

## REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

**29** [2.05, 2.05.6, 6].—P. L. BUTZER, J.-P. KAHANE & B. SZ.-NAGY, Editors, *Linear Operators and Approximation*, ISNM vol. 20, Birkhäuser Verlag, Basel, Switzerland, 1972, 506 pp., 25 cm. Price sfr.84.—.

This book contains the lectures presented at the conference on *Linear Operators and Approximation* held at the Oberwolfach Mathematical Research Institute, August 14–22, 1971. Four papers are included in addition to the 38 lectures presented. The fly leaf states that the book's goal is to "elucidate the actual state of research in the vast field of approximation and related topics", but the book's title better indicates the main thrust of the book. Thus, one does obtain a good sampling of current (in 1971) research in the area described by the key words: approximation theory, functional analysis and operator theory.

The number of papers and their length preclude individual mention of the papers. The average is at the high quality that one expects from an Oberwolfach conference. The papers are divided into five sections as follows:

1. Operator Theory (9 papers)
2. Topics in Functional Analysis (8 papers)
3. Approximation in Abstract Spaces (9 papers)
4. Harmonic Analysis and Approximation (7 papers)
5. Spline and Algebraic Approximation (9 papers).

There is a large and interesting set of new and unsolved problems which were collected during the conference.

The quality of production is high with few misprints although there is some puzzling use of small type on pages 475–477.

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**30** [2.05, 2.10, 2.15, 2.25, 2.30, 2.35, 4, 13.05, 13.15, 13.35].—P. R. GRAVES-MORRIS, Editor, *Padé Approximants and their Applications*, Proceedings of a Conference held at the University of Kent, Canterbury, England, July 17–21, 1972, Academic Press, London and New York, 1973, x + 354 pp., 23 cm. Price £5.50.

Padé approximants are a two-dimensional array of rational functions which are associated with a given power series by requiring that their Maclaurin expansions agree with the given series to as many terms as possible. Approximations of this type are encountered in many branches of classical analysis: the theory of meromorphic functions, continued fractions, the moment problem, operator theory, etc. The subject, for a long time exclusive hunting-ground for analysts, has recently attracted the attention not only of numerical analysts, but also of research workers in applied mathematics and the physical sciences. To numerical analysts the theory of Padé approximation has become of interest primarily as a source of useful approximation techniques. Beyond this, the theory has important bearings on nonlinear convergence acceleration procedures, e.g., the epsilon algorithm, and on stability theory for discretization methods in differential equations (a topic not covered in these proceedings). For theoretical physicists and chemists, Padé methods provide a means of extracting maximum information from the series expansions which they encounter. In statistical mechanics, e.g., they are used to predict the singular behavior of thermodynamic functions. Given this new activity in an old field, inevitably, attempts are being made at generalizing the basic idea of Padé approximation. Some of these consider series expansions other than power series (e.g., series in Legendre polynomials), others go beyond the boundaries of rational approximation, seeking approximants in the form of solutions of quadratic (or higher-degree) equations with polynomial coefficients. Still others use the orthogonality concept as their point of departure.

In the summer of 1972, an international symposium on Padé approximants was held at the University of Kent in England, with the purpose of bringing together people representing all these diverse points of view. The symposium was divided into a "School" and a "Conference". The "School" involved a number of introductory tutorial lectures, which are to be published separately by the Institute of Physics. The "Conference", on the other hand, consisted of a series of technical papers describing recent and current progress in the field. The volume under review is the proceedings of this conference. The contributions have largely the character of brief expository reviews. They are grouped into five sections, the first two containing papers on mathematical and numerical aspects, the remaining three on applications to theoretical physics and simulation and control theory. Unfortunately, no index of any kind is provided.

The individual authors and their titles are as follows.

