

calculate the values of the characters of degree 1, 300, 260, 819 on most of the conjugacy classes and the remaining values are uniquely determined on all conjugacy classes of  $PS\Omega^+(8, 3)$ .

**4. The Character Table.** Applying the automorphism group to the irreducible characters determined above yields 11 irreducible characters. A large number of generalized characters can be determined by forming tensor products and symmetric and alternating products of the known irreducible characters. Other generalized characters can be determined by inducing characters from subgroups isomorphic to  $PS\Omega(7, 3)$  and  $PS\Omega^+(8, 2)$ . The character table of  $PS\Omega^+(8, 2)$  can be found in Dye [1]. The permutation representation on the cosets of a subgroup isomorphic to  $PS\Omega^+(8, 2)$  is given as the final column of Table 1.  $PS\Omega^+(8, 3)$  contains at least 4 conjugacy classes of subgroups isomorphic to  $PS\Omega^+(8, 2)$ . The above generalized characters are sufficient to determine the entire rational character table; see Table 2 on the microfiche supplement. Only one rational character is not absolutely irreducible, the last of the 113 which is the sum of 2 absolutely irreducible characters.

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1. R. H. DYE, "The simple group  $FH(8, 2)$  of order  $2^{12} \cdot 3^5 \cdot 5^2 \cdot 7$  and the associated geometry of triality," *Proc. London Math. Soc.* (3), v. 18, 1968, pp. 521-562. MR 37 #1468.

2. B. FISCHER, "Finite groups generated by 3-transpositions. I," *Invent. Math.*, v. 13, 1971, pp. 232-246. MR 45 #3557.

3. D. C. HUNT, "Character tables of certain finite simple groups," *Bull. Austral. Math. Soc.*, v. 5, 1971, pp. 1-42. MR 46 #1896.

4. G. E. WALL, "On the conjugacy classes in the unitary, symplectic and orthogonal groups," *J. Austral. Math. Soc.*, v. 3, 1963, pp. 1-62. MR 27 #212.

## The Character Table of Fischer's Simple Group, $M(23)$

By David C. Hunt

**Abstract.** This paper describes the calculation of the character table of  $M(23)$ , the sporadic simple group discovered by B. Fischer [1].

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1. **Introduction.**  $M(23)$  is a group of order  $2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23 = 4,089,470,473,293,004,800$ .  $M(23)$  is the second of three simple groups discovered by Fischer whilst characterising groups generated by a conjugacy class of 3-transpositions [1].  $M(23)$  contains two conjugacy classes of subgroups of relatively small index.  $M^*$  is of index 31671 in  $M(23)$  and  $M^*$  factored by its centre of order 2 is isomorphic to  $M(22)$ , the smallest of Fischer's three groups.  $S$  is of index 137632 and  $S$  contains a subgroup  $P$  of index 6 in  $S$  with  $P$  isomorphic to the 8-dimensional orthogonal simple group  $PS\Omega^+(8, 3)$ . The character tables of  $M(22)$  and  $PS\Omega^+(8, 3)$  appear elsewhere (Hunt [2], [3]).

2. **Conjugacy Classes of  $M(23)$ .** Table 1 lists the 98 conjugacy classes of  $M(23)$  by number, name and order of centralizer. The table also gives the values of the permutation characters of degree 31671 and 137632 on each conjugacy class and also the conjugacy number of the square and the cube of each element in the group. The restriction of both permutation representations to the subgroups  $M^*$  and  $S$  can be found and hence all conjugacy classes in  $M(23)$  with representatives from  $M^*$  and  $S$  are determined. Other conjugacy classes are determined by the structure of the Sylow normalizers for large prime divisors of the group order. This determines all conjugacy classes up to a few alternatives which can be decided during the calculation of the characters.

3. **The Character Table.** The two permutation representations of degree 31671 and 137632 are both rank three and yield irreducible characters of degree 1, 782, 30888 and 106743. The values of these characters on almost all conjugacy classes can be found by restricting to the subgroups  $M^*$  and  $PS\Omega^+(8, 3)$ . The values on the remaining conjugacy classes are uniquely determined. A large number of characters can now be generated by forming tensor, alternating and symmetric products of known characters and by inducing characters from  $M^*$  and  $P$ . It is possible to determine all the irreducible characters from these generalized characters by forming linear combinations of them and also by restricting small linear combinations of irreducibles to the known subgroups and splitting them into irreducibles for the subgroups.

