The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

16[2.05, 4, 5, 6].—MARTIN H. SCHULTZ, *Spline Analysis*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1973, xiii + 156 pp., 24 cm. Price $10.50.

This book is intended as a unified, elementary introduction to polynomial spline functions and their applications in several important areas of numerical analysis, including the finite element method for approximating the solutions of differential equations. Since it is written for readers with a background in calculus and linear algebra, the approach is to illustrate these techniques by rigorously treating only simple model problems. Results in more general cases are left to the exercises and to further study. For example, only piecewise linear and cubic polynomials are used, and all the differential equations considered are of second order, defined on an interval, or a square.

Terminology and some basic results are provided in Chapter I. Chapters II, III and IV are concerned with spline interpolation, piecewise linear, piecewise cubic Hermite, and cubic spline, respectively. The one-dimensional case is considered first, and the existence, uniqueness and minimum energy properties of such interpolations are given. Local bases are developed for the piecewise linear and cubic Hermite cases. Unfortunately, however, the *B*-spline local basis for cubic splines is not introduced until later in the text. The two-dimensional (bivariate) interpolation problem on the unit square is then considered using Cartesian products of one-dimensional spline functions. A thorough discussion of error estimates is given.

Chapter V describes the method of degenerate kernels for approximating the solution of a Fredholm integral equation of Type II by the solution of a linear system of equations. Here, the kernel is approximated by a bivariate piecewise polynomial.

Chapter VI deals with least-squares approximation in one and two dimensions by spline functions. It is pointed out that, if the mesh spacing is uniform, and a normalized set of local basis functions is used, then the matrices involved are symmetric, positive definite, banded, and, in addition, have condition numbers that are uniformly bounded as the mesh spacing tends to zero. This latter fact should give least squares approximation by spline functions added impetus.

Chapters VII and VIII describe the Rayleigh-Ritz-Galerkin methods for approximating the solutions of boundary-value problems and eigenvalue problems. Chapter X deals with a Ritz procedure for the state regulator problem in optimal control. In each of these settings, the problem is first posed equivalently as finding the minimum of a real-valued functional over an infinite-dimensional function space. An approximation to the solution is obtained as the minimum of the functional over an appropriate space of piecewise polynomials (the finite element method)—or, equivalently, as the solution of a linear system of equations, involving positive-definite symmetric, sparse and well-conditioned matrices. In each case, existence, uniqueness and error estimates are given.

Chapter IX describes the Galerkin method for approximating the solution of a parabolic partial differential equation by the solution of a linear system of ordinary differential equations. The usual Padé approximation methods for solving this system are discussed, although somewhat tersely.

The only real criticism of the text is that, although discussed throughout, the practical aspects of spline methods are still not given adequate emphasis, especially, considering the intended audience. For example, numerical results are for the most part described only to motivate error estimates, and only for analytic objective functions. The treatment of the cubic spline interpolation problem is via continuity conditions rather than the more practical and easily generalized B-spline technique. And, finally, one of the most directly practical aspects of spline functions, their best estimation properties, is not treated. Aside from these relatively minor considerations, the book is a welcome addition to the literature in a very important and practical area of numerical analysis. It is the most readily accessible book on spline functions written to date, and the only text to treat the finite element methods in a unified elementary fashion.

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17[2.10, 2.15, 2.20, 2.30, 2.35, 2.40, 2.55].—JOSEF STOER, *Einführung in die Numerische Mathematik*. I, Springer-Verlag, Berlin-Heidelberg-New York, 1972, ix + 250 pp., 21 cm. Price $4.70 paper bound.

The "Heidelberger Taschenbücher", of which this is volume 105, is a series of text books in mathematics and the physical sciences which among its authors includes some of the most distinguished names in the respective fields. The volume under review, the first of a two-volume introduction into numerical mathematics, continues in the same tradition of expository excellence. Written at about the beginning graduate level, it makes a serious attempt to treat in depth those numerical techniques which can readily be implemented on digital computers and which are proven to be useful and reliable for high-speed computation. Accordingly, essential parts of key algorithms are often given as short programs in ALGOL 60. In addition, and more importantly, a great deal of emphasis has been placed on questions of numerical stability. Concepts such as condition, algorithmic stability, and "good-naturedness" of algorithms are rightly considered by the author to belong to the very core of numerical mathematics.

True to this spirit, the volume opens with a chapter on error analysis, developing the basic facts of machine arithmetic, rounding errors, and error propagation. It is here where the central concept of "good-natured algorithm" (due to F. L. Bauer) is introduced. Basically, this is a computing algorithm in which the influence of all intermediate rounding errors on the final result is not greater than the unavoidable error due to rounded input data. Examples of algorithms which enjoy "good-naturedness," and others which lack it, are given in this, as well as in subsequent, chapters. Chapter 2 takes up the problem of interpolation. It begins with the usual