#### I. MATHEMATICAL PROPERTIES OF PADÉ APPROXIMANTS

- J. L. Gammel, Review of two recent generalizations of the Padé approximant
- J. S. R. Chisholm, Convergence properties of Padé approximants
- W. J. Thron, Recent approaches to convergence theory of continued fractions
- J. Nuttall, Variational principles and Padé approximants
- D. Masson, Padé approximants and Hilbert spaces

- R. C. Johnson, Alternative approach to Padé approximants  
 J. Fleischer, Nonlinear Padé approximants for Legendre series  
 F. V. Atkinson, Orthogonal polynomials and lacunary approximants

## II. NUMERICAL ANALYSIS AND NUMERICAL METHODS

- George A. Baker, Jr., Recursive calculation of Padé approximants  
 P. J. S. Watson, Algorithms for differentiation and integration  
 Jacek Gilewicz, Numerical detection of the best Padé approximant and determination of the Fourier coefficients of the insufficiently sampled functions  
 A. C. Genz, Applications of the  $\epsilon$ -algorithm to quadrature problems  
 W. B. Gragg, On Hadamard's theory of polar singularities  
 William B. Jones, Truncation error bounds for continued fractions and Padé approximants  
 I. M. Longman, Use of Padé table for approximate Laplace transform inversion  
 A. Ronveaux, Padé approximant and homographic transformation of Riccati's phase equations  
 V. Zakian, Properties of  $I_{MN}$  approximants

## III. CRITICAL PHENOMENA AND PADÉ APPROXIMANTS

- George A. Baker, Jr., Generalised Padé approximant bounds for critical phenomena  
 Michael E. Fisher, Critical phenomena—series expansions and their analysis  
 G. S. Joyce and A. J. Guttmann, A new method of series analysis  
 C. Isenberg, A comparison of the vibrational properties of H.C.P. and F.C.C. crystals  
 F. Harbus and H. E. Stanley, Ising model antiferromagnets with tricritical points  
 F. Harbus, R. Krasnow, D. Lambeth, L. Liu and H. E. Stanley, Ising, planar and Heisenberg models with directional anisotropy  
 Sava Milosevic, Calculation of the equation of state near the critical point for the Heisenberg model using Padé approximants

## IV. ATOMIC, NUCLEAR AND PARTICLE PHYSICS AND PADÉ APPROXIMANTS

- A. K. Common, Applications of the moment problem  
 C. López and F. J. Ynduráin, The moment problem and stable extrapolations with an application to forward  $Kp$  dispersion relations  
 J. A. Tjon, Application of Padé approximants in the three-body problem  
 C. R. Garibotti, Padé approximants in potential scattering  
 P. R. Graves-Morris and J. F. Rennison, Padé approximants and the Lippmann Schwinger equation  
 D. Bessis, Padé approximants in quantum field theory  
 M. Pusterla, Model field theories and Padé approximants  
 G. Turchetti, Padé approximants in nucleon-nucleon dynamics

## V. SIMULATION AND CONTROL

M. I. Sobhy, Applications of Padé approximants in electrical network problems

J. B. Knowles, A. B. Keats and D. W. Leggett, The simulation of a continuously variable transport delay

Y. Shamash, Approximation of linear time-invariant systems

S. C. Chuang, Frequency domain approximation technique for optimal control

H. P. Debart, A Padé Chebyshev approximation in network theory

The reader interested in the applications of Padé approximation to theoretical physics may also wish to consult [1], which contains another recent cross-section of work in this field.

W. G.

1. G. A. BAKER, JR. & J. L. GAMMEL, Editors, *The Padé Approximant in Theoretical Physics*, Academic Press, New York and London, 1970.

31 [2.05, 2.35, 3.25, 13.35]. — L. COLLATZ & W. WETTERLING, Editors, *Numerische Methoden bei Optimierungsaufgaben* (in German and English), International Series of Numerical Mathematics, vol. 17, Birkhäuser Verlag, Basel, 1973, 136 pp., 25 cm. Price: approximately \$10.—.

Proceedings of a conference on Numerical Methods for Optimization Problems which was held November 14–20, 1971, in Oberwolfach, Germany. In addition to the papers appearing in the book, the following are given in the foreword as the most important points raised in discussions among the participants:

1. Many of the familiar methods used in optimization problems, being frequently the developments of those not oriented toward numerical analysis or computation, must be more closely examined than they have been in the past for their numerical fitness, and improved, if necessary.

2. In iterative methods, the determination of an initial approximation is frequently more difficult than the execution of the method itself. This should be taken into consideration when numerical algorithms are developed.

3. Several new (asymptotic) methods for integer programming have appeared. Still, the standard difficulty remains that their computational complexity is not constrained by a bound depending only upon the dimension of the problem.

4. New numerical methods which have been developed at universities are frequently unusable for the practitioner because the originators have not tested their methods sufficiently on applications problems and, consequently, cannot give adequate directions for their employment. This deficiency has to be remedied.