The table of conjugacy classes and the complete character table appear as Table 1 and Table 2 in the microfiche supplement.

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1. B. FISCHER, "Finite groups generated by 3-transpositions. I," *Invent. Math.*, v. 13, 1971, pp. 232-246. MR 45 #3557.

2. D. C. HUNT, "Character tables of certain finite simple groups," *Bull. Austral. Math. Soc.*, v. 5, 1971, pp. 1-42. MR 46 #1896.

3. D. C. HUNT, "The character table of an eight-dimensional orthogonal group," *Math. Comp.*, v. 28, 1974, pp. 659-660.

THE CHARACTER TABLE OF FISCHER'S SIMPLE GROUP,  $M(23)$

BY DAVID C. HUNT

Table 1 - Page 1

	Centralizer	Element	Square	Cube	$\chi_{31671}$	$\chi_{137632}$
1	$2^{18}.3^{13}.5^2.7.11.13.17.23$	1	1	1	31671	137632
2	$2^{18}.3^9.5^2.7.11.13$	2A	1	2	3511	14080
3	$2^{18}.3^6.5.7.11$	2B	1	3	695	1408
4	$2^{18}.3^5.5$	2C	1	4	183	416
5	23	23A	5	5	0	0
6	23	23B	6	6	0	0
7	17	17	7	7	0	0
8	2.3.13	13A	8	8	3	1
9	2.3.13	13B	9	9	3	1
10	2.13	26A	8	10	1	1
11	2.13	26B	9	11	1	1
12	3.13	39A	12	8	0	1
13	3.13	39B	13	9	0	1
14	$2^2.11$	11A	14	14	2	0
15	$2^2.11$	22A	14	15	2	0
16	$2^2.11$	22B	14	16	2	0
17	$2^2.11$	22C	14	17	2	0
18	$2^3.3.5.7$	7	18	18	10	5
19	$2^2.3.7.$	14A	18	19	4	3
20	$2^3.7$	14B	18	20	2	1
21	2.3.7	21	21	18	1	2
22	2.3.7	42	21	19	1	0
23	$2^2.7$	28	20	23	0	1
24	5.7	35	24	24	0	0
25	$2^4.3^2.5^2.7$	5	25	25	21	7
26	$2^4.3.5^2$	10A	25	26	11	5
27	$2^4.3.5$	10B	25	27	5	3

Table 1 - Page 2

Centralizer	Element	Square	Cube	$x_{31671}$	$x_{137632}$
28 $2^4.3.5^5$	10C	25	28	3	1
29 $2^3.3^2.5$	15A	29	25	6	4
30 $2.3^2.5$	15B	30	25	0	1
31 $2^3.3.5$	20A	27	31	3	3
32 $2^3.5$	20B	27	32	1	1
33 $2^2.3.5$	30A	29	26	2	2
34 $2^3.3.5$	30B	29	27	2	0
35 $2.3.5$	30C	30	28	0	1
36 $2^2.3.5$	60	34	31	0	0
37 $2^9.3^{10}.5.7.13$	3A	37	1	351	1444
38 $2^9.3^7.5.7$	6A	37	2	127	112
39 $2^9.3^5.5$	6B	37	3	47	40
40 $2^9.3^4$	6C	37	4	15	44
41 $2^6.3^3.5$	12A	39	90	15	10
42 $2^7.3^3$	12B	40	92	3	16
43 $2^7.3^2$	12C	40	93	3	4
44 $2^6.3^2$	12D	39	91	7	6
45 $2^4.3$	24A	43	94	1	2
46 $2^4.3$	24B	42	95	1	4
47 $2^{10}.3^{13}$	3B	47	1	324	580
48 $2^8.3^9$	6D	47	2	28	148
49 $2^9.3^6$	6E	47	3	20	4
50 $2^8.3^5$	6F	47	4	12	20
51 $2^{10}.3^5$	6G	47	4	36	68
52 $2^5.3^4$	12E	50	92	12	4
53 $2^7.3^3$	12F	51	92	12	4
54 $2^5.3^2$	12G	50	93	4	4

