Chapter IX describes the Galerkin method for approximating the solution of a parabolic partial differential equation by the solution of a linear system of ordinary differential equations. The usual Padé approximation methods for solving this system are discussed, although somewhat tersely.

The only real criticism of the text is that, although discussed throughout, the practical aspects of spline methods are still not given adequate emphasis, especially, considering the intended audience. For example, numerical results are for the most part described only to motivate error estimates, and only for analytic objective functions. The treatment of the cubic spline interpolation problem is via continuity conditions rather than the more practical and easily generalized *B*-spline technique. And, finally, one of the most directly practical aspects of spline functions, their best estimation properties, is not treated. Aside from these relatively minor considerations, the book is a welcome addition to the literature in a very important and practical area of numerical analysis. It is the most readily accessible book on spline functions written to date, and the only text to treat the finite element methods in a unified elementary fashion.

JOHN PIERCE

Department of Mathematics
University of Southern California
Los Angeles, California 90007

17[2.10, 2.15, 2.20, 2.30, 2.35, 2.40, 2.55].—JOSEF STOER, *Einführung in die Numerische Mathematik*. I, Springer-Verlag, Berlin-Heidelberg-New York, 1972, ix + 250 pp., 21 cm. Price $4.70 paper bound.

The "Heidelberger Taschenbücher", of which this is volume 105, is a series of text books in mathematics and the physical sciences which among its authors includes some of the most distinguished names in the respective fields. The volume under review, the first of a two-volume introduction into numerical mathematics, continues in the same tradition of expository excellence. Written at about the beginning graduate level, it makes a serious attempt to treat in depth those numerical techniques which can readily be implemented on digital computers and which are proven to be useful and reliable for high-speed computation. Accordingly, essential parts of key algorithms are often given as short programs in ALGOL 60. In addition, and more importantly, a great deal of emphasis has been placed on questions of numerical stability. Concepts such as condition, algorithmic stability, and "good-naturedness" of algorithms are rightly considered by the author to belong to the very core of numerical mathematics.

True to this spirit, the volume opens with a chapter on error analysis, developing the basic facts of machine arithmetic, rounding errors, and error propagation. It is here where the central concept of "good-natured algorithm" (due to F. L. Bauer) is introduced. Basically, this is a computing algorithm in which the influence of all intermediate rounding errors on the final result is not greater than the unavoidable error due to rounded input data. Examples of algorithms which enjoy "good-naturedness," and others which lack it, are given in this, as well as in subsequent, chapters. Chapter 2 takes up the problem of interpolation. It begins with the usual facts on polynomial interpolation, and then proceeds with an excellent treatment of rational interpolation, including, in particular, the important Neville-type algorithms due to the author. This is followed by trigonometric interpolation of equidistant data, which in turn leads naturally to questions of computing discrete and continuous Fourier transforms. For the former, the author describes the algorithm of Goertzel, which unfortunately is not always stable, a "good-natured" variant due to Reinsch, and the very effective and popular algorithm of Cooley and Tukey (unfortunately without a discussion of its stability). For the latter, he presents the reviewer's theory of attenuation factors. The chapter ends with a short introduction to spline interpolation, covering the minimum curvature property of cubic spline interpolants, constructive methods, and convergence properties. Chapter 3 is a modern treatment of numerical integration. Formulas of the Newton-Cotes type, while briefly discussed, are clearly de-emphasized in favor of extrapolation algorithms. These, then, are treated with unusual care, not only for the integration problem, but also for other discretization algorithms (e.g., numerical differentiation). With the theory of Gaussian quadrature formulas the author then returns to more classical grounds, although here, too, he includes recent results (due to Golub and Welsch) concerning representation of Christoffel numbers. Surprisingly, no mention is made of the book by Stroud and Secrest [1], which is currently the standard reference work on the subject. There is also a brief discussion on how to deal with singularities. Chapter 4 is devoted to the solution of systems of linear algebraic equations by direct methods: Gauss elimination, triangular decomposition, Gauss-Jordan algorithm, and the Cholesky decomposition. (Iterative methods are to be discussed in the second volume.) There follows an introduction into the theory of norms, in preparation for discussing the condition of linear systems and a-posteriori error estimates. One finds an interesting result of Prager and Oettli which permits one to decide whether or not a solution is acceptable, given the size of the residual. A detailed analysis of rounding errors in Gauss elimination and in the triangular decomposition method then follows, and it is shown that Gauss elimination is indeed a "good-natured" algorithm if the triangular factors are not too large (in a specified sense). The author then turns to orthogonal reduction methods, in particular, to the reduction of a matrix to triangular form by a sequence of Householder transformations. He also discusses the related Gram-Schmidt orthogonalization and its numerical properties, being careful to point out the possible pitfalls in the absence of "reorthogonalizations." As a natural application, least squares approximation to overdetermined linear systems is considered, for which the basic theory is given and the elegant numerical solution by means of Householder reductions. There is also an interesting discussion of the condition of this problem, as well as a short outlook on nonlinear least squares approximation. Chapter 5 is devoted to nonlinear equations and systems thereof. The discussion first centers around iteration in finite-dimensional Euclidean space. The principle of contraction mapping is developed and a convergence theorem for the Newton-Raphson-Kantorovich method. A useful modification of Newton's method, involving ideas of steepest descent, is also developed and shown to converge globally under appropriate hypotheses. The remaining parts of the chapter concentrate on algebraic equations, presenting various versions of Newton's method, an interesting "double-step" Newton's method for polynomials with only real zeros, Maehly's ingenious device of successive deflation, the use of Sturm sequences in combination with successive bisection and Bairstow's method.

