10 [9].—KARL C. RUBIN, *Table of* $A^{(k)}(n)$, Woodrow Wilson High School, Washington, D. C., 1973, ms. of 4 pages deposited in the UMT file.

This is an extension of Wagstaff's table [1] of $A^{(k)}(n)$. This is the cardinality of the largest subset of the natural numbers 1 to n wherein no k numbers are in arithmetic progression. Wagstaff computed these for $k = 3(1)8$ and for all $n = 1, 2, \cdots$ up to

$$A^{(3)}(53) = 17, \quad A^{(4)}(52) = 26, \quad A^{(5)}(74) = 48,$$

$$A^{(6)}(52) = 38, \quad A^{(7)}(53) = 42, \quad A^{(8)}(57) = 46.$$

Here, $k = 6(1)8$ are extended up to

$$A^{(6)}(80) = 55, \quad A^{(7)}(94) = 72, \quad A^{(8)}(80) = 64$$

using Wagstaff's method [2] on a SPC-16 minicomputer. The ratios

$$A^{(k)}(n)/n$$

for these three k have therefore been only reduced slightly. The conjecture is that they $\to 0$ as $n \to \infty$.

The author suggests that a further extension is "somewhat impractical" since "$A^{(6)}(80) = 55$ ran for several nights."

D. S.

1. SAMUEL S. WAGSTAFF, JR., *Math. Comp.*, v. 26, 1972, pp. 767–771.
2. S. S. WAGSTAFF, JR., *Math. Comp.*, v. 21, 1967, pp. 695–699.

11 [9].—HUGH WILLIAMS, LARRY HENDERSON & KEN WRIGHT, *Two Related Quadratic Surds Having Continued Fractions with Exceptionally Long Periods*, University of Manitoba, 1973, 177 computer sheets deposited in the UMT file.

It was known [1], [2] that the prime

$$p = 26437680473689$$

has two properties. (A) All numbers < 151 are quadratic residues of p. (B) The class number $h(p)$ of $Q(\sqrt{p})$ equals 1. It follows that the periodic continued fractions

$$\tfrac{1}{2}(\sqrt{p} - 5141757) = \frac{1}{1+}\,\frac{1}{3+}\,\frac{1}{940+}\,\frac{1}{3+}\cdots, \tag{1}$$

and

$$\sqrt{p} - 5141758 = \frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{1880+}\,\frac{1}{1+}\cdots \tag{2}$$

(which are related by Hurwitz's transformation) have periods that are exceptionally long. It turns out here that these periods are 18334815 and 18331889, respectively.

In the period for (1), 7609286 or 41.5018% of the partial quotients equal 1, 3117706 or 17.0042% equal 2, 1706864 or 9.3094% equal 3, and so on, until, finally, 1 partial quotient—the largest—equals 5141757. The first table deposited—73 pages long—lists these frequencies and percentages for each of the 4759 different partial quotients a that occur in (1).

The empirical percentages may be compared with the Gauss-Kuzmin law of almost all continued fractions wherein a occurs with the percentage

$$100 \log[1 + 1/(a^2 + 2a)]/\log 2.$$

For $a = 1, 2, 3$, this gives the values 41.5037, 16.9925, 9.3109, respectively, in close agreement with the empirical data.

In (1), each $a = 1, 2, 3, \cdots$ up to 1471 occurs. The first missing values are 1472, 1525, 1538, 1648, etc. The largest a in (1) are 5141757, 2570878, 1713918, 1285439, etc. These large a have an interesting distribution that we now explain.

All a are given by

$$a = [(\sqrt{p} + B_n)/2A_{n+1}]$$

where $(\pm A_n, B_n, \mp A_{n+1})$ is a reduced binary quadratic form of discriminant

$$p = B_n^2 + 4A_nA_{n+1}.$$

By the special properties of p mentioned above, each $A_{n+1} = k$ ($k = 1$ to 150) will occur in the period 2^r times, where r is the number of distinct primes dividing k. Thus, the first missing k is 151 since this is the first quadratic nonresidue of p, while $a = 13183$ occurs 16 times since $k = 2\cdot 3\cdot 5\cdot 13$ is divisible by four primes.

Since all $k < 151$ occur here, one might guess that large a occur here with a greater frequency than is predicted by the Gauss-Kuzmin law. Not so. The latter predicts $18334815 \log(1 + 1/100001)/\log 2 \approx 265$ partial quotients $a > 10^5$, while, in fact, there are only 163. The reason, of course, is that (1) is nonetheless a quadratic surd, and therefore cannot have rational approximations that are as good as a transcendental number has. Thus, no $a > 5141757 \approx \sqrt{p}$ can occur; none can occur between 5141757 and $2570878 \approx \frac{1}{2}\sqrt{p}$; etc.

The second table—76 pages long—gives the same data for (2). Here there are 4957 different values of a; the first missing a are 1262, 1388, 1612, 1621, etc.; the largest a are $10283516 \approx 2\sqrt{p}$, $3427838 \approx 2\sqrt{p}/3$, $2056703 \approx 2\sqrt{p}/5$, etc.; and Gauss-Kuzmin again holds well for the small a.

Also deposited is the 14 page computer program (for a 360/65) that was used. Apparently, each run took 6 and a fraction minutes central processor time for the $18.33\cdot 10^6$ partial quotients.

D. S.

1. Daniel Shanks, "The infrastructure of a real quadratic field and its applications," *Proceedings of the* 1972 *Number Theory Conference*, Boulder, Colorado, 1972, pp. 217–224.
2. Daniel Shanks, "Five number-theoretic algorithms," *Proceedings of the Second Manitoba Conference on Numerical Mathematics*, 1972, Winnipeg, Manitoba, Canada, pp. 51–70.