Table 1 - Page 3

Centralizer	Element	Square	Cube	$X_{31671}$	$X_{137632}$
55 $2^7.3^2$	12H	51	93	4	4
56 $2^5.3^4$	12I	49	90	0	4
57 $2^4.3$	24C	55	96	0	0
58 $2^3.3^6$	9A	58	47	18	13
59 $2^2.3^4$	18A	58	48	4	7
60 $2^3.3^3$	18B	58	49	2	1
61 $2^2.3^2$	36A	60	56	0	1
62 $2^3.3^6$	9B	62	47	9	13
63 $2^3.3^4$	18C	62	48	7	1
64 $2^3.3^3$	18D	62	49	5	1
65 $2^3.3^3$	18E	62	50	3	5
66 $2^3.3^7$	9C	66	47	0	13
67 $2^3.3^3$	18F	66	50	0	5
68 $2^2.3^2$	36B	67	52	0	1
69 $2.3^6$	9D	69	47	0	4
70 $2.3^6$	18G	69	50	0	2
71 $3^3$	27	71	66	0	1
72 $2^4.3^{10}$	3C	72	1	27	67
73 $2^4.3^7$	6H	72	2	1	13
74 $2^4.3^5$	6I	72	4	9	5
75 $2^4.3^4$	6J	72	4	3	11
76 $2^3.3^3$	12J	75	92	3	7
77 $2^3.3^2$	12K	75	93	1	1
78 $2.3^4$	9E	78	72	3	1
79 $2.3^4$	18H	78	73	1	1
80 $2^7.3^{10}.5$	3D	80	1	135	121
81 $2^7.3^7$	6K	80	2	37	49

Table 1 - Page 4

	Centralizer	Element	Square	Cube	X <sub>31671</sub>	X <sub>137632</sub>
82	$2^6.3^5$	6L	80	3	11	13
83	$2^6.3^4$	6M	80	4	9	5
84	$2^7.3^4$	6N	80	4	15	17
85	$2^7.3^5.5$	6O	80	4	45	41
86	$2^5.3^3$	12L	84	92	3	1
87	$2^5.3^2$	12M	82	91	1	1
88	$2^4.3^3$	12N	82	90	3	7
89	$2^4.3^2$	12O	82	91	1	3
90	$2^{11}.3^4.5.7$	4A	3	90	63	148
91	$2^{11}.3^2.5$	4B	3	91	31	36
92	$2^{12}.3^4$	4C	4	92	39	40
93	$2^{12}.3^2$	4D	4	93	7	16
94	$2^7.3$	8A	92	94	7	8
95	$2^7.3$	8B	92	95	7	4
96	$2^7.3$	8C	93	96	3	0
97	$2^5$	16A	96	97	1	0
98	$2^5$	16B	96	98	1	0

TABLE 2, PAGE NO. 1

1	2A	7B	2C	23A	23B	17	17A	13B	26A	76P
1	1702	178	144	1000	1000	1010	1200	1200	1000	1000
2	588	195	37	1000	1000	1010	1200	1200	1000	1000
3	508	462	168	1000	1000	1010	1200	1200	1000	1000
4	308	616	246	1000	1000	1010	1200	1200	1000	1000
5	609	191	46	1000	1000	1010	1200	1200	1000	1000
6	101	446	303	1000	1000	1010	1200	1200	1000	1000
7	179	738	435	1000	1000	1010	1200	1200	1000	1000
8	148	1057	345	1000	1000	1010	1200	1200	1000	1000
9	182	1500	800	1000	1000	1010	1200	1200	1000	1000
10	588	1700	350	1000	1000	1010	1200	1200	1000	1000
11	709	170	178	1000	1000	1010	1200	1200	1000	1000
12	308	432	120	1000	1000	1010	1200	1200	1000	1000
13	609	136	108	1000	1000	1010	1200	1200	1000	1000
14	101	708	108	1000	1000	1010	1200	1200	1000	1000
15	179	1034	102	1000	1000	1010	1200	1200	1000	1000
16	148	1276	176	1000	1000	1010	1200	1200	1000	1000
17	182	1522	168	1000	1000	1010	1200	1200	1000	1000
18	588	1700	190	1000	1000	1010	1200	1200	1000	1000
19	709	1700	272	1000	1000	1010	1200	1200	1000	1000
20	308	432	192	1000	1000	1010	1200	1200	1000	1000
21	609	136	162	1000	1000	1010	1200	1200	1000	1000
22	101	708	145	1000	1000	1010	1200	1200	1000	1000
23	179	1057	152	1000	1000	1010	1200	1200	1000	1000
24	148	1276	160	1000	1000	1010	1200	1200	1000	1000
25	182	1522	243	1000	1000	1010	1200	1200	1000	1000
26	588	1700	166	1000	1000	1010	1200	1200	1000	1000
27	709	1700	348	1000	1000	1010	1200	1200	1000	1000
28	308	432	119	1000	1000	1010	1200	1200	1000	1000
29	609	136	210	1000	1000	1010	1200	1200	1000	1000
30	101	708	145	1000	1000	1010	1200	1200	1000	1000
31	179	1057	149	1000	1000	1010	1200	1200	1000	1000
32	148	1276	105	1000	1000	1010	1200	1200	1000	1000
33	182	1522	210	1000	1000	1010	1200	1200	1000	1000
34	588	1700	105	1000	1000	1010	1200	1200	1000	1000
35	709	1700	105	1000	1000	1010	1200	1200	1000	1000





