

The Character Table of an Eight-Dimensional Orthogonal Group

By David C. Hunt

Abstract. This paper describes the calculation of the character table of the 8-dimensional orthogonal group of maximal index over the field with 3 elements. The group is of interest as it is a subgroup of relatively small index in the sporadic simple group $M(23)$ defined by B. Fischer [2]. The group also has an outer automorphism group of order 24, isomorphic to the symmetric group on 4 symbols.

1. Introduction. Let $O^+(n, q)$, n even, denote the group of orthogonal $n \times n$ matrices of maximal Witt index $n/2$ over the field $GF(q)$.

The order of $O^+(8, 3)$ is $2^{15} \cdot 3^{12} \cdot 5^2 \cdot 7 \cdot 13$. $O^+(8, 3)$ has a subgroup $S\Omega^+(8, 3)$ of index 4. The centre of $S\Omega^+(8, 3)$ is of order 2 and the factor group $PS\Omega^+(8, 3)$ is a simple group of order $2^{12} \cdot 3^{12} \cdot 5^2 \cdot 7 \cdot 13$. $PS\Omega^+(8, 3)$ is the simple Chevalley group known as $D_4(3)$.

2. Conjugacy Classes of $PS\Omega^+(8, 3)$. A theorem of Wall [4] gives the conjugacy classes in $O^+(8, 3)$. Those conjugacy classes containing elements of determinant 1 and which belong to the kernel of the spinorial norm lie in $S\Omega^+(8, 3)$. Some of those classes split into 2 or 4 conjugacy classes in $S\Omega^+(8, 3)$. The conjugacy classes in $PS\Omega^+(8, 3)$ come from forming the factor group of $S\Omega^+(8, 3)$ by its centre. $PS\Omega^+(8, 3)$ has 113 conjugacy classes and they are listed in Table 1 on the microfiche supplement.

3. Permutation Representations of Small Degree. $PS\Omega^+(8, 3)$ acts intransitively on the projective geometry $PG(8, 3)$ which consists of $(3^8 - 1)/(3 - 1) = 3280$ points. The group acts transitively on the points of lengths 0, 1 and 2, respectively. This gives a rank 3 permutation representation of degree 1120 and 2 different rank 3 permutation representations of degree 1080. The values of the corresponding permutation characters can easily be calculated for every conjugacy class in $PS\Omega^+(8, 3)$. The values of 2 of these permutation characters are given in columns of Table 1. The stabilizer of a point in the representation on 1080 points is $PS\Omega(7, 3)$. The character table of $PS\Omega(7, 3)$ has appeared in an earlier paper by the author (Hunt [3]). It is possible to determine how the conjugacy classes of $PS\Omega(7, 3)$ fuse as a subgroup of $PS\Omega^+(8, 3)$. Hence, it is possible to restrict the above permutation representations to $PS\Omega(7, 3)$ and split them into irreducible characters of $PS\Omega(7, 3)$. This set of irreducible characters partitions into 3 subsets which are the components of the 3 irreducible characters of $PS\Omega^+(8, 3)$ restricted to $PS\Omega(7, 3)$. Hence, it is possible to

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calculate the values of the characters of degree 1, 300, 260, 819 on most of the conjugacy classes and the remaining values are uniquely determined on all conjugacy classes of $PS\Omega^+(8, 3)$.

4. The Character Table. Applying the automorphism group to the irreducible characters determined above yields 11 irreducible characters. A large number of generalized characters can be determined by forming tensor products and symmetric and alternating products of the known irreducible characters. Other generalized characters can be determined by inducing characters from subgroups isomorphic to $PS\Omega(7, 3)$ and $PS\Omega^+(8, 2)$. The character table of $PS\Omega^+(8, 2)$ can be found in Dye [1]. The permutation representation on the cosets of a subgroup isomorphic to $PS\Omega^+(8, 2)$ is given as the final column of Table 1. $PS\Omega^+(8, 3)$ contains at least 4 conjugacy classes of subgroups isomorphic to $PS\Omega^+(8, 2)$. The above generalized characters are sufficient to determine the entire rational character table; see Table 2 on the microfiche supplement. Only one rational character is not absolutely irreducible, the last of the 113 which is the sum of 2 absolutely irreducible characters.

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1. R. H. DYE, "The simple group $FH(8, 2)$ of order $2^{12} \cdot 3^5 \cdot 5^2 \cdot 7$ and the associated geometry of triality," *Proc. London Math. Soc.* (3), v. 18, 1968, pp. 521-562. MR 37 #1468.

