4 [2.20].—CHIH-BING LING, *On Values of Roots of Monomial-Transcendental Equations*, Department of Mathematics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia. Ms. of 21 typewritten pp. deposited in the UMT file.

Herein are tabulated to 11S, in floating-point form, the first 50 real (or complex) roots of equations of the type $f(z) \pm z = 0$ where $f(z)$ represents one of the trigonometric functions tan z, cot z, sec z, csc z (or the corresponding hyperbolic functions) and the exponential function $e^{\pm z}$.

The introduction includes details of the underlying calculations and a list of references to various applications of the tables.

These tables supersede similar ones published jointly by the author and Yeung [1], which have been found to be generally unreliable, except for the tabulated roots of $\tan z \pm z = 0$.

The author has informed this reviewer that the present tabular values have been thoroughly checked by substitution in the appropriate equations. As a further check, the reviewer has successfully compared the roots of $\tan z = z$ as herein tabulated with the corresponding 40D values calculated by Robinson [2]. Also, the accuracy of the tables corresponding to $f(z) = e^{\pm z}$ has been confirmed by independent calculations to 13S by Fettis [3].

This set of tables may be considered a sequel to a table [4] by the same author, consisting of roots of similar equations where $f(z)$ is, respectively, sin z, cos z, and the corresponding hyperbolic functions.

J. W. W.

1. S.-F. YEUNG & C.-B. LING, "On values of roots of monomial-transcendental equations," *Hung-Ching Chow Sixty-fifth Anniversary Volume*, National Taiwan University, Taipei, Taiwan, December 1967, pp. 196–204.

2. H. P. ROBINSON, *Roots of* $\tan x = x$, Lawrence Berkeley Laboratory, University of California, Berkeley, California, December 1972, ms. deposited in the UMT file. (See *Math. Comp.*, v. 27, 1973, p. 999, RMT **44**.)

3. H. E. FETTIS, Private communication.

4. C.-B. LING, *Values of the Roots of Eight Equations of Algebraic-Transcendental Type*, Virginia Polytechnic Institute, Blacksburg, Virginia, June 1965, ms. deposited in the UMT file. (See *Math. Comp.*, v. 20, 1966, p. 175, RMT **16**.)

5 [3, 9, 10].—MORRIS NEWMAN, *Integral Matrices*, Academic Press, New York, 1972, xvii + 224 pp., 24 cm. Price $14.00.

This book is a gem. It definitely belongs in the library of anyone interested in rings, matrices, number theory, or group theory.

The first five chapters cover the basic material on equivalence, similarity, and congruence of matrices. By page 15, the Hermite normal for a matrix over a principal ideal domain (p.i.d.) appears. But typical of the rest of the book, Dr. Newman gives the reader something new and interesting even in dealing with such a classical result as this: in Theorem II.4, he discusses the number of classes with respect to left equivalence which have a fixed determinant. By page 22, we have the proof that every left ideal in the matrix ring over a p.i.d. is also principal. It is short, easy, and devoid of the usual excess baggage found in most presentations of this theorem. On page 33 there is a very nice argument showing that the Smith normal form is multiplicative, and again on page 35 we are treated to something new with a count of the number of two-sided equivalence classes of matrices with fixed determinant. An interesting application of the Smith form, virtually never found in linear algebra texts, is made on pages 37 and 38 to the solutions of linear diophantine equations.

By page 49, the standard similarity theory over a field is complete and the author goes on to discuss similarity over Z. Here he obtains the Latimer-MacDuffee theorem relating similarity classes over Z for which $f(A) = 0$ with ideal classes in $Z[\theta]$ where θ is a root of the monic integral irreducible polynomial $f(\lambda)$.

Chapter IV begins with the study of congruence of matrices over fields, including the characteristic-2 case. Witt's theorem and a statement of the Hasse-Minkowski theorem appear. A very nice argument is presented to prove that there are only finitely many congruence classes of symmetric integral matrices of a given determinant whose quadratic forms do not properly represent 0. Chapter V is entitled, "Combined similarity and equivalence", and contains a proof of the theorem of M. Hall, Jr. and H. J. Ryser that asserts that if two n-tuples over a field of characteristic not 2 have the same Euclidean square length, then they are orthogonal transforms of one another. Chapter VI provides a quick and self-contained introduction to the Minkowski geometry of numbers including a derivation of sharp bounds for the largest value for the arithmetic minimum of a positive definite form.

