*'s" in spelling various derivatives of the word "program". The Index should be carefully checked for incorrect page numbers. The statement on page 146, lines 9 and 10 from bottom, seems in rather poor taste.

Besides the insertion of the new chapter and end sheets, the main way that changes have been made is by the addition of six new footnotes on page 188 and some additions to the bibliography and chronological table.

We have then a hastily contrived second edition that continues to garble important history and present some incorrect facts. This could have been a very lively and factually correct reference work if the author had been more careful. As it is, the best advice is that this is an interesting and useful book but one must not believe something merely because it is so stated in this book; there are so many errors that one must check with other sources. The book should be rewritten entirely and the manuscript should be examined by qualified mathematicians and historians before being committed to the printed page; for despite the author's protestations at being neither mathematician nor historian, many readers will undoubtedly quote from the book as gospel truth. The history of $\pi$ may be embellished but it must first of all be correct.

Subject to the limitations we have discussed above, the book under review gives an interesting account of the calculations of $\pi$ from Biblical and Greek times down to the present day. The book is nicely illustrated and printed in a pleasing format.

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2 [2, 3, 4, 5, 7, 8].—A. M. COHEN, J. F. CUTTS, R. FIELDER, D. E. JONES, J. RIBBANS & E. STUART, *Numerical Analysis*, McGraw-Hill Book Co. (UK) Ltd., Maidenhead, Berkshire, England, xiv + 341 pp., 23 cm. Price £3.50.

"This book contains the material which now forms the basis of undergraduate honours courses in the College (Wales Institute of Science and Technology) and only assumes an elementary knowledge of real and complex variable theory. It aims to form a foundation for an appreciation of numerical work and to pave the way to understanding more advanced treatises and research papers."

The authors achieve their goal. They present careful descriptions of numerical methods and apply them to illustrative examples. Many of the mathematical analyses of the methods are found among the numerous problems at the end of each chapter. But, the level of some of these exercises may require a more than elementary facility with complex analysis. For example, in Chapter 2, on the roots of polynomials, one of the exercises asks for a proof of Rouché's theorem, with the hint to use the result of the previous problem in which the integral of $1/(z - a)$ is evaluated over the unit circle. To avoid this concise presentation of the wealth of material would have made the book impossibly long. Nevertheless, this softly (but sturdily) covered book will prove to be a valuable supplementary text for advanced undergraduate and beginning graduate students, who are learning about numerical methods for the first time. Much practical analysis is given in detail. The authors have a sense of humor and of reality as exemplified by the following quotation taken from the solution to a numerical exercise of Chapter 3: "One should not assume that questions are always correctly posed." Indeed, a student who was not aware of this principle might have wasted some effort in trying to establish the text's purposely incorrectly set problem. The book concludes with a large set of miscellaneous exercises and selected solutions for all of the text problems. The chapter headings are: Introduction; Polynomials and their zeros; Interpolation and differentiation; Orthogonal polynomials; Numerical integration (quadrature); Series summation; Function approximation; Direct methods for the solution of simultaneous linear equations;