2. B. FISCHER, "Finite groups generated by 3-transpositions. I," *Invent. Math.*, v. 13, 1971, pp. 232-246. MR 45 #3557.

3. D. C. HUNT, "Character tables of certain finite simple groups," *Bull. Austral. Math. Soc.*, v. 5, 1971, pp. 1-42. MR 46 #1896.

4. G. E. WALL, "On the conjugacy classes in the unitary, symplectic and orthogonal groups," *J. Austral. Math. Soc.*, v. 3, 1963, pp. 1-62. MR 27 #212.

The Character Table of Fischer's Simple Group, $M(23)$

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THE CHARACTER TABLE OF AN EIGHT DIMENSIONAL ORTHOGONAL GROUP

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Table 1 - Page 1

Element	Centralizer	Square	Cube	χ_{1120}	χ_{1080}	χ_{28431}
1A	$2^{12}.3^{12}.5^2.7.13$	1A	1A	1120	1080	28431
2A	$2^{10}.3^6.5.7$	1A	2A	112	128	567
2B	$2^{10}.3^6.5.7$	1A	2B	0	0	567
2C	$2^{10}.3^6.5.7$	1A	2C	0	0	567
2D	$2^{12}.3^4$	1A	2D	32	24	111
4A	$2^8.3^2.5$	2A	4A	12	16	15
4B	$2^6.3^2.5$	2B	4B	0	0	15
4C	$2^6.3^2.5$	2C	4C	0	0	15
4D	$2^9.3^3$	2D	4D	16	12	3
4E	$2^9.3$	2D	4E	0	4	27
8A	2^5	4D	8A	4	2	1
8B	2^5	4E	8B	0	2	1
3A	$2^7.3^{12}$	3A	1A	148	108	0
3B	$2^6.3^{10}.5$	3B	1A	121	135	0
3C	$2^6.3^{10}.5$	3C	1A	121	108	243
3D	$2^6.3^{10}.5$	3D	1A	40	0	0
3E	$2^6.3^{10}.5$	3E	1A	40	0	243
3F	$2^6.3^{10}.5$	3F	1A	40	0	0
3G	$2^6.3^{10}.5$	3G	1A	40	0	243
3H	$2^3.3^{10}$	3H	1A	13	27	243
3I	$2^3.3^{10}$	3I	1A	13	27	0
3J	$2^3.3^{10}$	3J	1A	13	0	0
3K	$2^3.3^{10}$	3K	1A	13	0	0
3L	2.3^8	3L	1A	22	9	0
3M	$2^3.3^8$	3M	1A	4	18	81
9A	$2^2.3^6$	9A	3A	13	9	0

Table 1 - Page 2

Element	Centralizer	Square	Cube	x_{1120}	x_{1080}	x_{28431}
9B	$2^2 \cdot 3^6$	9B	3A	13	18	0
9C	$2^2 \cdot 3^6$	9C	3A	4	0	0
9D	$2^2 \cdot 3^6$	9D	3A	4	0	0
9E	$2^2 \cdot 3^6$	9E	3A	4	0	0
9F	$2^2 \cdot 3^6$	9F	3A	4	0	0
9G	3^6	9G	3A	4	0	0
9H	3^6	9H	3A	4	0	0
9I	3^6	9I	3A	4	0	0
9J	3^6	9J	3A	4	0	0
9K	3^4	9K	3H	1	3	9
9L	3^4	9L	3I	1	3	0
9M	3^4	9M	3J	1	0	0
9N	3^4	9N	3E	1	0	0
6A	$2^6 \cdot 3^6$	3A	2A	4	20	0
6B	$2^6 \cdot 3^6$	3A	2B	0	0	0
6C	$2^6 \cdot 3^6$	3A	2C	0	0	0
6D	$2^5 \cdot 3^5$	3B	2A	13	11	0
6E	$2^5 \cdot 3^5$	3C	2A	13	20	27
6F	$2^5 \cdot 3^5$	3D	2B	0	0	0
6G	$2^5 \cdot 3^5$	3E	2B	0	0	27
6H	$2^5 \cdot 3^5$	3F	2C	0	0	0
6I	$2^5 \cdot 3^5$	3G	2C	0	0	27
6J	$2^3 \cdot 3^4$	3M	2A	4	2	9
6K	$2^3 \cdot 3^4$	3M	2B	0	0	9
6L	$2^3 \cdot 3^4$	3M	2C	0	0	9
6M	$2^7 \cdot 3^4$	3A	2D	20	12	0
6N	$2^6 \cdot 3^4$	3B	2D	17	15	0