39A 39B 11A 22A 22B 22C 7 14A 14B 21 42 2R 3S 5 17A 17B

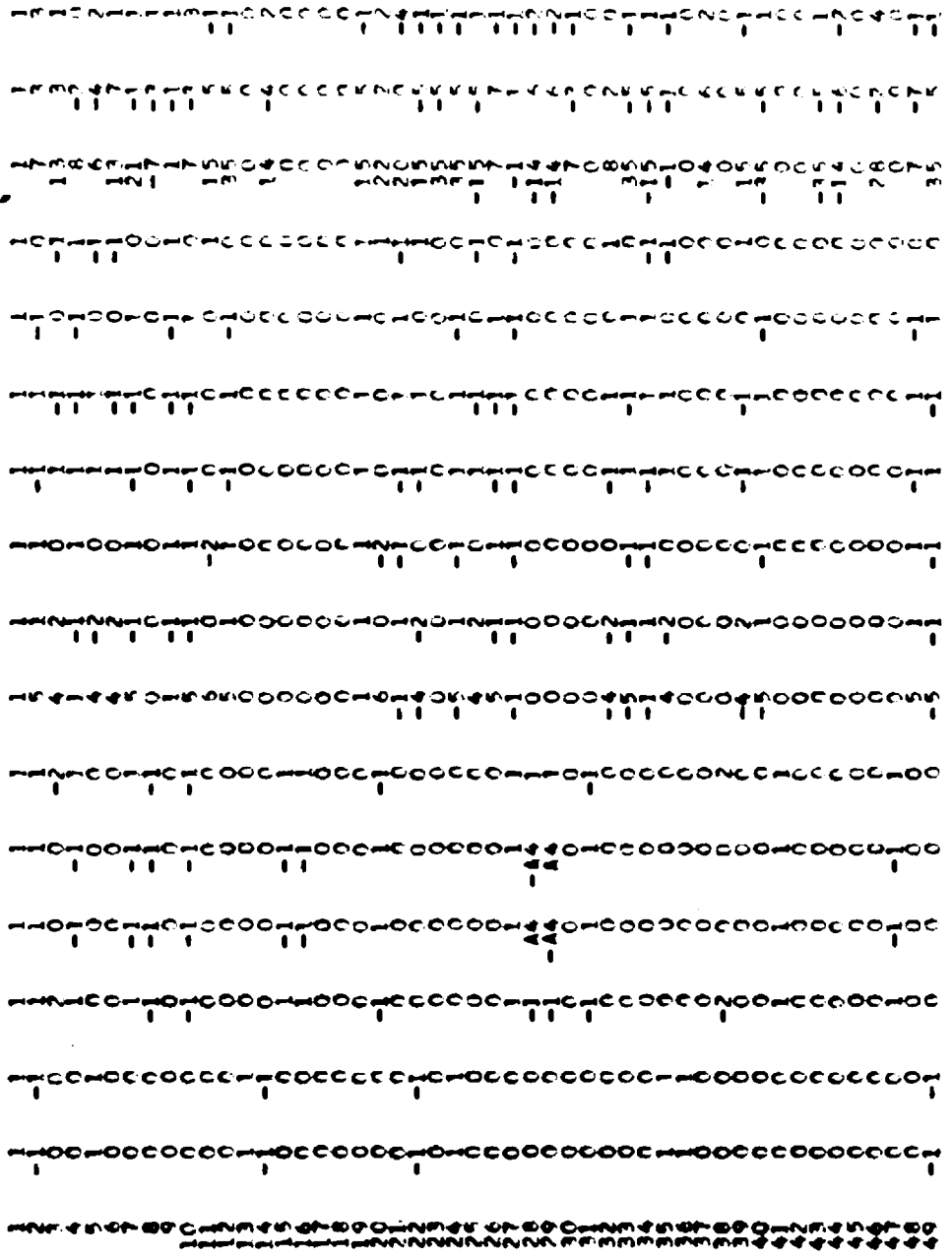






TABLE 7, PAGE NO. 6

10C	15A	15B	20A	20B	30A	30B	30C	60	3A	4A	6B	6C
55	55	55	55	55	55	55	55	55	1638	204	54	10
55	55	55	55	55	55	55	55	55	1636	276	54	24
55	55	55	55	55	55	55	55	55	1365	455	105	4
55	55	55	55	55	55	55	55	55	16948	220	4	27
55	55	55	55	55	55	55	55	55	29184	512	0	48
55	55	55	55	55	55	55	55	55	22750	910	0	0
55	55	55	55	55	55	55	55	55	22386	4	130	30
55	55	55	55	55	55	55	55	55	11648	0	128	0
55	55	55	55	55	55	55	55	55	7560	0	0	24
55	55	55	55	55	55	55	55	55	840	840	40	24
55	55	55	55	55	55	55	55	55	34	34	64	0
55	55	55	55	55	55	55	55	55	4125	384	64	0
55	55	55	55	55	55	55	55	55	4095	384	64	0
55	55	55	55	55	55	55	55	55	9477	1155	155	7
55	55	55	55	55	55	55	55	55	1682	1155	155	45
55	55	55	55	55	55	55	55	55	14914	720	81	27
55	55	55	55	55	55	55	55	55	17477	540	46	30