Chapter IX contains in about ten pages a mini course in group representation theory which is then used to study automorphs of positive definite quadratic forms and the finite subgroups of the two- and three-dimensional general linear groups over the integers. A circulant is a polynomial in the full cycle permutation matrix, and Chapter X contains a number of interesting results of O. Taussky, R. C. Thompson and the author on this fascinating subject. The last chapter resumes the study of quadratic forms and contains among other items Mordell's inequality for the Hermite constant and Minkowski's proof of the finiteness of the class number.

Chapters VII and VIII on general matrix groups and the classical modular group comprise about 60 pages in this 224-page book. The author includes much of his own fundamental work in this field here. Perhaps a list of the items he covers will convey some idea of the content: the general and special linear groups, the symplectic group, congruence groups over principal ideal rings, generators of the modular group, ranks of subgroups, an original proof of Wohlfahrt's theorem on congruence groups, parabolic class numbers, genus, and discrete subgroups of the special linear group over R.

Each of the eleven chapters ends with a problem set. Almost none of these are easy or routine, but some of the more excruciating ones include hints for solutions.

The theory of integral matrices is a large and difficult field. As the author states in his introduction, it "is a huge subject which extends into many different areas of mathematics." Despite this, Dr. Newman has written an accessible text which leads the reader up to the boundaries of current research in the subject. He has a friendly attitude towards a newcomer to the subject, both in his style of writing and his selection of explicit methods whenever possible. The book can be used for an advanced undergraduate or graduate course assuming only that the students have completed basic courses in modern algebra, matrix theory, and number theory.

Chapters I through V, IX and X could be the core of an advanced, one-semester course in matrix theory including elementary group representation theory. Selected topics from the remaining chapters could more than easily complete a one-year sequence.

This reviewer believes that *Integral Matrices* will certainly take its place among the very best in mathematical expositions: it deals with interesting material; it is packed with information; and it is intelligible.

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6 [7, 9].—ROBERT SPIRA, *Table of* $e^{\pi\sqrt{n}}$, Michigan State University, East Lansing, Michigan. Ms. of 9 typewritten pp. deposited in the UMT file.

This unpublished table consists of 15D values of $e^{\pi\sqrt{n}}$ for $n = 1(1)200$. Because of the increasing size of the integer parts of these numbers, the corresponding number of significant figures in the tabular entries ranges from 17 to 35. In the introduction we are informed that this table was calculated in order to test the author's general multiple-precision Fortran subroutines for the elementary functions. Each entry was computed in about four seconds on a CDC 3600 system, using 117S decimal arithmetic.

The author refers to a listing of decimal approximations to six of these numbers in the FMRC *Index* [1], and he notes his confirmation of terminal-digit errors in two of them, originally announced by Larsen [2].

This table should be of particular interest to number-theorists because of the known relation between the fractional part of $e^{\pi\sqrt{n}}$ and the number of classes of binary quadratic forms of determinant equal to $-n$, as mentioned by D. H. Lehmer [3].

J. W. W.

1. A. FLETCHER, J. C. P. MILLER, L. ROSENHEAD & L. J. COMRIE, *An Index of Mathematical Tables*, 2nd ed., Addison-Wesley Publishing Co., Reading, Massachusetts, 1962.
2. *Math Comp.*, v. 25, 1971, p. 200, MTE **474.**
3. *MTAC*, v. 1, 1943, pp. 30–31, QR **1.**

7 [9].—R. P. BRENT, *The Distribution of Prime Gaps in Intervals up to* 10^{16}, Australian National University, 1973, iv + 62 pp. deposited in the UMT file.

These tables are analogous to the Table 2 of Brent's paper [1]. For all primes p such that $N < p < N'$, the number of gaps

$$p_{i+1} - p_i = g$$

are tabulated for each $g = 2, 4, 6, \cdots$ that occurs in (N, N'). The *estimated* total