Table 1 - Page 3

Element	Centralizer	Square	Cube	X_{1120}	X_{1080}	X_{28431}
6O	$2^6 \cdot 3^4$	3C	2D	17	12	3
6P	$2^6 \cdot 3^4$	3D	2D	8	0	0
6Q	$2^6 \cdot 3^4$	3E	2D	8	0	3
6R	$2^6 \cdot 3^4$	3F	2D	8	0	0
6S	$2^6 \cdot 3^4$	3G	2D	8	0	3
6T	$2^3 \cdot 3^4$	3H	2D	5	3	3
6U	$2^3 \cdot 3^4$	3I	2D	5	3	0
6V	$2^3 \cdot 3^4$	3J	2D	5	0	0
6W	$2^3 \cdot 3^4$	3K	2D	5	0	0
6X	$2^3 \cdot 3^4$	3M	2D	2	6	27
6Y	$2^3 \cdot 3^4$	3N	2D	2	0	3
6Z	$2^3 \cdot 3^4$	3N	2D	2	0	3
6AA	$2^3 \cdot 3^4$	3N	2D	2	6	3
6AB	$2 \cdot 3^4$	3L	2D	2	3	0
12A	$2^3 \cdot 3^2$	6D	4A	3	1	0
12B	$2^3 \cdot 3^2$	6E	4A	3	4	3
12C	$2^3 \cdot 3^2$	6F	4B	0	0	0
12D	$2^3 \cdot 3^2$	6G	4B	0	0	3
12E	$2^3 \cdot 3^2$	6H	4C	0	0	0
12F	$2^3 \cdot 3^2$	6I	4C	0	0	3
12G	$2^6 \cdot 3^3$	6M	4D	16	12	0
12H	$2^6 \cdot 3^3$	6N	4D	4	0	0
12I	$2^6 \cdot 3^3$	6N	4D	4	0	0
12J	$2^4 \cdot 3^3$	6N	4D	1	3	0
12K	$2^4 \cdot 3^3$	6O	4D	1	0	3
12L	$2^4 \cdot 3^3$	6P	4D	4	0	0
12M	$2^4 \cdot 3^3$	6Q	4D	4	0	3

Table 1 - Page 4

Element	Centralizer	Square	Cube	X_{1120}	X_{1080}	X_{28431}
12N	$2^4.3^3$	6R	4D	4	0	0
12O	$2^4.3^3$	6S	4D	4	0	3
12P	$2^6.3$	6M	4E	0	4	0
12Q	$2^2.3^3$	6T	4D	1	3	3
12R	$2^2.3^3$	6U	4D	1	3	0
12S	$2^2.3^3$	6V	4D	1	0	0
12T	$2^2.3^3$	6W	4D	1	0	0
18A	$2^2.3^3$	9A	6A	1	5	0
18B	$2^2.3^3$	9B	6A	1	2	0
18C	$2^2.3^3$	9C	6B	0	0	0
18D	$2^2.3^3$	9D	6B	0	0	0
18E	$2^2.3^3$	9E	6C	0	0	0
18F	$2^2.3^3$	9F	6C	0	0	0
5A	$2^3.3^2.5^2$	5A	5A	10	15	6
5B	$2^3.3^2.5^2$	5B	5B	0	0	6
5C	$2^3.3^2.5^2$	5C	5C	0	0	6
10A	$2^3.5$	5A	10A	2	3	2
10B	$2^3.5$	5B	10B	0	0	2
10C	$2^3.5$	5C	10C	0	0	2
20A	$2^2.5$	10A	20A	2	1	0
20B	$2^2.5$	10B	20B	0	0	0
20C	$2^2.5$	10C	20C	0	0	0
15A	$3^2.5$	15A	5A	1	0	0
15B	$3^2.5$	15B	5A	1	3	3
15C	$3^2.5$	15C	5B	0	0	0
15D	$3^2.5$	15D	5B	0	0	3
15E	$3^2.5$	15E	5C	0	0	0

Table 1 - Page 5

Element	Centralizer	Square	Cube	x_{1120}	x_{1080}	x_{28431}
15F	$3^2.5$	15F	5C	0	0	3
7A	$2^2.7$	7A	7A	0	2	4
14A	$2^2.7$	7A	14A	0	2	0
14B	$2^2.7$	7A	14B	0	0	0
14C	$2^2.7$	7A	14C	0	0	0
13A	13	13B	13A	2	1	0
13B	13	13A	13B	2	1	0

9E
 9F
 9G
 9H
 9I
 9J
 9K
 9L
 9M
 9N
 9O
 9P
 9Q
 9R
 9S
 9T
 9U
 9V
 9W
 9X
 9Y
 9Z

12L
12K
12J
12I
12H
12G
12F
12E
12D
12C
12B
12A
6AB
6AA
6Z
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