55	55	55	55	55	55	55	55	55	17647	840	0	0
55	55	55	55	55	55	55	55	55	10205	405	120	40
55	55	55	55	55	55	55	55	55	12285	1374	126	27
55	55	55	55	55	55	55	55	55	22464	1575	126	27
55	55	55	55	55	55	55	55	55	27347	384	64	0
55	55	55	55	55	55	55	55	55	4280	384	64	0
55	55	55	55	55	55	55	55	55	14683	456	107	8
55	55	55	55	55	55	55	55	55	5005	405	107	8
55	55	55	55	55	55	55	55	55	7280	840	140	27
55	55	55	55	55	55	55	55	55	15912	720	140	27
55	55	55	55	55	55	55	55	55	15808	1560	180	16
55	55	55	55	55	55	55	55	55	2460	1450	172	24
55	55	55	55	55	55	55	55	55	4268	1560	172	24
55	55	55	55	55	55	55	55	55	81742	1332	220	12
55	55	55	55	55	55	55	55	55	14742	1332	220	12
55	55	55	55	55	55	55	55	55	1983	1332	220	12
55	55	55	55	55	55	55	55	55	17920	1332	220	12
55	55	55	55	55	55	55	55	55	12285	1332	220	12
55	55	55	55	55	55	55	55	55	11020	1332	220	12
55	55	55	55	55	55	55	55	55	1134	1332	220	12