Questions of the condition of roots are also briefly considered. The chapter concludes with some interpolatory methods: the regula falsi, secant method, Muller's method, and with their convergence properties, and, finally, with convergence-accelerating processes such as those of Aitken and Steffensen.

Each chapter is followed by a good set of exercises and by a list of selected references. The text makes occasional references to the items in these bibliographies, but additional notes on some of the sources would have been helpful. The index at the end of the book, on the whole, seems adequate, but the term "good-natured algorithm" is conspicuously missing.

Unfortunately, the book is not error-free; in fact, there are quite a few of them. However, most of the errors are of a trivial nature and can easily be rectified by an alert reader.

From the above outline of content, it should be apparent that we have here before us a text which is thoroughly up-to-date, original both in the selection of topics and in their mathematical treatment, a book, in short, which contains the essence of a good many years of experience in computing. Adding to this the extreme clarity and conciseness of exposition makes this indeed one of the outstanding introductory texts in numerical analysis. It is only to be hoped that an English translation will be available in the not too distant future.

W. G.

1. A. H. STROUD & D. SECREST, *Gaussian Quadrature Formulas*, Prentice-Hall, Englewood Cliffs, N. J., 1964.

18[2.10].—JAMES L. PHILLIPS & RICHARD J. HANSON, *Gauss Quadrature Rules with B-Spline Weight Functions*, 28 pages of tables and 4 pages of explanatory text, reproduced on the microfiche card attached to this issue.

The abscissas and weights of n-point Gaussian quadrature rules for integrals

$$\int_{-1}^{1} N(i, k; t)y(t)\, dt$$

are tabulated to 14S for $n = 1(1)17$, $k = 2, 4$, $i = 1(1)k$. Here $N(i, k; t)$ is a normalized B-spline of order k (degree $k - 1$) with support on $(-1, 1)$. Translates and reflections of the k B-splines $N(1, k; t), \cdots, N(k, k; t)$ provide a basis for the space of splines of order k defined on an interval $[a, b]$ with respect to a partition of equally spaced interior knots and end knots of multiplicity k.

The first 17 coefficients in the three term recurrence formula for polynomials orthonormal on $(-1, 1)$ with respect to the weight function $N(i, k; t)$ are given to 14S for the same values of i and k.

Details of the underlying calculations on an IBM 360/67 at Washington State University are also furnished.

JAMES L. PHILLIPS
RICHARD J. HANSON

Computer Science Department
Washington State University
Pullman, Washington 99163