TABLE 2, PAGE NO. 8

	72A	72R	72C	72D	24A	24R	3B	6D	6E	6F	6G	72F	72G
01	06	2N	2N	00	00	00	02	10	54	10	15R	10	20
52	15	00	00	00	00	00	210	10	132	10	15R	10	20
53	00	00	00	00	00	00	216	10	57	10	15R	10	20
54	00	00	00	00	00	00	250	10	174	10	15R	10	20
55	00	00	00	00	00	00	252	10	00	10	15R	10	20
56	00	00	00	00	00	00	254	10	00	10	15R	10	20
57	00	00	00	00	00	00	1536	10	00	10	15R	10	20
58	00	00	00	00	00	00	222	10	00	10	15R	10	20
59	00	00	00	00	00	00	242	10	00	10	15R	10	20
60	00	00	00	00	00	00	178	10	00	10	15R	10	20
61	00	00	00	00	00	00	243	10	00	10	15R	10	20
62	00	00	00	00	00	00	268	10	00	10	15R	10	20
63	00	00	00	00	00	00	162	10	00	10	15R	10	20
64	00	00	00	00	00	00	162	10	00	10	15R	10	20
65	00	00	00	00	00	00	102	10	00	10	15R	10	20
66	00	00	00	00	00	00	102	10	00	10	15R	10	20
67	00	00	00	00	00	00	102	10	00	10	15R	10	20
68	00	00	00	00	00	00	102	10	00	10	15R	10	20
69	00	00	00	00	00	00	102	10	00	10	15R	10	20
70	00	00	00	00	00	00	102	10	00	10	15R	10	20
71	00	00	00	00	00	00	102	10	00	10	15R	10	20
72	00	00	00	00	00	00	102	10	00	10	15R	10	20
73	00	00	00	00	00	00	102	10	00	10	15R	10	20
74	00	00	00	00	00	00	102	10	00	10	15R	10	20
75	00	00	00	00	00	00	102	10	00	10	15R	10	20
76	00	00	00	00	00	00	102	10	00	10	15R	10	20
77	00	00	00	00	00	00	102	10	00	10	15R	10	20
78	00	00	00	00	00	00	102	10	00	10	15R	10	20
79	00	00	00	00	00	00	102	10	00	10	15R	10	20
80	00	00	00	00	00	00	102	10	00	10	15R	10	20
81	00	00	00	00	00	00	102	10	00	10	15R	10	20
82	00	00	00	00	00	00	102	10	00	10	15R	10	20
83	00	00	00	00	00	00	102	10	00	10	15R	10	20
84	00	00	00	00	00	00	102	10	00	10	15R	10	20
85	00	00	00	00	00	00	102	10	00	10	15R	10	20
86	00	00	00	00	00	00	102	10	00	10	15R	10	20
87	00	00	00	00	00	00	102	10	00	10	15R	10	20
88	00	00	00	00	00	00	102	10	00	10	15R	10	20
89	00	00	00	00	00	00	102	10	00	10	15R	10	20
90	00	00	00	00	00	00	102	10	00	10	15R	10	20
91	00	00	00	00	00	00	102	10	00	10	15R	10	20
92	00	00	00	00	00	00	102	10	00	10	15R	10	20
93	00	00	00	00	00	00	102	10	00	10	15R	10	20
94	00	00	00	00	00	00	102	10	00	10	15R	10	20
95	00	00	00	00	00	00	102	10	00	10	15R	10	20
96	00	00	00	00	00	00	102	10	00	10	15R	10	20
97	00	00	00	00	00	00	102	10	00	10	15R	10	20
98	00	00	00	00	00	00	102	10	00	10	15R	10	20
99	00	00	00	00	00	00	102	10	00	10	15R	10	20
00	00	00	00	00	00	00	102	10	00	10	15R	10	20







TABLE 2, PAGE NO. 11

27	#####
3C	#####
6H	#####
6I	#####
6J	#####
12J	#####
12K	#####
9E	#####
1AH	#####
3D	#####
6K	#####
6L	#####
6M	#####
6N	#####

TABLE 2, PAGE NO. 12

	27	3C	6H	6I	6J	12J	12K	9E	18H	3C	6K	6L	6M	6N
5		1	2	5	1	5	1	1	1	5	1	2	4	5
6		1	2	5	1	5	1	1	1	5	1	2	4	5
7		1	2	5	1	5	1	1	1	5	1	2	4	5
8		1	2	5	1	5	1	1	1	5	1	2	4	5
9		1	2	5	1	5	1	1	1	5	1	2	4	5
10		1	2	5	1	5	1	1	1	5	1	2	4	5
11		1	2	5	1	5	1	1	1	5	1	2	4	5
12		1	2	5	1	5	1	1	1	5	1	2	4	5
13		1	2	5	1	5	1	1	1	5	1	2	4	5
14		1	2	5	1	5	1	1	1	5	1	2	4	5
15		1	2	5	1	5	1	1	1	5	1	2	4	5
16		1	2	5	1	5	1	1	1	5	1	2	4	5
17		1	2	5	1	5	1	1	1	5	1	2	4	5
18		1	2	5	1	5	1	1	1	5	1	2	4	5
19		1	2	5	1	5	1	1	1	5	1	2	4	5
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